Computer algebra independent integration tests

Summer 2022 edition

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/166-6.2.2-e-x-^m-a+b-x^n-^p-cosh

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [111]. This is test number [166].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (111)	0.00 (0)
Mathematica	100.00 (111)	0.00 (0)
Fricas	100.00 (111)	0.00 (0)
Maple	100.00 (111)	0.00 (0)
Giac	63.96 (71)	36.04 (40)
Maxima	57.66 (64)	42.34 (47)
Sympy	23.42 (26)	76.58 (85)
Mupad	18.02 (20)	81.98 (91)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

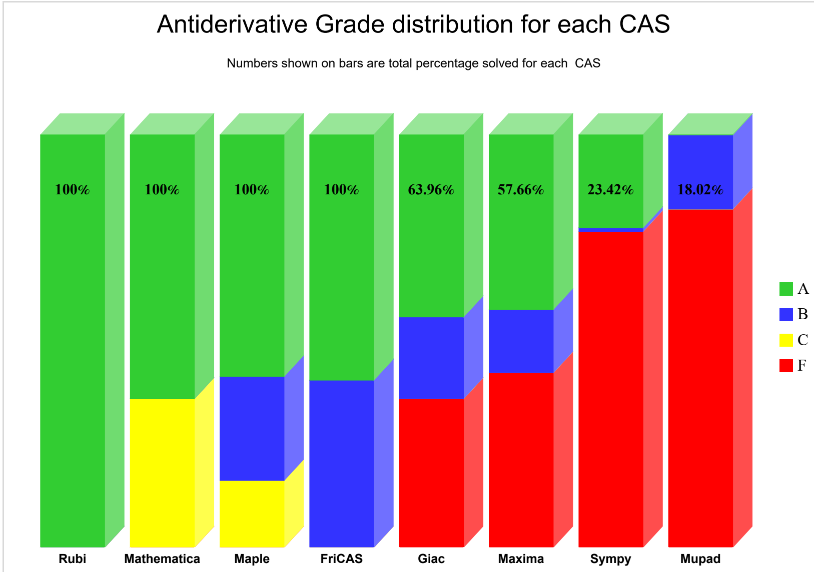
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

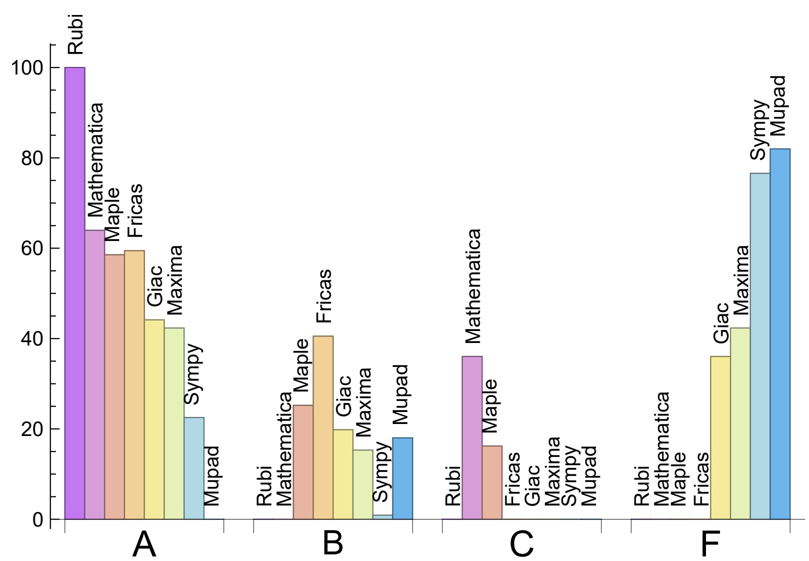
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	63.96	0.00	36.04	0.00
Fricas	59.46	40.54	0.00	0.00
Maple	58.56	25.23	16.22	0.00
Giac	44.14	19.82	0.00	36.04
Maxima	42.34	15.32	0.00	42.34
Sympy	22.52	0.90	0.00	76.58
Mupad	N/A	18.02	0.00	81.98

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	0	0.00 %	0.00 %	0.00 %
Giac	40	92.50 %	0.00 %	7.50 %
Maxima	47	87.23 %	12.77 %	0.00 %
Sympy	85	67.06 %	32.94 %	0.00 %
Mupad	91	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

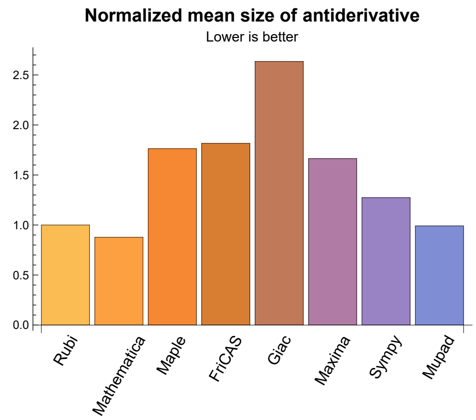
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

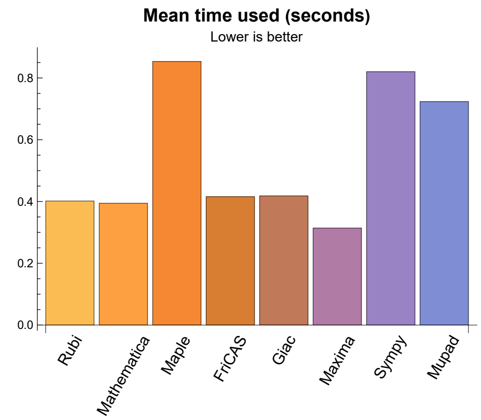
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.40	268.55	1.00	178.00	1.00
Mathematica	0.39	233.19	0.88	140.00	0.86
Maple	0.85	412.83	1.76	288.00	1.69
Maxima	0.31	183.64	1.66	170.00	1.71
Fricas	0.42	652.41	1.82	200.00	1.68
Sympy	0.82	138.96	1.27	127.50	1.23
Giac	0.42	398.31	2.63	199.00	1.66
Mupad	0.72	118.50	0.99	115.00	0.97

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0.0
0.5
1.0
1.5
2.0
2.5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0.0
0.2
0.4
0.6
0.8
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

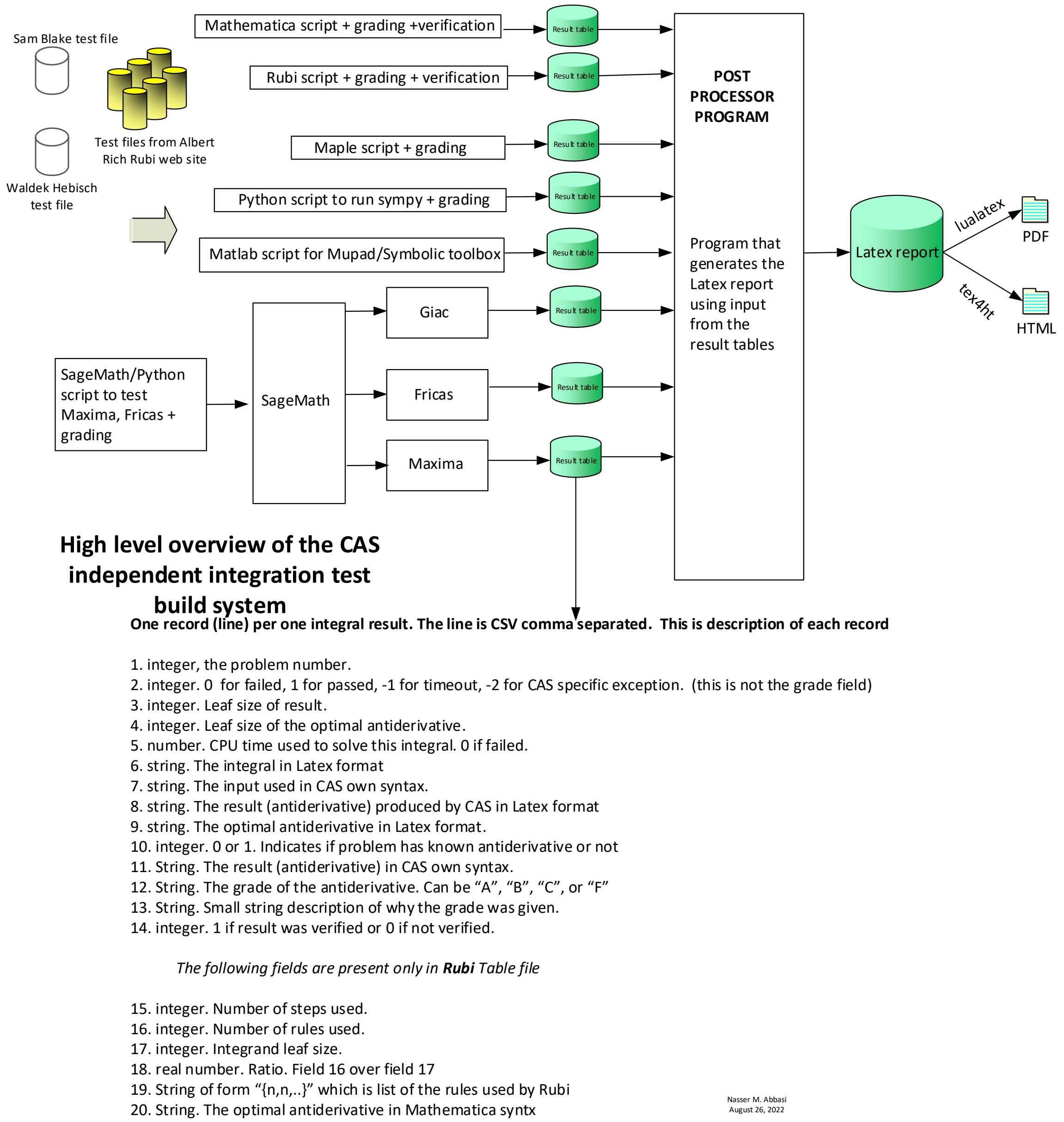
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { }

C grade: { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F grade: { }

2.1.3 Maple

A grade: { 3, 4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 37, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 83, 84, 85, 86, 91, 92, 93 }

B grade: { 1, 2, 10, 11, 12, 33, 34, 35, 36, 38, 39, 40, 41, 42, 49, 50, 51, 52, 72, 78, 79, 80, 81, 82, 87, 88, 89, 90 }

C grade: { 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 22, 24, 25, 26, 27, 28, 29, 30, 31, 36, 40, 41, 43, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 79, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { 2, 3, 4, 5, 11, 12, 13, 19, 20, 21, 23, 42, 44, 52, 80, 81, 83 }

C grade: { }

F grade: { 32, 33, 34, 35, 37, 38, 39, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 34, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 60, 62, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { 30, 33, 35, 36, 37, 38, 39, 57, 58, 59, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

C grade: { }

F grade: { }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 10, 11, 13, 40, 41, 42, 43, 44, 49, 50, 51, 52, 79, 80, 81, 82, 83, 87, 88, 89 }

B grade: { 12 }

C grade: { }

F grade: { 6, 7, 8, 9, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 79, 80, 81, 82, 83, 85, 86, 87, 88, 92 }

B grade: { 12, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 52, 53, 84, 89, 90, 91, 93 }

C grade: { }

F grade: { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 10, 11, 12, 40, 41, 42, 43, 49, 50, 51, 79, 80, 81, 82, 87, 88 }

C grade: { }

F grade: { 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	82	356	232	85	151	152	122
N.S.	1	1.00	0.66	2.87	1.87	0.69	1.22	1.23	0.98
time (sec)	N/A	0.253	0.115	0.680	0.272	0.407	0.332	0.410	0.958

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	65	224	196	67	117	116	92
N.S.	1	1.00	0.69	2.38	2.09	0.71	1.24	1.23	0.98
time (sec)	N/A	0.173	0.098	0.653	0.268	0.369	0.218	0.421	0.962

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	45	122	160	48	82	79	62
N.S.	1	1.00	0.70	1.91	2.50	0.75	1.28	1.23	0.97
time (sec)	N/A	0.097	0.074	0.750	0.281	0.419	0.139	0.443	0.922

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	53	68	30	46	46	35
N.S.	1	1.00	0.96	1.89	2.43	1.07	1.64	1.64	1.25
time (sec)	N/A	0.016	0.044	0.651	0.269	0.451	0.090	0.451	0.062

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	39	52	97	54	34	47	-1
N.S.	1	1.00	1.39	1.86	3.46	1.93	1.21	1.68	-0.04
time (sec)	N/A	0.108	0.026	0.845	0.342	0.352	2.002	0.421	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	59	77	82	76	0	72	-1
N.S.	1	1.00	1.26	1.64	1.74	1.62	0.00	1.53	-0.02
time (sec)	N/A	0.165	0.103	0.852	0.328	0.349	0.000	0.439	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	78	139	66	116	0	134	-1
N.S.	1	1.00	0.89	1.58	0.75	1.32	0.00	1.52	-0.01
time (sec)	N/A	0.200	0.122	0.928	0.350	0.489	0.000	0.411	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	205	78	143	0	199	-1
N.S.	1	1.00	0.83	1.55	0.59	1.08	0.00	1.51	-0.01
time (sec)	N/A	0.233	0.206	0.996	0.364	0.342	0.000	0.406	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	140	271	82	161	0	266	-1
N.S.	1	1.00	0.84	1.63	0.49	0.97	0.00	1.60	-0.01
time (sec)	N/A	0.277	0.246	1.023	0.337	0.398	0.000	0.414	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	100	463	329	127	228	236	168
N.S.	1	1.00	0.54	2.52	1.79	0.69	1.24	1.28	0.91
time (sec)	N/A	0.261	0.164	0.622	0.288	0.405	0.367	0.418	0.161

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	87	283	275	95	172	171	125
N.S.	1	1.00	0.65	2.11	2.05	0.71	1.28	1.28	0.93
time (sec)	N/A	0.148	0.124	0.631	0.282	0.360	0.228	0.408	0.923

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	56	147	135	64	112	112	82
N.S.	1	1.00	1.14	3.00	2.76	1.31	2.29	2.29	1.67
time (sec)	N/A	0.034	0.091	0.605	0.283	0.414	0.141	0.392	0.903

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	51	121	175	94	73	113	-1
N.S.	1	1.00	0.82	1.95	2.82	1.52	1.18	1.82	-0.02
time (sec)	N/A	0.137	0.162	0.798	0.348	0.345	2.233	0.418	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	62	118	136	122	0	119	-1
N.S.	1	1.00	0.89	1.69	1.94	1.74	0.00	1.70	-0.01
time (sec)	N/A	0.188	0.156	0.763	0.327	0.341	0.000	0.433	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	93	181	126	153	0	181	-1
N.S.	1	1.00	0.77	1.50	1.04	1.26	0.00	1.50	-0.01
time (sec)	N/A	0.243	0.235	0.686	0.344	0.370	0.000	0.402	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	154	287	117	194	0	285	-1
N.S.	1	1.00	0.90	1.67	0.68	1.13	0.00	1.66	-0.01
time (sec)	N/A	0.300	0.281	0.698	0.340	0.344	0.000	0.424	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	206	396	128	231	0	395	-1
N.S.	1	1.00	0.83	1.60	0.52	0.93	0.00	1.59	-0.00
time (sec)	N/A	0.360	0.355	0.704	0.346	0.360	0.000	0.405	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	159	442	437	236	0	407	-1
N.S.	1	1.00	0.73	2.02	2.00	1.08	0.00	1.86	-0.00
time (sec)	N/A	0.344	0.400	0.967	0.311	0.366	0.000	0.425	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	118	292	328	190	0	256	-1
N.S.	1	1.00	0.79	1.95	2.19	1.27	0.00	1.71	-0.01
time (sec)	N/A	0.247	0.309	0.909	0.319	0.371	0.000	0.406	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	89	184	233	156	0	148	-1
N.S.	1	1.00	0.89	1.84	2.33	1.56	0.00	1.48	-0.01
time (sec)	N/A	0.192	0.194	0.875	0.311	0.378	0.000	0.413	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	64	114	156	118	0	83	-1
N.S.	1	1.00	0.94	1.68	2.29	1.74	0.00	1.22	-0.01
time (sec)	N/A	0.120	0.093	0.815	0.310	0.386	0.000	0.422	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	81	57	95	0	56	-1
N.S.	1	1.00	0.96	1.59	1.12	1.86	0.00	1.10	-0.02
time (sec)	N/A	0.054	0.056	0.745	0.299	0.414	0.000	0.423	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	108	155	123	0	75	-1
N.S.	1	1.00	0.86	1.48	2.12	1.68	0.00	1.03	-0.01
time (sec)	N/A	0.183	0.087	0.721	0.332	0.373	0.000	0.410	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	101	172	192	179	0	129	-1
N.S.	1	1.00	0.89	1.52	1.70	1.58	0.00	1.14	-0.01
time (sec)	N/A	0.266	0.244	0.717	0.356	0.443	0.000	0.410	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	178	281	242	275	0	248	-1
N.S.	1	1.00	0.94	1.48	1.27	1.45	0.00	1.31	-0.01
time (sec)	N/A	0.345	0.290	0.727	0.376	0.365	0.000	0.402	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	173	431	406	371	0	2979	-1
N.S.	1	1.00	0.75	1.87	1.76	1.61	0.00	12.90	-0.00
time (sec)	N/A	0.404	0.746	0.987	0.336	0.376	0.000	0.456	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	156	325	312	333	0	1991	-1
N.S.	1	1.00	0.86	1.79	1.71	1.83	0.00	10.94	-0.01
time (sec)	N/A	0.325	0.626	0.937	0.344	0.417	0.000	0.456	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	115	254	236	274	0	1308	-1
N.S.	1	1.00	0.78	1.73	1.61	1.86	0.00	8.90	-0.01
time (sec)	N/A	0.286	0.496	0.853	0.337	0.362	0.000	0.456	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	97	215	178	200	0	994	-1
N.S.	1	1.00	0.78	1.72	1.42	1.60	0.00	7.95	-0.01
time (sec)	N/A	0.221	0.272	0.738	0.324	0.373	0.000	0.458	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	65	132	81	149	0	615	-1
N.S.	1	1.00	0.92	1.86	1.14	2.10	0.00	8.66	-0.01
time (sec)	N/A	0.084	0.187	0.711	0.317	0.376	0.000	0.441	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	139	254	227	270	0	1329	-1
N.S.	1	1.00	0.93	1.69	1.51	1.80	0.00	8.86	-0.01
time (sec)	N/A	0.307	0.806	0.714	0.382	0.368	0.000	0.457	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	183	312	0	377	0	3353	-1
N.S.	1	1.00	0.98	1.68	0.00	2.03	0.00	18.03	-0.01
time (sec)	N/A	0.367	0.905	0.737	0.000	0.465	0.000	0.472	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	236	571	0	566	0	879	-1
N.S.	1	1.00	0.89	2.16	0.00	2.14	0.00	3.33	-0.00
time (sec)	N/A	0.474	0.671	0.914	0.000	0.371	0.000	0.405	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	153	527	0	475	0	741	-1
N.S.	1	1.00	0.63	2.19	0.00	1.97	0.00	3.07	-0.00
time (sec)	N/A	0.412	0.625	0.752	0.000	0.367	0.000	0.421	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	158	435	0	373	0	529	-1
N.S.	1	1.00	0.89	2.44	0.00	2.10	0.00	2.97	-0.01
time (sec)	N/A	0.282	0.399	0.740	0.000	0.444	0.000	0.411	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	88	276	95	253	0	298	-1
N.S.	1	1.00	0.85	2.65	0.91	2.43	0.00	2.87	-0.01
time (sec)	N/A	0.100	0.398	0.721	0.295	0.342	0.000	0.420	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	451	488	0	601	0	837	-1
N.S.	1	1.00	1.72	1.86	0.00	2.29	0.00	3.19	-0.00
time (sec)	N/A	0.425	0.866	0.728	0.000	0.407	0.000	0.422	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	541	643	0	762	0	1006	-1
N.S.	1	1.00	1.82	2.16	0.00	2.56	0.00	3.38	-0.00
time (sec)	N/A	0.520	0.977	0.756	0.000	0.430	0.000	0.404	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	627	760	0	892	0	1169	-1
N.S.	1	1.00	1.66	2.02	0.00	2.37	0.00	3.10	-0.00
time (sec)	N/A	0.634	1.214	0.767	0.000	0.392	0.000	0.429	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	92	447	250	95	168	174	116
N.S.	1	1.00	0.66	3.22	1.80	0.68	1.21	1.25	0.83
time (sec)	N/A	0.184	0.128	0.689	0.263	0.425	0.491	0.422	0.991

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	74	298	214	78	134	138	93
N.S.	1	1.00	0.68	2.73	1.96	0.72	1.23	1.27	0.85
time (sec)	N/A	0.136	0.104	0.661	0.273	0.392	0.308	0.418	0.934

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	57	183	213	60	99	101	70
N.S.	1	1.00	0.72	2.32	2.70	0.76	1.25	1.28	0.89
time (sec)	N/A	0.087	0.092	0.648	0.274	0.374	0.224	0.436	0.975

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	40	97	86	42	65	70	47
N.S.	1	1.00	0.78	1.90	1.69	0.82	1.27	1.37	0.92
time (sec)	N/A	0.051	0.061	0.651	0.269	0.516	0.127	0.408	0.925

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	81	122	73	49	76	-1
N.S.	1	1.00	1.34	1.98	2.98	1.78	1.20	1.85	-0.02
time (sec)	N/A	0.077	0.090	0.911	0.305	0.405	1.936	0.399	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	81	80	82	0	80	-1
N.S.	1	1.00	1.00	1.93	1.90	1.95	0.00	1.90	-0.02
time (sec)	N/A	0.090	0.075	0.862	0.319	0.465	0.000	0.397	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	80	110	90	107	0	109	-1
N.S.	1	1.00	1.08	1.49	1.22	1.45	0.00	1.47	-0.01
time (sec)	N/A	0.127	0.130	0.924	0.345	0.409	0.000	0.421	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	95	172	73	127	0	170	-1
N.S.	1	1.00	0.90	1.64	0.70	1.21	0.00	1.62	-0.01
time (sec)	N/A	0.172	0.180	0.933	0.349	0.466	0.000	0.417	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	127	238	76	152	0	237	-1
N.S.	1	1.00	0.85	1.60	0.51	1.02	0.00	1.59	-0.01
time (sec)	N/A	0.219	0.248	0.997	0.325	0.372	0.000	0.413	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	138	738	383	155	286	304	182
N.S.	1	1.00	0.59	3.15	1.64	0.66	1.22	1.30	0.78
time (sec)	N/A	0.266	0.218	0.615	0.282	0.395	0.703	0.413	1.031

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	113	513	353	126	226	239	148
N.S.	1	1.00	0.61	2.79	1.92	0.68	1.23	1.30	0.80
time (sec)	N/A	0.201	0.152	0.609	0.284	0.379	0.471	0.413	0.982

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	85	332	189	98	172	180	114
N.S.	1	1.00	0.62	2.44	1.39	0.72	1.26	1.32	0.84
time (sec)	N/A	0.130	0.117	0.582	0.274	0.388	0.312	0.423	0.118

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	82	226	235	130	121	222	-1
N.S.	1	1.00	0.75	2.05	2.14	1.18	1.10	2.02	-0.01
time (sec)	N/A	0.134	0.259	0.838	0.320	0.419	2.466	0.412	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	190	179	127	0	197	-1
N.S.	1	1.00	1.00	2.00	1.88	1.34	0.00	2.07	-0.01
time (sec)	N/A	0.127	0.173	0.879	0.316	0.390	0.000	0.407	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	97	188	165	164	0	206	-1
N.S.	1	1.00	0.85	1.65	1.45	1.44	0.00	1.81	-0.01
time (sec)	N/A	0.155	0.272	0.857	0.341	0.357	0.000	0.406	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	114	222	135	171	0	236	-1
N.S.	1	1.00	0.86	1.67	1.02	1.29	0.00	1.77	-0.01
time (sec)	N/A	0.183	0.277	0.830	0.328	0.374	0.000	0.419	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	124	291	139	194	0	294	-1
N.S.	1	1.00	0.71	1.66	0.79	1.11	0.00	1.68	-0.01
time (sec)	N/A	0.241	0.298	0.789	0.348	0.359	0.000	0.411	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	274	369	0	605	0	0	-1
N.S.	1	1.00	1.00	1.35	0.00	2.22	0.00	0.00	-0.00
time (sec)	N/A	0.508	0.333	1.171	0.000	0.358	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	210	268	0	502	0	0	-1
N.S.	1	1.00	1.00	1.28	0.00	2.40	0.00	0.00	-0.00
time (sec)	N/A	0.262	0.298	0.987	0.000	0.390	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	213	259	0	496	0	0	-1
N.S.	1	1.00	0.94	1.15	0.00	2.19	0.00	0.00	-0.00
time (sec)	N/A	0.272	0.211	0.886	0.000	0.369	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	171	200	0	219	0	0	-1
N.S.	1	1.00	0.97	1.13	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.148	0.730	0.000	0.390	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	180	212	0	316	0	0	-1
N.S.	1	1.00	0.85	1.00	0.00	1.48	0.00	0.00	-0.00
time (sec)	N/A	0.190	0.171	0.712	0.000	0.416	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	187	227	0	249	0	0	-1
N.S.	1	1.00	0.95	1.15	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.271	0.283	0.714	0.000	0.434	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	243	288	0	599	0	0	-1
N.S.	1	1.00	0.98	1.16	0.00	2.41	0.00	0.00	-0.00
time (sec)	N/A	0.367	0.288	0.721	0.000	0.497	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	257	330	0	583	0	0	-1
N.S.	1	1.00	0.95	1.22	0.00	2.16	0.00	0.00	-0.00
time (sec)	N/A	0.352	0.422	0.743	0.000	0.386	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	621	532	0	1179	0	0	-1
N.S.	1	1.00	1.38	1.18	0.00	2.63	0.00	0.00	-0.00
time (sec)	N/A	0.604	1.093	1.144	0.000	0.659	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	582	495	0	931	0	0	-1
N.S.	1	1.00	1.35	1.15	0.00	2.16	0.00	0.00	-0.00
time (sec)	N/A	0.496	0.495	0.898	0.000	0.453	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	364	491	0	1162	0	0	-1
N.S.	1	1.00	0.88	1.18	0.00	2.79	0.00	0.00	-0.00
time (sec)	N/A	0.420	0.700	0.832	0.000	0.435	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	239	291	0	641	0	0	-1
N.S.	1	1.00	1.00	1.22	0.00	2.68	0.00	0.00	-0.00
time (sec)	N/A	0.237	0.402	0.783	0.000	0.521	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	590	503	0	1162	0	0	-1
N.S.	1	1.00	1.24	1.06	0.00	2.44	0.00	0.00	-0.00
time (sec)	N/A	0.603	0.557	0.731	0.000	0.413	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	501	546	0	992	0	0	-1
N.S.	1	1.00	1.15	1.26	0.00	2.28	0.00	0.00	-0.00
time (sec)	N/A	0.615	0.845	0.749	0.000	0.489	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	500	500	675	595	0	1310	0	0	-1
N.S.	1	1.00	1.35	1.19	0.00	2.62	0.00	0.00	-0.00
time (sec)	N/A	0.908	0.703	0.784	0.000	0.481	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	648	820	0	1620	0	0	-1
N.S.	1	1.00	1.36	1.72	0.00	3.40	0.00	0.00	-0.00
time (sec)	N/A	0.749	1.424	1.103	0.000	0.440	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	746	746	932	1064	0	2047	0	0	-1
N.S.	1	1.00	1.25	1.43	0.00	2.74	0.00	0.00	-0.00
time (sec)	N/A	0.822	1.898	0.932	0.000	0.375	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	637	743	0	1607	0	0	-1
N.S.	1	1.00	1.24	1.45	0.00	3.14	0.00	0.00	-0.00
time (sec)	N/A	0.604	1.352	0.837	0.000	0.575	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	856	856	933	1064	0	2116	0	0	-1
N.S.	1	1.00	1.09	1.24	0.00	2.47	0.00	0.00	-0.00
time (sec)	N/A	0.883	1.775	0.756	0.000	0.391	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	730	730	1046	1090	0	2076	0	0	-1
N.S.	1	1.00	1.43	1.49	0.00	2.84	0.00	0.00	-0.00
time (sec)	N/A	1.260	1.715	0.783	0.000	0.539	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	874	874	1009	1178	0	2346	0	0	-1
N.S.	1	1.00	1.15	1.35	0.00	2.68	0.00	0.00	-0.00
time (sec)	N/A	2.116	1.936	0.790	0.000	0.521	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	791	791	998	1294	0	2363	0	0	-1
N.S.	1	1.00	1.26	1.64	0.00	2.99	0.00	0.00	-0.00
time (sec)	N/A	1.411	2.630	0.820	0.000	0.490	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	100	551	268	105	185	192	152
N.S.	1	1.00	0.65	3.58	1.74	0.68	1.20	1.25	0.99
time (sec)	N/A	0.209	0.132	0.672	0.281	0.423	0.684	0.405	1.046

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	84	389	267	87	151	156	122
N.S.	1	1.00	0.68	3.14	2.15	0.70	1.22	1.26	0.98
time (sec)	N/A	0.158	0.110	0.661	0.277	0.373	0.448	0.413	0.960

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	66	257	196	68	116	119	92
N.S.	1	1.00	0.70	2.73	2.09	0.72	1.23	1.27	0.98
time (sec)	N/A	0.108	0.089	0.660	0.260	0.385	0.300	0.403	0.924

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	49	158	104	53	82	88	65
N.S.	1	1.00	0.74	2.39	1.58	0.80	1.24	1.33	0.98
time (sec)	N/A	0.068	0.074	0.658	0.268	0.418	0.189	0.407	0.098

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	49	113	140	86	66	109	-1
N.S.	1	1.00	0.88	2.02	2.50	1.54	1.18	1.95	-0.02
time (sec)	N/A	0.093	0.147	0.902	0.315	0.347	2.052	0.405	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	110	102	90	0	111	-1
N.S.	1	1.00	1.00	2.00	1.85	1.64	0.00	2.02	-0.02
time (sec)	N/A	0.093	0.105	0.925	0.312	0.413	0.000	0.419	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	86	114	87	101	0	118	-1
N.S.	1	1.00	1.25	1.65	1.26	1.46	0.00	1.71	-0.01
time (sec)	N/A	0.098	0.115	0.926	0.322	0.362	0.000	0.413	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	73	143	95	118	0	141	-1
N.S.	1	1.00	0.80	1.57	1.04	1.30	0.00	1.55	-0.01
time (sec)	N/A	0.150	0.193	0.935	0.329	0.407	0.000	0.402	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	139	818	383	161	284	303	301
N.S.	1	1.00	0.59	3.50	1.64	0.69	1.21	1.29	1.29
time (sec)	N/A	0.277	0.196	0.612	0.267	0.376	0.977	0.405	0.354

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	111	592	243	130	226	244	182
N.S.	1	1.00	0.60	3.18	1.31	0.70	1.22	1.31	0.98
time (sec)	N/A	0.207	0.179	0.618	0.293	0.399	0.690	0.404	0.241

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	108	335	289	161	168	331	-1
N.S.	1	1.00	0.68	2.09	1.81	1.01	1.05	2.07	-0.01
time (sec)	N/A	0.207	0.348	0.864	0.322	0.368	3.193	0.408	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	143	296	235	160	0	308	-1
N.S.	1	1.00	1.00	2.07	1.64	1.12	0.00	2.15	-0.01
time (sec)	N/A	0.197	0.233	0.891	0.327	0.413	0.000	0.408	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	136	265	203	161	0	280	-1
N.S.	1	1.00	0.96	1.88	1.44	1.14	0.00	1.99	-0.01
time (sec)	N/A	0.163	0.226	0.895	0.324	0.361	0.000	0.397	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	135	261	188	187	0	279	-1
N.S.	1	1.00	0.90	1.74	1.25	1.25	0.00	1.86	-0.01
time (sec)	N/A	0.208	0.408	0.913	0.329	0.402	0.000	0.407	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	150	292	154	196	0	315	-1
N.S.	1	1.00	0.90	1.75	0.92	1.17	0.00	1.89	-0.01
time (sec)	N/A	0.251	0.359	0.934	0.339	0.372	0.000	0.415	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	213	925	0	989	0	0	-1
N.S.	1	1.00	0.57	2.48	0.00	2.65	0.00	0.00	-0.00
time (sec)	N/A	0.720	0.370	1.156	0.000	0.433	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	198	671	0	977	0	0	-1
N.S.	1	1.00	0.55	1.87	0.00	2.73	0.00	0.00	-0.00
time (sec)	N/A	0.476	0.196	1.069	0.000	0.404	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	170	423	0	500	0	0	-1
N.S.	1	1.00	0.60	1.49	0.00	1.77	0.00	0.00	-0.00
time (sec)	N/A	0.360	0.171	1.014	0.000	0.414	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	180	280	0	671	0	0	-1
N.S.	1	1.00	0.52	0.81	0.00	1.94	0.00	0.00	-0.00
time (sec)	N/A	0.304	0.168	1.013	0.000	0.441	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	180	143	0	673	0	0	-1
N.S.	1	1.00	0.52	0.41	0.00	1.95	0.00	0.00	-0.00
time (sec)	N/A	0.300	0.109	1.001	0.000	0.451	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	186	138	0	530	0	0	-1
N.S.	1	1.00	0.61	0.46	0.00	1.75	0.00	0.00	-0.00
time (sec)	N/A	0.372	0.211	0.992	0.000	0.406	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	215	187	0	1154	0	0	-1
N.S.	1	1.00	0.56	0.49	0.00	3.03	0.00	0.00	-0.00
time (sec)	N/A	0.442	0.322	1.017	0.000	0.483	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	237	240	0	1251	0	0	-1
N.S.	1	1.00	0.58	0.59	0.00	3.05	0.00	0.00	-0.00
time (sec)	N/A	0.478	0.297	1.040	0.000	0.444	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	718	718	363	877	0	2050	0	0	-1
N.S.	1	1.00	0.51	1.22	0.00	2.86	0.00	0.00	-0.00
time (sec)	N/A	0.793	0.169	1.149	0.000	0.525	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	203	594	0	1276	0	0	-1
N.S.	1	1.00	0.54	1.59	0.00	3.42	0.00	0.00	-0.00
time (sec)	N/A	0.439	0.133	1.097	0.000	0.411	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	695	695	387	395	0	2135	0	0	-1
N.S.	1	1.00	0.56	0.57	0.00	3.07	0.00	0.00	-0.00
time (sec)	N/A	0.926	0.146	1.062	0.000	0.515	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	739	739	387	226	0	2048	0	0	-1
N.S.	1	1.00	0.52	0.31	0.00	2.77	0.00	0.00	-0.00
time (sec)	N/A	1.003	0.149	1.017	0.000	0.406	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	697	697	411	338	0	1773	0	0	-1
N.S.	1	1.00	0.59	0.48	0.00	2.54	0.00	0.00	-0.00
time (sec)	N/A	1.031	0.432	1.050	0.000	0.479	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	784	784	397	2448	0	2980	0	0	-1
N.S.	1	1.00	0.51	3.12	0.00	3.80	0.00	0.00	-0.00
time (sec)	N/A	1.159	0.446	1.612	0.000	0.469	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1105	1105	675	1927	0	4691	0	0	-1
N.S.	1	1.00	0.61	1.74	0.00	4.25	0.00	0.00	-0.00
time (sec)	N/A	1.355	0.363	1.405	0.000	0.527	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	776	776	429	1456	0	2962	0	0	-1
N.S.	1	1.00	0.55	1.88	0.00	3.82	0.00	0.00	-0.00
time (sec)	N/A	2.018	0.471	1.305	0.000	0.504	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	781	781	423	994	0	2972	0	0	-1
N.S.	1	1.00	0.54	1.27	0.00	3.81	0.00	0.00	-0.00
time (sec)	N/A	1.034	0.332	1.223	0.000	0.496	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1147	1147	669	810	0	4691	0	0	-1
N.S.	1	1.00	0.58	0.71	0.00	4.09	0.00	0.00	-0.00
time (sec)	N/A	2.385	0.345	1.134	0.000	0.499	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [49] had the largest ratio of [19]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	11	4	1.00	15	0.267
2	A	9	4	1.00	15	0.267
3	A	7	4	1.00	13	0.308
4	A	2	2	1.00	12	0.167
5	A	6	5	1.00	15	0.333
6	A	9	5	1.00	15	0.333
7	A	11	5	1.00	15	0.333
8	A	13	5	1.00	15	0.333
9	A	15	5	1.00	15	0.333
10	A	14	4	1.00	17	0.235
11	A	11	4	1.00	15	0.267
12	A	3	2	1.00	14	0.143
13	A	8	7	1.00	17	0.412
14	A	10	6	1.00	17	0.353
15	A	14	5	1.00	17	0.294
16	A	17	5	1.00	17	0.294
17	A	20	5	1.00	17	0.294
18	A	15	7	1.00	17	0.412
19	A	11	7	1.00	17	0.412
20	A	8	7	1.00	17	0.412
21	A	6	5	1.00	15	0.333
22	A	3	3	1.00	14	0.214
23	A	8	4	1.00	17	0.235
24	A	12	5	1.00	17	0.294
25	A	17	5	1.00	17	0.294

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	15	8	1.00	17	0.471
27	A	12	8	1.00	17	0.471
28	A	10	6	1.00	17	0.353
29	A	9	5	1.00	15	0.333
30	A	4	4	1.00	14	0.286
31	A	12	5	1.00	17	0.294
32	A	16	5	1.00	17	0.294
33	A	15	6	1.00	17	0.353
34	A	14	5	1.00	17	0.294
35	A	11	5	1.00	15	0.333
36	A	5	4	1.00	14	0.286
37	A	17	5	1.00	17	0.294
38	A	21	5	1.00	17	0.294
39	A	26	5	1.00	17	0.294
40	A	12	3	1.00	17	0.176
41	A	10	3	1.00	17	0.176
42	A	8	3	1.00	15	0.200
43	A	6	3	1.00	14	0.214
44	A	7	6	1.00	17	0.353
45	A	7	6	1.00	17	0.353
46	A	10	5	1.00	17	0.294
47	A	12	5	1.00	17	0.294
48	A	14	5	1.00	17	0.294
49	A	17	3	1.00	19	0.158
50	A	14	3	1.00	17	0.176
51	A	11	3	1.00	16	0.188
52	A	11	6	1.00	19	0.316
53	A	10	7	1.00	19	0.368
54	A	12	7	1.00	19	0.368
55	A	13	6	1.00	19	0.316
56	A	17	5	1.00	19	0.263
57	A	14	7	1.00	19	0.368
58	A	12	6	1.00	19	0.316
59	A	11	6	1.00	19	0.316
60	A	8	4	1.00	17	0.235

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	8	4	1.00	16	0.250
62	A	13	4	1.00	19	0.210
63	A	14	6	1.00	19	0.316
64	A	18	5	1.00	19	0.263
65	A	24	9	1.00	19	0.474
66	A	20	8	1.00	19	0.421
67	A	17	6	1.00	19	0.316
68	A	9	5	1.00	17	0.294
69	A	18	5	1.00	16	0.312
70	A	22	6	1.00	19	0.316
71	A	32	6	1.00	19	0.316
72	A	27	8	1.00	19	0.421
73	A	28	7	1.00	19	0.368
74	A	19	6	1.00	17	0.353
75	A	28	5	1.00	16	0.312
76	A	41	7	1.00	19	0.368
77	A	60	6	1.00	19	0.316
78	A	46	7	1.00	19	0.368
79	A	13	4	1.00	17	0.235
80	A	11	4	1.00	17	0.235
81	A	9	4	1.00	15	0.267
82	A	7	4	1.00	14	0.286
83	A	8	6	1.00	17	0.353
84	A	8	7	1.00	17	0.412
85	A	8	6	1.00	17	0.353
86	A	11	5	1.00	17	0.294
87	A	17	4	1.00	17	0.235
88	A	14	4	1.00	16	0.250
89	A	14	7	1.00	19	0.368
90	A	13	8	1.00	19	0.421
91	A	12	8	1.00	19	0.421
92	A	14	7	1.00	19	0.368
93	A	15	7	1.00	19	0.368
94	A	15	6	1.00	19	0.316
95	A	14	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	11	4	1.00	19	0.210
97	A	11	4	1.00	17	0.235
98	A	11	4	1.00	16	0.250
99	A	16	4	1.00	19	0.210
100	A	17	5	1.00	19	0.263
101	A	18	6	1.00	19	0.316
102	A	23	6	1.00	19	0.316
103	A	12	5	1.00	19	0.263
104	A	34	7	1.00	17	0.412
105	A	36	8	1.00	16	0.500
106	A	41	8	1.00	19	0.421
107	A	36	8	1.00	19	0.421
108	A	47	9	1.00	19	0.474
109	A	71	10	1.00	19	0.526
110	A	37	9	1.00	19	0.474
111	A	89	9	1.00	17	0.529

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^3(a+bx)\cosh(c+dx)\,dx$

Optimal. Leaf size=124

$$-\frac{6a\cosh(c+dx)}{d^4}-\frac{24bx\cosh(c+dx)}{d^4}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{24b\sinh(c+dx)}{d^5}+\frac{6ax\sinh}{d}$$

```
[Out] -6*a*cosh(d*x+c)/d^4-24*b*x*cosh(d*x+c)/d^4-3*a*x^2*cosh(d*x+c)/d^2-4*b*x^3
*cosh(d*x+c)/d^2+24*b*sinh(d*x+c)/d^5+6*a*x*sinh(d*x+c)/d^3+12*b*x^2*sinh(d
*x+c)/d^3+a*x^3*sinh(d*x+c)/d+b*x^4*sinh(d*x+c)/d
```

Rubi [A]

time = 0.25, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6874, 3377, 2718, 2717}

$$-\frac{6a\cosh(c+dx)}{d^4}+\frac{6ax\sinh(c+dx)}{d^3}-\frac{3ax^2\cosh(c+dx)}{d^2}+\frac{ax^3\sinh(c+dx)}{d}+\frac{24b\sinh(c+dx)}{d^5}-\frac{24bx\cosh(c+dx)}{d^4}+\frac{12bx^2\sinh(c+dx)}{d^3}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{bx^4\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] (-6*a*Cosh[c + d*x])/d^4 - (24*b*x*Cosh[c + d*x])/d^4 - (3*a*x^2*Cosh[c + d
*x])/d^2 - (4*b*x^3*Cosh[c + d*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (6*a*x*
Sinh[c + d*x])/d^3 + (12*b*x^2*Sinh[c + d*x])/d^3 + (a*x^3*Sinh[c + d*x])/d
 + (b*x^4*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^3(a+bx)\cosh(c+dx)\,dx = \int \left(ax^3\cosh(c+dx)+bx^4\cosh(c+dx)\right)\,dx$$
$$= a\int x^3\cosh(c+dx)\,dx + b\int x^4\cosh(c+dx)\,dx$$
$$= \frac{ax^3\sinh(c+dx)}{d}+\frac{bx^4\sinh(c+dx)}{d}-\frac{(3a)\int x^2\sinh(c+dx)\,dx}{d}-\frac{(4b)\int x^3}{}$$
$$= -\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{ax^3\sinh(c+dx)}{d}+\frac{bx^4\sinh(c+}{d}$$
$$= -\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{6ax\sinh(c+dx)}{d^3}+\frac{12bx^2\sinh(}{d^3}$$
$$= -\frac{6a\cosh(c+dx)}{d^4}-\frac{24bx\cosh(c+dx)}{d^4}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c}{d^2}$$
$$= -\frac{6a\cosh(c+dx)}{d^4}-\frac{24bx\cosh(c+dx)}{d^4}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c}{d^2}$$

Mathematica [A]
time = 0.12, size = 82, normalized size = 0.66

$$\frac{-d(3a(2+d^2x^2)+4bx(6+d^2x^2))\cosh(c+dx)+(ad^2x(6+d^2x^2)+b(24+12d^2x^2+d^4x^4))\sinh(c+dx)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] (-(d*(3*a*(2 + d^2*x^2) + 4*b*x*(6 + d^2*x^2))*Cosh[c + d*x]) + (a*d^2*x*(6
 + d^2*x^2) + b*(24 + 12*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(124) = 248$.
time = 0.68, size = 356, normalized size = 2.87

method	result
risch	$\frac{(b x^4 d^4+a d^4 x^3-4b d^3 x^3-3a d^3 x^2+12b d^2 x^2+6a d^2 x-24bdx-6ad+24b)e^{dx+c}}{2d^5}-\frac{(b x^4 d^4+a d^4 x^3+4b d^3 x^3+3a d^3 x^2+1}{}$
meijerg	$-\frac{16ib\cosh(c)\sqrt{\pi}\left(-\frac{ixd\left(\frac{5d^2x^2}{2}+15\right)\cosh(dx)}{10\sqrt{\pi}}+\frac{i\left(\frac{5}{8}d^4x^4+\frac{15}{2}d^2x^2+15\right)\sinh(dx)}{10\sqrt{\pi}}\right)}{d^5}-\frac{16b\sinh(c)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d\right.}{}\right.}{}$
derivativedivides	$\frac{-\frac{4b c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}+\frac{6b c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d}-\frac{4bc\left((dx+c)^3\sinh(dx+c)-\right.}{}}{}$

default	$\frac{-\frac{4b\,c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}+\frac{6b\,c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d}-\frac{4bc\left((dx+c)^3\sinh(dx+c\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(-4*b/d*c^3*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+6*b/d*c^2*((d*x+c)^2*si
nh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-4*b/d*c*((d*x+c)^3*sinh(d*x+
c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))+b/d*((d*x+c
)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)
*cosh(d*x+c)+24*sinh(d*x+c))+b/d*c^4*sinh(d*x+c)-a*c^3*sinh(d*x+c)+3*a*c^2*
((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-3*a*c*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*co
sh(d*x+c)+2*sinh(d*x+c))+a*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6
*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c)))
```

Maxima [A]

time = 0.27, size = 232, normalized size = 1.87

$$-\frac{1}{40}\,d\left(\frac{5\,(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2e^c-24\,dxe^c+24\,e^c)ae^{(dx)}}{d^5}+\frac{5\,(d^4x^4+4\,d^3x^3+12\,d^2x^2+24\,dx+24)ae^{(-dx-c)}}{d^5}+\frac{4\,(d^5x^5e^c-5\,d^4x^4e^c+20\,d^3x^3e^c-60\,d^2x^2e^c+120\,dxe^c-120\,e^c)be^{(dx)}}{d^6}+\frac{4\,(d^5x^5+5\,d^4x^4+20\,d^3x^3+60\,d^2x^2+120\,dx+120)be^{(-dx-c)}}{d^6}\right)+\frac{1}{20}\,(4\,bx^5+5\,ax^4)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/40*d*(5*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*
e^c)*a*e^(d*x)/d^5 + 5*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a*e
^(-d*x - c)/d^5 + 4*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*
x^2*e^c + 120*d*x*e^c - 120*e^c)*b*e^(d*x)/d^6 + 4*(d^5*x^5 + 5*d^4*x^4 + 2
0*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*b*e^(-d*x - c)/d^6) + 1/20*(4*b*x^5
 + 5*a*x^4)*cosh(d*x + c)
```

Fricas [A]

time = 0.41, size = 85, normalized size = 0.69

$$-\frac{(4\,bd^3x^3+3\,ad^3x^2+24\,bdx+6\,ad)\cosh\left(dx+c\right)-(bd^4x^4+ad^4x^3+12\,bd^2x^2+6\,ad^2x+24\,b)\sinh\left(dx+c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((4*b*d^3*x^3 + 3*a*d^3*x^2 + 24*b*d*x + 6*a*d)*cosh(d*x + c) - (b*d^4*x^4
 + a*d^4*x^3 + 12*b*d^2*x^2 + 6*a*d^2*x + 24*b)*sinh(d*x + c))/d^5
```

Sympy [A]

time = 0.33, size = 151, normalized size = 1.22

$$\begin{cases} \frac{ax^3\sinh\left(c+dx\right)}{d}-\frac{3ax^2\cosh\left(c+dx\right)}{d^2}+\frac{6ax\sinh\left(c+dx\right)}{d^3}-\frac{6a\cosh\left(c+dx\right)}{d^4}+\frac{bx^4\sinh\left(c+dx\right)}{d}-\frac{4bx^3\cosh\left(c+dx\right)}{d^2}+\frac{12bx^2\sinh\left(c+dx\right)}{d^3}-\frac{24bx\cosh\left(c+dx\right)}{d^4}+\frac{24b\sinh\left(c+dx\right)}{d^5} & \text{for}\: d \neq 0 \\ \left(\frac{ax^4}{4}+\frac{bx^5}{5}\right)\cosh\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x**3*sinh(c + d*x)/d - 3*a*x**2*cosh(c + d*x)/d**2 + 6*a*x*sin
h(c + d*x)/d**3 - 6*a*cosh(c + d*x)/d**4 + b*x**4*sinh(c + d*x)/d - 4*b*x**
3*cosh(c + d*x)/d**2 + 12*b*x**2*sinh(c + d*x)/d**3 - 24*b*x*cosh(c + d*x)/
d**4 + 24*b*sinh(c + d*x)/d**5, Ne(d, 0)), ((a*x**4/4 + b*x**5/5)*cosh(c),
True))
```

Giac [A]
time = 0.41, size = 152, normalized size = 1.23

$$\frac{(bd^4x^4 + ad^4x^3 - 4\,bd^3x^3 - 3\,ad^3x^2 + 12\,bd^2x^2 + 6\,ad^2x - 24\,bdx - 6\,ad + 24\,b)e^{(dx+c)}}{2\,d^5} - \frac{(bd^4x^4 + ad^4x^3 + 4\,bd^3x^3 + 3\,ad^3x^2 + 12\,bd^2x^2 + 6\,ad^2x + 24\,bdx + 6\,ad + 24\,b)e^{(-dx-c)}}{2\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^4*x^4 + a*d^4*x^3 - 4*b*d^3*x^3 - 3*a*d^3*x^2 + 12*b*d^2*x^2 + 6*a
*d^2*x - 24*b*d*x - 6*a*d + 24*b)*e^(d*x + c)/d^5 - 1/2*(b*d^4*x^4 + a*d^4*
x^3 + 4*b*d^3*x^3 + 3*a*d^3*x^2 + 12*b*d^2*x^2 + 6*a*d^2*x + 24*b*d*x + 6*a
*d + 24*b)*e^(-d*x - c)/d^5
```

Mupad [B]
time = 0.96, size = 122, normalized size = 0.98

$$\frac{12\,b\,x^2\,\sinh(c+d\,x)+6\,a\,x\,\sinh(c+d\,x)}{d^3}-\frac{6\,a\cosh(c+d\,x)+24\,b\,x\cosh(c+d\,x)}{d^4}-\frac{3\,a\,x^2\cosh(c+d\,x)+4\,b\,x^3\cosh(c+d\,x)}{d^2}+\frac{a\,x^3\sinh(c+d\,x)+b\,x^4\sinh(c+d\,x)}{d}+\frac{24\,b\sinh(c+d\,x)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(c + d*x)*(a + b*x),x)
```

```
[Out] (12*b*x^2*sinh(c + d*x) + 6*a*x*sinh(c + d*x))/d^3 - (6*a*cosh(c + d*x) + 2
4*b*x*cosh(c + d*x))/d^4 - (3*a*x^2*cosh(c + d*x) + 4*b*x^3*cosh(c + d*x))/
d^2 + (a*x^3*sinh(c + d*x) + b*x^4*sinh(c + d*x))/d + (24*b*sinh(c + d*x))/
d^5
```

3.2 $\int x^2(a+bx)\cosh(c+dx)\,dx$

Optimal. Leaf size=94

$$-\frac{6b\cosh(c+dx)}{d^4}-\frac{2ax\cosh(c+dx)}{d^2}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{2a\sinh(c+dx)}{d^3}+\frac{6bx\sinh(c+dx)}{d^3}+\frac{ax^2\sinh(c+$$

```
[Out] -6*b*cosh(d*x+c)/d^4-2*a*x*cosh(d*x+c)/d^2-3*b*x^2*cosh(d*x+c)/d^2+2*a*sinh
(d*x+c)/d^3+6*b*x*sinh(d*x+c)/d^3+a*x^2*sinh(d*x+c)/d+b*x^3*sinh(d*x+c)/d
```

Rubi [A]
time = 0.17, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6874, 3377, 2717, 2718}

$$\frac{2a\sinh(c+dx)}{d^3}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{ax^2\sinh(c+dx)}{d}-\frac{6b\cosh(c+dx)}{d^4}+\frac{6bx\sinh(c+dx)}{d^3}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{bx^3\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] (-6*b*Cosh[c + d*x])/d^4 - (2*a*x*Cosh[c + d*x])/d^2 - (3*b*x^2*Cosh[c + d*
x])/d^2 + (2*a*Sinh[c + d*x])/d^3 + (6*b*x*Sinh[c + d*x])/d^3 + (a*x^2*Sinh
[c + d*x])/d + (b*x^3*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2(a+bx)\cosh(c+dx)\,dx = \int \left(ax^2\cosh(c+dx)+bx^3\cosh(c+dx)\right)\,dx$$
$$= a\int x^2\cosh(c+dx)\,dx + b\int x^3\cosh(c+dx)\,dx$$
$$= \frac{ax^2\sinh(c+dx)}{d} + \frac{bx^3\sinh(c+dx)}{d} - \frac{(2a)\int x\sinh(c+dx)\,dx}{d} - \frac{(3b)\int x^2$$
$$= -\frac{2ax\cosh(c+dx)}{d^2} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{ax^2\sinh(c+dx)}{d} + \frac{bx^3\sinh(c+}{d}$$
$$= -\frac{2ax\cosh(c+dx)}{d^2} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{2a\sinh(c+dx)}{d^3} + \frac{6bx\sinh(c+}{d^3}$$
$$= -\frac{6b\cosh(c+dx)}{d^4} - \frac{2ax\cosh(c+dx)}{d^2} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{2a\sinh(c+d}{d^3}$$

Mathematica [A]
time = 0.10, size = 65, normalized size = 0.69

$$\frac{-((2ad^2x+3b(2+d^2x^2))\cosh(c+dx))+d(a(2+d^2x^2)+bx(6+d^2x^2))\sinh(c+dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] (-((2*a*d^2*x + 3*b*(2 + d^2*x^2))*Cosh[c + d*x]) + d*(a*(2 + d^2*x^2) + b*
x*(6 + d^2*x^2))*Sinh[c + d*x])/d^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(94) = 188.
time = 0.65, size = 224, normalized size = 2.38

method	result
risch	$\frac{(b\,d^3x^3+a\,d^3x^2-3b\,d^2x^2-2a\,d^2x+6bdx+2ad-6b)\,e^{dx+c}}{2d^4} - \frac{(b\,d^3x^3+a\,d^3x^2+3b\,d^2x^2+2a\,d^2x+6bdx+2ad+6b)\,e^{-dx-c}}{2d^4}$
meijerg	$\frac{8b\cosh(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(\frac{3d^2x^2}{2}+3\right)\cosh(dx)}{4\sqrt{\pi}}+\frac{dx\left(\frac{d^2x^2}{2}+3\right)\sinh(dx)}{4\sqrt{\pi}}\right)}{d^4} - \frac{8ib\sinh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{5d^2x^2}{2}+15\right)\cosh(dx)}{20\sqrt{\pi}}\right.}{d^4}$
derivativedivides	$\frac{\frac{3b\,c^2((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}-\frac{3bc\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d}+\frac{b\left((dx+c)^3\sinh(dx+c)-3(dx-\right.}{}}{}$
default	$\frac{\frac{3b\,c^2((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}-\frac{3bc\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d}+\frac{b\left((dx+c)^3\sinh(dx+c)-3(dx-\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(3*b/d*c^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-3*b/d*c*((d*x+c)^2*sinh(
d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+b/d*((d*x+c)^3*sinh(d*x+c)-3*(d
*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))-b/d*c^3*sinh(d*x+c
)+a*c^2*sinh(d*x+c)-2*a*c*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+a*((d*x+c)^2*si
nh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(94) = 188$.
time = 0.27, size = 196, normalized size = 2.09

$$-\frac{1}{24}\,d\left(\frac{4\,(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c)ae^{(dx)}}{d^4}+\frac{4\,(d^3x^3+3\,d^2x^2+6\,dx+6)ae^{(-dx-c)}}{d^4}+\frac{3\,(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2e^c-24\,dxe^c+24\,e^c)be^{(dx)}}{d^5}+\frac{3\,(d^4x^4+4\,d^3x^3+12\,d^2x^2+24\,dx+24)be^{(-dx-c)}}{d^5}\right)+\frac{1}{12}\,(3\,bx^4+4\,ax^3)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/24*d*(4*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*e^(d*x)/d^4
+ 4*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*e^(-d*x - c)/d^4 + 3*(d^4*x^4*e^c -
 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*b*e^(d*x)/d^5 + 3*(d
^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*b*e^(-d*x - c)/d^5) + 1/12*(
3*b*x^4 + 4*a*x^3)*cosh(d*x + c)
```

Fricas [A]
time = 0.37, size = 67, normalized size = 0.71

$$-\frac{(3\,bd^2x^2+2\,ad^2x+6\,b)\cosh\left(dx+c\right)-(bd^3x^3+ad^3x^2+6\,bdx+2\,ad)\sinh\left(dx+c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((3*b*d^2*x^2 + 2*a*d^2*x + 6*b)*cosh(d*x + c) - (b*d^3*x^3 + a*d^3*x^2 +
6*b*d*x + 2*a*d)*sinh(d*x + c))/d^4
```

Sympy [A]
time = 0.22, size = 117, normalized size = 1.24

$$\begin{cases}\frac{ax^2\sinh(c+dx)}{d}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{2a\sinh(c+dx)}{d^3}+\frac{bx^3\sinh(c+dx)}{d}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{6bx\sinh(c+dx)}{d^3}-\frac{6b\cosh(c+dx)}{d^4} & \text{for } d\neq 0\\ \left(\frac{ax^3}{3}+\frac{bx^4}{4}\right)\cosh(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x**2*sinh(c + d*x)/d - 2*a*x*cosh(c + d*x)/d**2 + 2*a*sinh(c +
 d*x)/d**3 + b*x**3*sinh(c + d*x)/d - 3*b*x**2*cosh(c + d*x)/d**2 + 6*b*x*s
inh(c + d*x)/d**3 - 6*b*cosh(c + d*x)/d**4, Ne(d, 0)), ((a*x**3/3 + b*x**4/
4)*cosh(c), True))
```

Giac [A]
time = 0.42, size = 116, normalized size = 1.23

$$\frac{(bd^3x^3 + ad^3x^2 - 3\,bd^2x^2 - 2\,ad^2x + 6\,bdx + 2\,ad - 6\,b)e^{(dx+c)}}{2\,d^4} - \frac{(bd^3x^3 + ad^3x^2 + 3\,bd^2x^2 + 2\,ad^2x + 6\,bdx + 2\,ad + 6\,b)e^{(-dx-c)}}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^3*x^3 + a*d^3*x^2 - 3*b*d^2*x^2 - 2*a*d^2*x + 6*b*d*x + 2*a*d - 6*
b)*e^(d*x + c)/d^4 - 1/2*(b*d^3*x^3 + a*d^3*x^2 + 3*b*d^2*x^2 + 2*a*d^2*x +
 6*b*d*x + 2*a*d + 6*b)*e^(-d*x - c)/d^4
```

Mupad [B]
time = 0.96, size = 92, normalized size = 0.98

$$\frac{2\,a\sinh(c+d\,x) + 6\,b\,x\sinh(c+d\,x)}{d^3} - \frac{2\,a\,x\cosh(c+d\,x) + 3\,b\,x^2\cosh(c+d\,x)}{d^2} + \frac{a\,x^2\sinh(c+d\,x) + b\,x^3\sinh(c+d\,x)}{d} - \frac{6\,b\cosh(c+d\,x)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(c + d*x)*(a + b*x),x)
```

```
[Out] (2*a*sinh(c + d*x) + 6*b*x*sinh(c + d*x))/d^3 - (2*a*x*cosh(c + d*x) + 3*b*
x^2*cosh(c + d*x))/d^2 + (a*x^2*sinh(c + d*x) + b*x^3*sinh(c + d*x))/d - (6
*b*cosh(c + d*x))/d^4
```

3.3 $\int x(a+bx)\cosh(c+dx)\,dx$

Optimal. Leaf size=64

$$-\frac{a\cosh(c+dx)}{d^2}-\frac{2bx\cosh(c+dx)}{d^2}+\frac{2b\sinh(c+dx)}{d^3}+\frac{ax\sinh(c+dx)}{d}+\frac{bx^2\sinh(c+dx)}{d}$$

```
[Out] -a*cosh(d*x+c)/d^2-2*b*x*cosh(d*x+c)/d^2+2*b*sinh(d*x+c)/d^3+a*x*sinh(d*x+c
)/d+b*x^2*sinh(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6874, 3377, 2718, 2717}

$$-\frac{a\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}+\frac{2b\sinh(c+dx)}{d^3}-\frac{2bx\cosh(c+dx)}{d^2}+\frac{bx^2\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] -((a*Cosh[c + d*x])/d^2) - (2*b*x*Cosh[c + d*x])/d^2 + (2*b*Sinh[c + d*x])/
d^3 + (a*x*Sinh[c + d*x])/d + (b*x^2*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x(a+bx)\cosh(c+dx)\,dx = \int \left(ax\cosh(c+dx) + bx^2\cosh(c+dx)\right)\,dx$$
$$= a\int x\cosh(c+dx)\,dx + b\int x^2\cosh(c+dx)\,dx$$
$$= \frac{ax\sinh(c+dx)}{d} + \frac{bx^2\sinh(c+dx)}{d} - \frac{a\int\sinh(c+dx)\,dx}{d} - \frac{(2b)\int x\sinh(c \ldots}{d}$$
$$= -\frac{a\cosh(c+dx)}{d^2} - \frac{2bx\cosh(c+dx)}{d^2} + \frac{ax\sinh(c+dx)}{d} + \frac{bx^2\sinh(c+dx)}{d} + \ldots$$
$$= -\frac{a\cosh(c+dx)}{d^2} - \frac{2bx\cosh(c+dx)}{d^2} + \frac{2b\sinh(c+dx)}{d^3} + \frac{ax\sinh(c+dx)}{d} + \ldots$$

Mathematica [A]
time = 0.07, size = 45, normalized size = 0.70

$$\frac{-d(a+2bx)\cosh(c+dx) + (ad^2x + b(2+d^2x^2))\sinh(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] (-(d*(a + 2*b*x)*Cosh[c + d*x]) + (a*d^2*x + b*(2 + d^2*x^2))*Sinh[c + d*x]
)/d^3
```

Maple [A]
time = 0.75, size = 122, normalized size = 1.91

method	result
risch	$\frac{(b\,d^2x^2+a\,d^2x-2bdx-ad+2b)\mathrm{e}^{dx+c}}{2d^3} - \frac{(b\,d^2x^2+a\,d^2x+2bdx+ad+2b)\mathrm{e}^{-dx-c}}{2d^3}$
derivativedivides	$\frac{\frac{b\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d} - \frac{2bc((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d} + a((dx+c)\sinh(dx+c)-\cosh(d \ldots}{d^2}$
default	$\frac{\frac{b\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d} - \frac{2bc((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d} + a((dx+c)\sinh(dx+c)-\cosh(d \ldots}{d^2}$
meijerg	$\frac{4ib\cosh(c)\sqrt{\pi}\left(\frac{ixd\cosh(dx)}{2\sqrt{\pi}} - \frac{i\left(\frac{3d^2x^2}{2}+3\right)\sinh(dx)}{6\sqrt{\pi}}\right)}{d^3} + \frac{4b\sinh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}} + \frac{\left(\frac{d^2x^2}{2}+1\right)\cosh(dx)}{2\sqrt{\pi}} - \frac{dx\sinh(dx)}{2\sqrt{\pi}} \ldots\right.}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(b/d*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-2*b/
d*c*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+a*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+b
/d*c^2*sinh(d*x+c)-c*a*sinh(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(64) = 128$.
time = 0.28, size = 160, normalized size = 2.50

$$-\frac{1}{12}\,d\left(\frac{3\,(d^2x^2e^c - 2\,dxe^c + 2\,e^c)ae^{(dx)}}{d^3} + \frac{3\,(d^2x^2 + 2\,dx + 2)ae^{(-dx-c)}}{d^3} + \frac{2\,(d^3x^3e^c - 3\,d^2x^2e^c + 6\,dxe^c - 6\,e^c)be^{(dx)}}{d^4} + \frac{2\,(d^3x^3 + 3\,d^2x^2 + 6\,dx + 6)be^{(-dx-c)}}{d^4}\right) + \frac{1}{6}\,(2\,bx^3 + 3\,ax^2)\cosh\left(dx + c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/12*d*(3*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*e^(d*x)/d^3 + 3*(d^2*x^2 + 2
*d*x + 2)*a*e^(-d*x - c)/d^3 + 2*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c -
 6*e^c)*b*e^(d*x)/d^4 + 2*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*b*e^(-d*x - c)/
d^4) + 1/6*(2*b*x^3 + 3*a*x^2)*cosh(d*x + c)
```

Fricas [A]
time = 0.42, size = 48, normalized size = 0.75

$$-\frac{(2\,bdx + ad)\cosh\left(dx + c\right) - (bd^2x^2 + ad^2x + 2\,b)\sinh\left(dx + c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((2*b*d*x + a*d)*cosh(d*x + c) - (b*d^2*x^2 + a*d^2*x + 2*b)*sinh(d*x + c)
)/d^3
```

Sympy [A]
time = 0.14, size = 82, normalized size = 1.28

$$\begin{cases} \frac{ax\sinh\left(c+dx\right)}{d} - \frac{a\cosh\left(c+dx\right)}{d^2} + \frac{bx^2\sinh\left(c+dx\right)}{d} - \frac{2bx\cosh\left(c+dx\right)}{d^2} + \frac{2b\sinh\left(c+dx\right)}{d^3} & \text{for}\ d \neq 0 \\ \left(\frac{ax^2}{2} + \frac{bx^3}{3}\right)\cosh\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x*sinh(c + d*x)/d - a*cosh(c + d*x)/d**2 + b*x**2*sinh(c + d*x
)/d - 2*b*x*cosh(c + d*x)/d**2 + 2*b*sinh(c + d*x)/d**3, Ne(d, 0)), ((a*x**
2/2 + b*x**3/3)*cosh(c), True))
```

Giac [A]
time = 0.44, size = 79, normalized size = 1.23

$$\frac{(bd^2x^2 + ad^2x - 2\,bdx - ad + 2\,b)e^{(dx+c)}}{2\,d^3} - \frac{(bd^2x^2 + ad^2x + 2\,bdx + ad + 2\,b)e^{(-dx-c)}}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^2*x^2 + a*d^2*x - 2*b*d*x - a*d + 2*b)*e^(d*x + c)/d^3 - 1/2*(b*d^
2*x^2 + a*d^2*x + 2*b*d*x + a*d + 2*b)*e^(-d*x - c)/d^3
```

Mupad [B]
time = 0.92, size = 62, normalized size = 0.97

$$\frac{b\,x^2\,\sinh(c+d\,x)+a\,x\,\sinh(c+d\,x)}{d}-\frac{a\cosh(c+d\,x)+2\,b\,x\cosh(c+d\,x)}{d^2}+\frac{2\,b\sinh(c+d\,x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(c + d*x)*(a + b*x),x)
```

```
[Out] (b*x^2*sinh(c + d*x) + a*x*sinh(c + d*x))/d - (a*cosh(c + d*x) + 2*b*x*cosh
(c + d*x))/d^2 + (2*b*sinh(c + d*x))/d^3
```

3.4 $\int (a+bx)\cosh(c+dx)\,dx$

Optimal. Leaf size=28

$$-\frac{b\cosh(c+dx)}{d^2}+\frac{(a+bx)\sinh(c+dx)}{d}$$

```
[Out] -b*cosh(d*x+c)/d^2+(b*x+a)*sinh(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3377, 2718}

$$\frac{(a+bx)\sinh(c+dx)}{d}-\frac{b\cosh(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] -((b*Cosh[c + d*x])/d^2) + ((a + b*x)*Sinh[c + d*x])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\cosh(c+dx)\,dx &= \frac{(a+bx)\sinh(c+dx)}{d}-\frac{b\int \sinh(c+dx)\,dx}{d}\\ &= -\frac{b\cosh(c+dx)}{d^2}+\frac{(a+bx)\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 27, normalized size = 0.96

$$\frac{-b\cosh(c+dx)+d(a+bx)\sinh(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Cosh[c + d*x],x]
[Out] (-(b*Cosh[c + d*x]) + d*(a + b*x)*Sinh[c + d*x])/d^2
```

Maple [A]
time = 0.65, size = 53, normalized size = 1.89

method	result
risch	$\frac{(bdx+ad-b)e^{dx+c}}{2d^2}-\frac{(bdx+ad+b)e^{-dx-c}}{2d^2}$
derivativedivides	$\frac{\frac{b((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}-\frac{bc\sinh(dx+c)}{d}+a\sinh(dx+c)}{d}$
default	$\frac{\frac{b((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}-\frac{bc\sinh(dx+c)}{d}+a\sinh(dx+c)}{d}$
meijerg	$-\frac{2b\cosh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{b\sinh(c)(\cosh(dx)dx-\sinh(dx))}{d^2}+\frac{a\cosh(c)\sinh(dx)}{d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*(b/d*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-b/d*c*sinh(d*x+c)+a*sinh(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(28) = 56$.
time = 0.27, size = 68, normalized size = 2.43

$$\frac{ae^{(dx+c)}}{2\,d}+\frac{(dxe^{c}-e^{c})be^{(dx)}}{2\,d^2}-\frac{(dx+1)be^{(-dx-c)}}{2\,d^2}-\frac{ae^{(-dx-c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c),x, algorithm="maxima")
[Out] 1/2*a*e^(d*x + c)/d + 1/2*(d*x*e^c - e^c)*b*e^(d*x)/d^2 - 1/2*(d*x + 1)*b*e
^(-d*x - c)/d^2 - 1/2*a*e^(-d*x - c)/d
```

Fricas [A]
time = 0.45, size = 30, normalized size = 1.07

$$-\frac{b\cosh\left(dx+c\right)-\left(bdx+ad\right)\sinh\left(dx+c\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c),x, algorithm="fricas")
[Out] -(b*cosh(d*x + c) - (b*d*x + a*d)*sinh(d*x + c))/d^2
```

Sympy [A]
time = 0.09, size = 46, normalized size = 1.64

$$\begin{cases} \frac{a\sinh{(c+dx)}}{d} + \frac{bx\sinh{(c+dx)}}{d} - \frac{b\cosh{(c+dx)}}{d^2} & \text{for } d \neq 0 \\ \left(ax + \frac{bx^2}{2}\right)\cosh{(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)/d + b*x*sinh(c + d*x)/d - b*cosh(c + d*x)/d**2,
Ne(d, 0)), ((a*x + b*x**2/2)*cosh(c), True))
```

Giac [A]
time = 0.45, size = 46, normalized size = 1.64

$$\frac{(bdx + ad - b)e^{(dx+c)}}{2\,d^2} - \frac{(bdx + ad + b)e^{(-dx-c)}}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x + a*d - b)*e^(d*x + c)/d^2 - 1/2*(b*d*x + a*d + b)*e^(-d*x - c)/
d^2
```

Mupad [B]
time = 0.06, size = 35, normalized size = 1.25

$$\frac{a\sinh(c + d\,x) + b\,x\sinh(c + d\,x)}{d} - \frac{b\cosh(c + d\,x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b*x),x)
```

```
[Out] (a*sinh(c + d*x) + b*x*sinh(c + d*x))/d - (b*cosh(c + d*x))/d^2
```

3.5 $\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx$

Optimal. Leaf size=28

$$a\cosh(c)\text{Chi}(dx)+\frac{b\sinh(c+dx)}{d}+a\sinh(c)\text{Shi}(dx)$$

[Out] a*Chi(d*x)*cosh(c)+a*Shi(d*x)*sinh(c)+b*sinh(d*x+c)/d

Rubi [A]
time = 0.11, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6874, 2717, 3384, 3379, 3382}

$$a\cosh(c)\text{Chi}(dx)+a\sinh(c)\text{Shi}(dx)+\frac{b\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x)*Cosh[c + d*x])/x,x]

[Out] a*Cosh[c]*CoshIntegral[d*x] + (b*Sinh[c + d*x])/d + a*Sinh[c]*SinhIntegral[d*x]

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx &= \int \left(b\cosh(c+dx)+\frac{a\cosh(c+dx)}{x}\right)dx \\
&= a\int \frac{\cosh(c+dx)}{x}\,dx + b\int \cosh(c+dx)\,dx \\
&= \frac{b\sinh(c+dx)}{d} + (a\cosh(c))\int \frac{\cosh(dx)}{x}\,dx + (a\sinh(c))\int \frac{\sinh(dx)}{x}\,dx \\
&= a\cosh(c)\text{Chi}(dx) + \frac{b\sinh(c+dx)}{d} + a\sinh(c)\text{Shi}(dx)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 1.39

$$a\cosh(c)\text{Chi}(dx)+\frac{b\cosh(dx)\sinh(c)}{d}+\frac{b\cosh(c)\sinh(dx)}{d}+a\sinh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x,x]
```

```
[Out] a*Cosh[c]*CoshIntegral[d*x] + (b*Cosh[d*x]*Sinh[c])/d + (b*Cosh[c]*Sinh[d*x
])/d + a*Sinh[c]*SinhIntegral[d*x]
```

Maple [A]
time = 0.84, size = 52, normalized size = 1.86

method	result
risch	$-\frac{a\,\mathrm{e}^{-c}\,\text{expIntegral}(1,dx)}{2}-\frac{b\,\mathrm{e}^{-dx-c}}{2d}-\frac{a\,\mathrm{e}^{c}\,\text{expIntegral}(1,-dx)}{2}+\frac{b\,\mathrm{e}^{dx+c}}{2d}$
meijerg	$\frac{b\cosh(c)\sinh(dx)}{d}-\frac{b\sinh(c)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(dx)}{\sqrt{\pi}}\right)}{d}+\frac{a\cosh(c)\sqrt{\pi}\left(\frac{2\,\text{hyperbolicCosineIntegral}(dx)-2\ln(dx)-2\gamma}{\sqrt{\pi}}+\frac{2\gamma+2\ln(x)\cdots}{\sqrt{\pi}\cdots}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*cosh(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*exp(-c)*Ei(1,d*x)-1/2*b/d*exp(-d*x-c)-1/2*a*exp(c)*Ei(1,-d*x)+1/2*b/
d*exp(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(28) = 56$.
time = 0.34, size = 97, normalized size = 3.46

$$-\frac{1}{2}\left(b\left(\frac{(dxe^{c}-e^{c})e^{(dx)}}{d^{2}}+\frac{(dx+1)e^{(-dx-c)}}{d^{2}}\right)+\frac{2\,a\cosh\left(dx+c\right)\log\left(x\right)}{d}-\frac{\left(\mathrm{Ei}(-dx)\,e^{(-c)}+\mathrm{Ei}(dx)\,e^{c}\right)a}{d}\right)d+\left(bx+a\log\left(x\right)\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2) + 2*a*co
sh(d*x + c)*log(x)/d - (Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*a/d)*d + (b*x + a*lo
g(x))*cosh(d*x + c)
```

Fricas [A]
time = 0.35, size = 54, normalized size = 1.93

$$\frac{\left(ad\mathrm{Ei}(dx)+ad\mathrm{Ei}(-dx)\right)\cosh\left(c\right)+2\,b\sinh\left(dx+c\right)+\left(ad\mathrm{Ei}(dx)-ad\mathrm{Ei}(-dx)\right)\sinh\left(c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] 1/2*((a*d*Ei(d*x) + a*d*Ei(-d*x))*cosh(c) + 2*b*sinh(d*x + c) + (a*d*Ei(d*x
) - a*d*Ei(-d*x))*sinh(c))/d
```

Sympy [A]
time = 2.00, size = 34, normalized size = 1.21

$$a\sinh\left(c\right)\mathrm{Shi}\left(dx\right)+a\cosh\left(c\right)\mathrm{Chi}\left(dx\right)+b\left(\begin{cases}\frac{\sinh\left(c+dx\right)}{d} & \text{for}\: d\neq 0\\x\cosh\left(c\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x,x)
```

```
[Out] a*sinh(c)*Shi(d*x) + a*cosh(c)*Chi(d*x) + b*Piecewise((sinh(c + d*x)/d, Ne(
d, 0)), (x*cosh(c), True))
```

Giac [A]
time = 0.42, size = 47, normalized size = 1.68

$$\frac{ad\mathrm{Ei}(-dx)\,e^{(-c)}+ad\mathrm{Ei}(dx)\,e^{c}+be^{(dx+c)}-be^{(-dx-c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(a*d*Ei(-d*x)*e^(-c) + a*d*Ei(d*x)*e^c + b*e^(d*x + c) - b*e^(-d*x - c)
)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$a\,\mathrm{coshint}(d\,x)\,\cosh(c)+a\,\mathrm{sinhint}(d\,x)\,\sinh(c)+\frac{b\sinh(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x))/x,x)
```

```
[Out] a*coshint(d*x)*cosh(c) + a*sinhint(d*x)*sinh(c) + (b*sinh(c + d*x))/d
```

3.6 $\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx$

Optimal. Leaf size=47

$$-\frac{a\cosh(c+dx)}{x}+b\cosh(c)\mathrm{Chi}(dx)+ad\mathrm{Chi}(dx)\sinh(c)+ad\cosh(c)\mathrm{Shi}(dx)+b\sinh(c)\mathrm{Shi}(dx)$$

```
[Out] b*Chi(d*x)*cosh(c)-a*cosh(d*x+c)/x+a*d*cosh(c)*Shi(d*x)+a*d*Chi(d*x)*sinh(c
)+b*Shi(d*x)*sinh(c)
```

Rubi [A]
time = 0.17, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6874, 3378, 3384, 3379, 3382}

$$ad\sinh(c)\mathrm{Chi}(dx)+ad\cosh(c)\mathrm{Shi}(dx)-\frac{a\cosh(c+dx)}{x}+b\cosh(c)\mathrm{Chi}(dx)+b\sinh(c)\mathrm{Shi}(dx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a*Cosh[c + d*x])/x) + b*Cosh[c]*CoshIntegral[d*x] + a*d*CoshIntegral[d*x
]*Sinh[c] + a*d*Cosh[c]*SinhIntegral[d*x] + b*Sinh[c]*SinhIntegral[d*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx &= \int \left(\frac{a\cosh(c+dx)}{x^2}+\frac{b\cosh(c+dx)}{x}\right)dx\\ &= a\int \frac{\cosh(c+dx)}{x^2}\,dx+b\int \frac{\cosh(c+dx)}{x}\,dx\\ &= -\frac{a\cosh(c+dx)}{x}+(ad)\int \frac{\sinh(c+dx)}{x}\,dx+(b\cosh(c))\int \frac{\cosh(dx)}{x}\,dx+(\\ &= -\frac{a\cosh(c+dx)}{x}+b\cosh(c)\text{Chi}(dx)+b\sinh(c)\text{Shi}(dx)+(ad\cosh(c))\int \frac{\sin}{}\\ &= -\frac{a\cosh(c+dx)}{x}+b\cosh(c)\text{Chi}(dx)+ad\text{Chi}(dx)\sinh(c)+ad\cosh(c)\text{Shi}(d\end{aligned}$$

Mathematica [A]
time = 0.10, size = 59, normalized size = 1.26

$$-\frac{a\cosh(c)\cosh(dx)}{x}+b\cosh(c)\text{Chi}(dx)-\frac{a\sinh(c)\sinh(dx)}{x}+b\sinh(c)\text{Shi}(dx)+ad(\text{Chi}(dx)\sinh(c)+\cosh(c)\text{Shi}(dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x^2,x]
[Out] -((a*Cosh[c]*Cosh[d*x])/x) + b*Cosh[c]*CoshIntegral[d*x] - (a*Sinh[c]*Sinh[
d*x])/x + b*Sinh[c]*SinhIntegral[d*x] + a*d*(CoshIntegral[d*x]*Sinh[c] + Co
sh[c]*SinhIntegral[d*x])
```

Maple [A]
time = 0.85, size = 77, normalized size = 1.64

method	result
risch	$-\frac{a\,e^{-dx-c}}{2x}+\frac{da\,e^{-c}\,\text{expIntegral}(1,dx)}{2}-\frac{b\,e^{-c}\,\text{expIntegral}(1,dx)}{2}-\frac{a\,e^{dx+c}}{2x}-\frac{da\,e^{c}\,\text{expIntegral}(1,-dx)}{2}-\frac{b\,e^{c}\,\text{expIntegral}(}{2}$
meijerg	$\frac{b\cosh(c)\sqrt{\pi}\left(\frac{2\,\text{hyperbolicCosineIntegral}(dx)-2\ln(dx)-2\gamma}{\sqrt{\pi}}+\frac{2\gamma+2\ln(x)+2\ln(id)}{\sqrt{\pi}}\right)}{2}+b\,\text{hyperbolicSineIntegral}\,(dx)\sinh\,($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*cosh(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*exp(-d*x-c)/x+1/2*d*a*exp(-c)*Ei(1,d*x)-1/2*b*exp(-c)*Ei(1,d*x)-1/2*
a/x*exp(d*x+c)-1/2*d*a*exp(c)*Ei(1,-d*x)-1/2*b*exp(c)*Ei(1,-d*x)
```

Maxima [A]
time = 0.33, size = 82, normalized size = 1.74

$$-\frac{1}{2}\left(\left(\mathrm{Ei}(-dx)\,e^{(-c)}-\mathrm{Ei}(dx)\,e^{c}\right)a+\frac{2\,b\cosh\left(dx+c\right)\log\left(x\right)}{d}-\frac{\left(\mathrm{Ei}(-dx)\,e^{(-c)}+\mathrm{Ei}(dx)\,e^{c}\right)b}{d}\right)d+\left(b\log\left(x\right)-\frac{a}{x}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*((Ei(-d*x)*e^(-c) - Ei(d*x)*e^c)*a + 2*b*cosh(d*x + c)*log(x)/d - (Ei(
-d*x)*e^(-c) + Ei(d*x)*e^c)*b/d)*d + (b*log(x) - a/x)*cosh(d*x + c)
```

Fricas [A]
time = 0.35, size = 76, normalized size = 1.62

$$-\frac{2\,a\cosh\left(dx+c\right)-\left((ad+b)x\mathrm{Ei}(dx)-(ad-b)x\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-\left((ad+b)x\mathrm{Ei}(dx)+(ad-b)x\mathrm{Ei}(-dx)\right)\sinh\left(c\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*cosh(d*x + c) - ((a*d + b)*x*Ei(d*x) - (a*d - b)*x*Ei(-d*x))*cosh
(c) - ((a*d + b)*x*Ei(d*x) + (a*d - b)*x*Ei(-d*x))*sinh(c))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+bx)\cosh\left(c+dx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x)*cosh(c + d*x)/x**2, x)
```

Giac [A]
time = 0.44, size = 72, normalized size = 1.53

$$-\frac{adx\mathrm{Ei}(-dx)\,e^{(-c)}-adx\mathrm{Ei}(dx)\,e^{c}-bx\mathrm{Ei}(-dx)\,e^{(-c)}-bx\mathrm{Ei}(dx)\,e^{c}+ae^{(dx+c)}+ae^{(-dx-c)}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*d*x*Ei(-d*x)*e^(-c) - a*d*x*Ei(d*x)*e^c - b*x*Ei(-d*x)*e^(-c) - b*x
*Ei(d*x)*e^c + a*e^(d*x + c) + a*e^(-d*x - c))/x
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(c + d\,x)\,\,(a + b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x))/x^2,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x))/x^2, x)
```

3.7 $\int \frac{(a+bx)\cosh(c+dx)}{x^3}\, dx$

Optimal. Leaf size=88

$$-\frac{a\cosh(c+dx)}{2x^2}-\frac{b\cosh(c+dx)}{x}+\frac{1}{2}ad^2\cosh(c)\mathrm{Chi}(dx)+bd\mathrm{Chi}(dx)\sinh(c)-\frac{ad\sinh(c+dx)}{2x}+bd\cosh(c)\mathrm{Shi}($$

```
[Out] 1/2*a*d^2*Chi(d*x)*cosh(c)-1/2*a*cosh(d*x+c)/x^2-b*cosh(d*x+c)/x+b*d*cosh(c
)*Shi(d*x)+b*d*Chi(d*x)*sinh(c)+1/2*a*d^2*Shi(d*x)*sinh(c)-1/2*a*d*sinh(d*x
+c)/x
```

Rubi [A]
time = 0.20, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\frac{1}{2}ad^2\cosh(c)\mathrm{Chi}(dx)+\frac{1}{2}ad^2\sinh(c)\mathrm{Shi}(dx)-\frac{a\cosh(c+dx)}{2x^2}-\frac{ad\sinh(c+dx)}{2x}+bd\sinh(c)\mathrm{Chi}(dx)+bd\cosh(c)\mathrm{Shi}(dx)-\frac{b\cosh(c+dx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Cosh[c + d*x])/x^3,x]
```

```
[Out] -1/2*(a*Cosh[c + d*x])/x^2 - (b*Cosh[c + d*x])/x + (a*d^2*Cosh[c]*CoshInteg
ral[d*x])/2 + b*d*CoshIntegral[d*x]*Sinh[c] - (a*d*Sinh[c + d*x])/(2*x) + b
*d*Cosh[c]*SinhIntegral[d*x] + (a*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx = \int \left(\frac{a\cosh(c+dx)}{x^3}+\frac{b\cosh(c+dx)}{x^2}\right)dx$$
$$= a\int \frac{\cosh(c+dx)}{x^3}\,dx + b\int \frac{\cosh(c+dx)}{x^2}\,dx$$
$$= -\frac{a\cosh(c+dx)}{2x^2} - \frac{b\cosh(c+dx)}{x} + \frac{1}{2}(ad)\int \frac{\sinh(c+dx)}{x^2}\,dx + (bd)\int \frac{\sinh}{}$$
$$= -\frac{a\cosh(c+dx)}{2x^2} - \frac{b\cosh(c+dx)}{x} - \frac{ad\sinh(c+dx)}{2x} + \frac{1}{2}(ad^2)\int \frac{\cosh(c+}{x}$$
$$= -\frac{a\cosh(c+dx)}{2x^2} - \frac{b\cosh(c+dx)}{x} + bd\text{Chi}(dx)\sinh(c) - \frac{ad\sinh(c+dx)}{2x} +$$
$$= -\frac{a\cosh(c+dx)}{2x^2} - \frac{b\cosh(c+dx)}{x} + \frac{1}{2}ad^2\cosh(c)\text{Chi}(dx) + bd\text{Chi}(dx)\sinh($$

Mathematica [A]
time = 0.12, size = 78, normalized size = 0.89

$$\frac{-a\cosh(c+dx)-2bx\cosh(c+dx)+dx^2\text{Chi}(dx)(ad\cosh(c)+2b\sinh(c))-adx\sinh(c+dx)+dx^2(2b\cosh(c)+ad\sinh(c))\text{Shi}(dx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x^3,x]
```

```
[Out] (-(a*Cosh[c + d*x]) - 2*b*x*Cosh[c + d*x] + d*x^2*CoshIntegral[d*x]*(a*d*Co
sh[c] + 2*b*Sinh[c]) - a*d*x*Sinh[c + d*x] + d*x^2*(2*b*Cosh[c] + a*d*Sinh[
c])*SinhIntegral[d*x])/(2*x^2)
```

Maple [A]
time = 0.93, size = 139, normalized size = 1.58

method	result
risch	$\frac{da\,e^{-dx-c}}{4x}-\frac{a\,e^{-dx-c}}{4x^2}-\frac{d^2a\,e^{-c}\,\text{expIntegral}(1,dx)}{4}-\frac{b\,e^{-dx-c}}{2x}+\frac{db\,e^{-c}\,\text{expIntegral}(1,dx)}{2}-\frac{a\,e^{dx+c}}{4x^2}-\frac{da\,e^{dx+c}}{4x}-\frac{d^2a}{}$

meijerg	$\frac{idb\cosh(c)\sqrt{\pi}\left(\frac{4i\cosh(dx)}{dx\sqrt{\pi}}-\frac{4i\,\text{hyperbolicSineIntegral}(dx)}{\sqrt{\pi}}\right)}{4}+\frac{db\sinh(c)\sqrt{\pi}\left(\frac{4}{\sqrt{\pi}}-\frac{4\sinh(dx)}{\sqrt{\pi}\,dx}+\frac{4\,\text{hyperbolicCosineIntegral}(dx)-4\ln}{\sqrt{\pi}}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*cosh(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d*a*exp(-d*x-c)/x-1/4*a*exp(-d*x-c)/x^2-1/4*d^2*a*exp(-c)*Ei(1,d*x)-1/2
*b*exp(-d*x-c)/x+1/2*d*b*exp(-c)*Ei(1,d*x)-1/4*a/x^2*exp(d*x+c)-1/4*d*a/x*e
xp(d*x+c)-1/4*d^2*a*exp(c)*Ei(1,-d*x)-1/2*b/x*exp(d*x+c)-1/2*d*b*exp(c)*Ei(
1,-d*x)
```

Maxima [A]
time = 0.35, size = 66, normalized size = 0.75

$$\frac{1}{4}\left(ade^{(-c)}\Gamma(-1,dx)+ade^{c}\Gamma(-1,-dx)-2\,b\text{Ei}(-dx)\,e^{(-c)}+2\,b\text{Ei}(dx)\,e^{c}\right)d-\frac{(2\,bx+a)\cosh\left(dx+c\right)}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a*d*e^(-c)*gamma(-1, d*x) + a*d*e^c*gamma(-1, -d*x) - 2*b*Ei(-d*x)*e^(
-c) + 2*b*Ei(d*x)*e^c)*d - 1/2*(2*b*x + a)*cosh(d*x + c)/x^2
```

Fricas [A]
time = 0.49, size = 116, normalized size = 1.32

$$-\frac{2\,adx\sinh\left(dx+c\right)+2\left(2\,bx+a\right)\cosh\left(dx+c\right)-\left((ad^{2}+2\,bd)x^{2}\text{Ei}(dx)+(ad^{2}-2\,bd)x^{2}\text{Ei}(-dx)\right)\cosh\left(c\right)-\left((ad^{2}+2\,bd)x^{2}\text{Ei}(dx)-(ad^{2}-2\,bd)x^{2}\text{Ei}(-dx)\right)\sinh\left(c\right)}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d*x*sinh(d*x + c) + 2*(2*b*x + a)*cosh(d*x + c) - ((a*d^2 + 2*b*d
)*x^2*Ei(d*x) + (a*d^2 - 2*b*d)*x^2*Ei(-d*x))*cosh(c) - ((a*d^2 + 2*b*d)*x^
2*Ei(d*x) - (a*d^2 - 2*b*d)*x^2*Ei(-d*x))*sinh(c))/x^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 134, normalized size = 1.52

$$\frac{ad^2x^2\text{Ei}(-dx)\,e^{(-c)}+ad^2x^2\text{Ei}(dx)\,e^c-2\,bdx^2\text{Ei}(-dx)\,e^{(-c)}+2\,bdx^2\text{Ei}(dx)\,e^c-adxe^{(dx+c)}+adxe^{(-dx-c)}-2\,bxe^{(dx+c)}-2\,bxe^{(-dx-c)}-ae^{(dx+c)}-ae^{(-dx-c)}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^2*x^2*Ei(-d*x)*e^(-c) + a*d^2*x^2*Ei(d*x)*e^c - 2*b*d*x^2*Ei(-d*x)
*e^(-c) + 2*b*d*x^2*Ei(d*x)*e^c - a*d*x*e^(d*x + c) + a*d*x*e^(-d*x - c) -
2*b*x*e^(d*x + c) - 2*b*x*e^(-d*x - c) - a*e^(d*x + c) - a*e^(-d*x - c))/x^
2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x))/x^3,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x))/x^3, x)
```

3.8 $\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx$

Optimal. Leaf size=132

$$-\frac{a\cosh(c+dx)}{3x^3}-\frac{b\cosh(c+dx)}{2x^2}-\frac{ad^2\cosh(c+dx)}{6x}+\frac{1}{2}bd^2\cosh(c)\mathrm{Chi}(dx)+\frac{1}{6}ad^3\mathrm{Chi}(dx)\sinh(c)-\frac{ad\sinh(c}{6x^2}$$

```
[Out] 1/2*b*d^2*Chi(d*x)*cosh(c)-1/3*a*cosh(d*x+c)/x^3-1/2*b*cosh(d*x+c)/x^2-1/6*
a*d^2*cosh(d*x+c)/x+1/6*a*d^3*cosh(c)*Shi(d*x)+1/6*a*d^3*Chi(d*x)*sinh(c)+1
/2*b*d^2*Shi(d*x)*sinh(c)-1/6*a*d*sinh(d*x+c)/x^2-1/2*b*d*sinh(d*x+c)/x
```

Rubi [A]
time = 0.23, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,
Rules used = {6874, 3378, 3384, 3379, 3382}

$$\frac{1}{6}ad^3\sinh(c)\mathrm{Chi}(dx)+\frac{1}{6}ad^3\cosh(c)\mathrm{Shi}(dx)-\frac{ad^2\cosh(c+dx)}{6x}-\frac{a\cosh(c+dx)}{3x^3}-\frac{ad\sinh(c+dx)}{6x^2}+\frac{1}{2}bd^2\cosh(c)\mathrm{Chi}(dx)+\frac{1}{2}bd^2\sinh(c)\mathrm{Shi}(dx)-\frac{b\cosh(c+dx)}{2x^2}-\frac{bd\sinh(c+dx)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Cosh[c + d*x])/x^4,x]
```

```
[Out] -1/3*(a*Cosh[c + d*x])/x^3 - (b*Cosh[c + d*x])/(2*x^2) - (a*d^2*Cosh[c + d*
x])/(6*x) + (b*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (a*d^3*CoshIntegral[d*x]*
Sinh[c])/6 - (a*d*Sinh[c + d*x])/(6*x^2) - (b*d*Sinh[c + d*x])/(2*x) + (a*d
^3*Cosh[c]*SinhIntegral[d*x])/6 + (b*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx &= \int \left(\frac{a\cosh(c+dx)}{x^4}+\frac{b\cosh(c+dx)}{x^3}\right)dx \\
&= a\int \frac{\cosh(c+dx)}{x^4}\,dx+b\int \frac{\cosh(c+dx)}{x^3}\,dx \\
&= -\frac{a\cosh(c+dx)}{3x^3}-\frac{b\cosh(c+dx)}{2x^2}+\frac{1}{3}(ad)\int \frac{\sinh(c+dx)}{x^3}\,dx+\frac{1}{2}(bd)\int \frac{\mathrm{si}}{} \\
&= -\frac{a\cosh(c+dx)}{3x^3}-\frac{b\cosh(c+dx)}{2x^2}-\frac{ad\sinh(c+dx)}{6x^2}-\frac{bd\sinh(c+dx)}{2x}+\frac{1}{6} \\
&= -\frac{a\cosh(c+dx)}{3x^3}-\frac{b\cosh(c+dx)}{2x^2}-\frac{ad^2\cosh(c+dx)}{6x}-\frac{ad\sinh(c+dx)}{6x^2}- \\
&= -\frac{a\cosh(c+dx)}{3x^3}-\frac{b\cosh(c+dx)}{2x^2}-\frac{ad^2\cosh(c+dx)}{6x}+\frac{1}{2}bd^2\cosh(c)\mathrm{Chi}(dx \\
&= -\frac{a\cosh(c+dx)}{3x^3}-\frac{b\cosh(c+dx)}{2x^2}-\frac{ad^2\cosh(c+dx)}{6x}+\frac{1}{2}bd^2\cosh(c)\mathrm{Chi}(dx
\end{aligned}
$$

Mathematica [A]
time = 0.21, size = 110, normalized size = 0.83

$$
-\frac{2a\cosh(c+dx)+3bx\cosh(c+dx)+ad^2x^2\cosh(c+dx)-d^2x^3\mathrm{Chi}(dx)(3b\cosh(c)+ad\sinh(c))+adx\sinh(c+dx)+3bdx^2\sinh(c+dx)-d^2x^3(ad\cosh(c)+3b\sinh(c))\mathrm{Shi}(dx)}{6x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x^4,x]
```

```
[Out] -1/6*(2*a*Cosh[c + d*x] + 3*b*x*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x] - d
^2*x^3*CoshIntegral[d*x]*(3*b*Cosh[c] + a*d*Sinh[c]) + a*d*x*Sinh[c + d*x]
+ 3*b*d*x^2*Sinh[c + d*x] - d^2*x^3*(a*d*Cosh[c] + 3*b*Sinh[c])*SinhIntegra
l[d*x])/x^3
```

Maple [A]
time = 1.00, size = 205, normalized size = 1.55

method	result
risch	$-\frac{d^{2} a \,{\mathrm e}^{-d x -c}}{12 x}+\frac{d a \,{\mathrm e}^{-d x -c}}{12 x^{2}}-\frac{a \,{\mathrm e}^{-d x -c}}{6 x^{3}}+\frac{d^{3} a \,{\mathrm e}^{-c} \operatorname{expIntegral}(1,d x)}{12}+\frac{d b \,{\mathrm e}^{-d x -c}}{4 x}-\frac{b \,{\mathrm e}^{-d x -c}}{4 x^{2}}-\frac{d^{2} b \,{\mathrm e}^{-c} \operatorname{expIntegral}(1,d x)}{4}$
meijerg	$-\frac{d^{2} b \cosh(c) \sqrt{\pi}\, \left(-\frac{4 \left(\frac{9 d^{2} x^{2}}{2}+3\right)}{3 \sqrt{\pi}\, d^{2} x^{2}}+\frac{4 \cosh(d x)}{\sqrt{\pi}\, d^{2} x^{2}}+\frac{4 \sinh(d x)}{\sqrt{\pi}\, d x}-\frac{4 (\operatorname{hyperbolicCosineIntegral}(d x)-\ln(d x)-\gamma)}{\sqrt{\pi}}-\frac{2 (2 \gamma -3+2 \ln(x)+2 \ln(i d))}{\sqrt{\pi}}+\frac{}{\sqrt{?}}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*cosh(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*d^2*a*exp(-d*x-c)/x+1/12*d*a*exp(-d*x-c)/x^2-1/6*a*exp(-d*x-c)/x^3+1/
12*d^3*a*exp(-c)*Ei(1,d*x)+1/4*d*b*exp(-d*x-c)/x-1/4*b*exp(-d*x-c)/x^2-1/4*
d^2*b*exp(-c)*Ei(1,d*x)-1/6*a/x^3*exp(d*x+c)-1/12*d*a/x^2*exp(d*x+c)-1/12*d
^2*a/x*exp(d*x+c)-1/12*d^3*a*exp(c)*Ei(1,-d*x)-1/4*b/x^2*exp(d*x+c)-1/4*d*b
/x*exp(d*x+c)-1/4*d^2*b*exp(c)*Ei(1,-d*x)
```

Maxima [A]
time = 0.36, size = 78, normalized size = 0.59

$$\frac{1}{12} \, {\left(2 \, a d^{2} e^{\left(-c\right)} \Gamma\left(-2, d x\right) - 2 \, a d^{2} e^{c} \Gamma\left(-2, -d x\right) + 3 \, b d e^{\left(-c\right)} \Gamma\left(-1, d x\right) + 3 \, b d e^{c} \Gamma\left(-1, -d x\right)\right)} d - \frac{{\left(3 \, b x + 2 \, a\right)} \cosh\left(d x + c\right)}{6 \, x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/12*(2*a*d^2*e^(-c)*gamma(-2, d*x) - 2*a*d^2*e^c*gamma(-2, -d*x) + 3*b*d*e
^(-c)*gamma(-1, d*x) + 3*b*d*e^c*gamma(-1, -d*x))*d - 1/6*(3*b*x + 2*a)*cos
h(d*x + c)/x^3
```

Fricas [A]
time = 0.34, size = 143, normalized size = 1.08

$$-\frac{2 \, {\left(a d^{2} x^{2} + 3 \, b x + 2 \, a\right)} \cosh\left(d x + c\right) - {\left({\left(a d^{3} + 3 \, b d^{2}\right)} x^{3} {\rm Ei}\left(d x\right) - {\left(a d^{3} - 3 \, b d^{2}\right)} x^{3} {\rm Ei}\left(-d x\right)\right)} \cosh\left(c\right) + 2 \, {\left(3 \, b d x^{2} + a d x\right)} \sinh\left(d x + c\right) - {\left({\left(a d^{3} + 3 \, b d^{2}\right)} x^{3} {\rm Ei}\left(d x\right) + {\left(a d^{3} - 3 \, b d^{2}\right)} x^{3} {\rm Ei}\left(-d x\right)\right)} \sinh\left(c\right)}{12 \, x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(a*d^2*x^2 + 3*b*x + 2*a)*cosh(d*x + c) - ((a*d^3 + 3*b*d^2)*x^3*E
i(d*x) - (a*d^3 - 3*b*d^2)*x^3*Ei(-d*x))*cosh(c) + 2*(3*b*d*x^2 + a*d*x)*si
nh(d*x + c) - ((a*d^3 + 3*b*d^2)*x^3*Ei(d*x) + (a*d^3 - 3*b*d^2)*x^3*Ei(-d*
x))*sinh(c))/x^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 199, normalized size = 1.51

$$-\frac{ad^3x^3\mathrm{Ei}(-dx)\,e^{(-c)} - ad^3x^3\mathrm{Ei}(dx)\,e^c - 3\,bd^2x^3\mathrm{Ei}(-dx)\,e^{(-c)} - 3\,bd^2x^3\mathrm{Ei}(dx)\,e^c + ad^2x^2e^{(dx+c)} + ad^2x^2e^{(-dx-c)} + 3\,bdx^2e^{(dx+c)} - 3\,bdx^2e^{(-dx-c)} + adxe^{(dx+c)} - adxe^{(-dx-c)} + 3\,bxe^{(dx+c)} + 3\,bxe^{(-dx-c)} + 2\,ae^{(dx+c)} + 2\,ae^{(-dx-c)}}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(a*d^3*x^3*Ei(-d*x)*e^(-c) - a*d^3*x^3*Ei(d*x)*e^c - 3*b*d^2*x^3*Ei(-
d*x)*e^(-c) - 3*b*d^2*x^3*Ei(d*x)*e^c + a*d^2*x^2*e^(d*x + c) + a*d^2*x^2*e
^(-d*x - c) + 3*b*d*x^2*e^(d*x + c) - 3*b*d*x^2*e^(-d*x - c) + a*d*x*e^(d*x
 + c) - a*d*x*e^(-d*x - c) + 3*b*x*e^(d*x + c) + 3*b*x*e^(-d*x - c) + 2*a*e
^(d*x + c) + 2*a*e^(-d*x - c))/x^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x))/x^4,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x))/x^4, x)
```

3.9 $\int \frac{(a+bx)\cosh(c+dx)}{x^5} dx$

Optimal. Leaf size=166

$$-\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{bd^2\cosh(c+dx)}{6x}+\frac{1}{24}ad^4\cosh(c)\text{Chi}(dx)+\frac{1}{6}bd^3\text{Chi}(dx$$

```
[Out] 1/24*a*d^4*Chi(d*x)*cosh(c)-1/4*a*cosh(d*x+c)/x^4-1/3*b*cosh(d*x+c)/x^3-1/2
4*a*d^2*cosh(d*x+c)/x^2-1/6*b*d^2*cosh(d*x+c)/x+1/6*b*d^3*cosh(c)*Shi(d*x)+
1/6*b*d^3*Chi(d*x)*sinh(c)+1/24*a*d^4*Shi(d*x)*sinh(c)-1/12*a*d*sinh(d*x+c)
/x^3-1/6*b*d*sinh(d*x+c)/x^2-1/24*a*d^3*sinh(d*x+c)/x
```

Rubi [A]
time = 0.28, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\frac{1}{24}ad^4\cosh(c)\text{Chi}(dx)+\frac{1}{24}ad^4\sinh(c)\text{Shi}(dx)-\frac{ad^3\sinh(c+dx)}{24x}-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{a\cosh(c+dx)}{4x^4}-\frac{ad\sinh(c+dx)}{12x^3}+\frac{1}{6}bd^3\sinh(c)\text{Chi}(dx)+\frac{1}{6}bd^3\cosh(c)\text{Shi}(dx)-\frac{bd^2\cosh(c+dx)}{6x}-\frac{b\cosh(c+dx)}{3x^3}-\frac{bd\sinh(c+dx)}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Cosh[c + d*x])/x^5,x]
```

```
[Out] -1/4*(a*Cosh[c + d*x])/x^4 - (b*Cosh[c + d*x])/(3*x^3) - (a*d^2*Cosh[c + d*
x])/(24*x^2) - (b*d^2*Cosh[c + d*x])/(6*x) + (a*d^4*Cosh[c]*CoshIntegral[d*
x])/24 + (b*d^3*CoshIntegral[d*x]*Sinh[c])/6 - (a*d*Sinh[c + d*x])/(12*x^3)
 - (b*d*Sinh[c + d*x])/(6*x^2) - (a*d^3*Sinh[c + d*x])/(24*x) + (b*d^3*Cosh
[c]*SinhIntegral[d*x])/6 + (a*d^4*Sinh[c]*SinhIntegral[d*x])/24
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx &= \int \left(\frac{a\cosh(c+dx)}{x^5}+\frac{b\cosh(c+dx)}{x^4}\right)dx \\
&= a\int \frac{\cosh(c+dx)}{x^5}\,dx+b\int \frac{\cosh(c+dx)}{x^4}\,dx \\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{3x^3}+\frac{1}{4}(ad)\int \frac{\sinh(c+dx)}{x^4}\,dx+\frac{1}{3}(bd)\int \frac{\mathrm{si}}{} \\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{3x^3}-\frac{ad\sinh(c+dx)}{12x^3}-\frac{bd\sinh(c+dx)}{6x^2}+\frac{1}{12} \\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{bd^2\cosh(c+dx)}{6x}- \\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{bd^2\cosh(c+dx)}{6x}- \\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{bd^2\cosh(c+dx)}{6x}+ \\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{bd^2\cosh(c+dx)}{6x}+
\end{aligned}$$

Mathematica [A]
time = 0.25, size = 140, normalized size = 0.84

$$-\frac{6a\cosh(c+dx)+8bx\cosh(c+dx)+ad^2x^2\cosh(c+dx)+4bd^2x^3\cosh(c+dx)-d^3x^4\mathrm{Chi}(dx)(ad\cosh(c)+4b\sinh(c))+2adx\sinh(c+dx)+4bdx^2\sinh(c+dx)+ad^3x^3\sinh(c+dx)-d^3x^4(4b\cosh(c)+ad\sinh(c))\mathrm{Shi}(dx)}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x^5,x]
```

```
[Out] -1/24*(6*a*Cosh[c + d*x] + 8*b*x*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x] +
4*b*d^2*x^3*Cosh[c + d*x] - d^3*x^4*CoshIntegral[d*x]*(a*d*Cosh[c] + 4*b*Si
```

```
nh[c]) + 2*a*d*x*Sinh[c + d*x] + 4*b*d*x^2*Sinh[c + d*x] + a*d^3*x^3*Sinh[c
 + d*x] - d^3*x^4*(4*b*Cosh[c] + a*d*Sinh[c])*SinhIntegral[d*x])/x^4
```

Maple [A]
time = 1.02, size = 271, normalized size = 1.63

method	result
risch	$\frac{d^{3} a\,{\mathrm e}^{-d x -c}}{48 x}-\frac{d^{2} a\,{\mathrm e}^{-d x -c}}{48 x^{2}}+\frac{d a\,{\mathrm e}^{-d x -c}}{24 x^{3}}-\frac{a\,{\mathrm e}^{-d x -c}}{8 x^{4}}-\frac{d^{4} a\,{\mathrm e}^{-c} \operatorname{expIntegral}(1, d x)}{48}-\frac{d^{2} b\,{\mathrm e}^{-d x -c}}{12 x}+\frac{d b\,{\mathrm e}^{-d x -c}}{12 x^{2}}-\frac{b\,{\mathrm e}^{-d x -c}}{6 x^{3}} -$
meijerg	$-\frac{i d^{3} b \cosh(c) \sqrt{\pi}\, \left(-\frac{8 i \left(d^{2} x^{2}+2\right) \cosh(d x)}{3 d^{3} x^{3} \sqrt{\pi}}-\frac{8 i \sinh(d x)}{3 d^{2} x^{2} \sqrt{\pi}}+\frac{8 i \operatorname{hyperbolicSineIntegral}(d x)}{3 \sqrt{\pi}}\right)}{16}-\frac{d^{3} b \sinh(c) \sqrt{\pi}\, \left(-\frac{8 \left(\frac{55 d^{2} x^{2}}{2}+45\right)}{45 \sqrt{\pi}\, d^{2} x^{2}}+\frac{8}{3}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*cosh(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*d^3*a*exp(-d*x-c)/x-1/48*d^2*a*exp(-d*x-c)/x^2+1/24*d*a*exp(-d*x-c)/x^
3-1/8*a*exp(-d*x-c)/x^4-1/48*d^4*a*exp(-c)*Ei(1,d*x)-1/12*d^2*b*exp(-d*x-c)
/x+1/12*d*b*exp(-d*x-c)/x^2-1/6*b*exp(-d*x-c)/x^3+1/12*d^3*b*exp(-c)*Ei(1,d
*x)-1/8*a/x^4*exp(d*x+c)-1/24*d*a/x^3*exp(d*x+c)-1/48*d^2*a/x^2*exp(d*x+c)-
1/48*d^3*a/x*exp(d*x+c)-1/48*d^4*a*exp(c)*Ei(1,-d*x)-1/6*b/x^3*exp(d*x+c)-1
/12*d*b/x^2*exp(d*x+c)-1/12*d^2*b/x*exp(d*x+c)-1/12*d^3*b*exp(c)*Ei(1,-d*x)
```

Maxima [A]
time = 0.34, size = 82, normalized size = 0.49

$$\frac{1}{24} \left(3\, a d^{3} e^{(-c)} \Gamma\left(-3, d x\right) + 3\, a d^{3} e^{c} \Gamma\left(-3, -d x\right) + 4\, b d^{2} e^{(-c)} \Gamma\left(-2, d x\right) - 4\, b d^{2} e^{c} \Gamma\left(-2, -d x\right)\right) d - \frac{\left(4\, b x + 3\, a\right) \cosh\left(d x + c\right)}{12\, x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] 1/24*(3*a*d^3*e^(-c)*gamma(-3, d*x) + 3*a*d^3*e^c*gamma(-3, -d*x) + 4*b*d^2
*e^(-c)*gamma(-2, d*x) - 4*b*d^2*e^c*gamma(-2, -d*x))*d - 1/12*(4*b*x + 3*a
)*cosh(d*x + c)/x^4
```

Fricas [A]
time = 0.40, size = 161, normalized size = 0.97

$$-\frac{2 \, {\left(4 \, b d^{2} x^{3} + a d^{2} x^{2} + 8 \, b x + 6 \, a\right)} \cosh\left(d x + c\right) - {\left({\left(a d^{4} + 4 \, b d^{3}\right)} x^{4} {\rm Ei}\left(d x\right) + {\left(a d^{4} - 4 \, b d^{3}\right)} x^{4} {\rm Ei}\left(-d x\right)\right)} \cosh\left(c\right) + 2 \, {\left(a d^{3} x^{3} + 4 \, b d x^{2} + 2 \, a d x\right)} \sinh\left(d x + c\right) - {\left({\left(a d^{4} + 4 \, b d^{3}\right)} x^{4} {\rm Ei}\left(d x\right) - {\left(a d^{4} - 4 \, b d^{3}\right)} x^{4} {\rm Ei}\left(-d x\right)\right)} \sinh\left(c\right)}{48 \, x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(2*(4*b*d^2*x^3 + a*d^2*x^2 + 8*b*x + 6*a)*cosh(d*x + c) - ((a*d^4 +
4*b*d^3)*x^4*Ei(d*x) + (a*d^4 - 4*b*d^3)*x^4*Ei(-d*x))*cosh(c) + 2*(a*d^3*x
```

```
^3 + 4*b*d*x^2 + 2*a*d*x)*sinh(d*x + c) - ((a*d^4 + 4*b*d^3)*x^4*Ei(d*x) -
(a*d^4 - 4*b*d^3)*x^4*Ei(-d*x))*sinh(c))/x^4
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x**5,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 266, normalized size = 1.60

$$\frac{ad^4x^4\mathrm{Ei}(-dx)\,e^{(-c)} + ad^4x^4\mathrm{Ei}(dx)\,e^c - 4\,bd^3x^4\mathrm{Ei}(-dx)\,e^{(-c)} + 4\,bd^3x^4\mathrm{Ei}(dx)\,e^c - ad^3x^3e^{(dx+c)} + ad^3x^3e^{(-dx-c)} - 4\,bd^2x^3e^{(dx+c)} - 4\,bd^2x^3e^{(-dx-c)} - ad^2x^2e^{(dx+c)} - ad^2x^2e^{(-dx-c)} - 4\,bdx^2e^{(dx+c)} + 4\,bdx^2e^{(-dx-c)} - 2\,adxe^{(dx+c)} + 2\,adxe^{(-dx-c)} - 8\,bxe^{(dx+c)} - 8\,bxe^{(-dx-c)} - 6\,ae^{(dx+c)} - 6\,ae^{(-dx-c)}}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] 1/48*(a*d^4*x^4*Ei(-d*x)*e^(-c) + a*d^4*x^4*Ei(d*x)*e^c - 4*b*d^3*x^4*Ei(-d
*x)*e^(-c) + 4*b*d^3*x^4*Ei(d*x)*e^c - a*d^3*x^3*e^(d*x + c) + a*d^3*x^3*e^
(-d*x - c) - 4*b*d^2*x^3*e^(d*x + c) - 4*b*d^2*x^3*e^(-d*x - c) - a*d^2*x^2
*e^(d*x + c) - a*d^2*x^2*e^(-d*x - c) - 4*b*d*x^2*e^(d*x + c) + 4*b*d*x^2*e
^(-d*x - c) - 2*a*d*x*e^(d*x + c) + 2*a*d*x*e^(-d*x - c) - 8*b*x*e^(d*x + c
) - 8*b*x*e^(-d*x - c) - 6*a*e^(d*x + c) - 6*a*e^(-d*x - c))/x^4
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)}{x^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x))/x^5,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x))/x^5, x)
```

3.10 $\int x^2(a+bx)^2\cosh(c+dx)\,dx$

Optimal. Leaf size=184

$$-\frac{12ab\cosh(c+dx)}{d^4}-\frac{24b^2x\cosh(c+dx)}{d^4}-\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+\frac{24b}{}$$

```
[Out] -12*a*b*cosh(d*x+c)/d^4-24*b^2*x*cosh(d*x+c)/d^4-2*a^2*x*cosh(d*x+c)/d^2-6*
a*b*x^2*cosh(d*x+c)/d^2-4*b^2*x^3*cosh(d*x+c)/d^2+24*b^2*sinh(d*x+c)/d^5+2*
a^2*sinh(d*x+c)/d^3+12*a*b*x*sinh(d*x+c)/d^3+12*b^2*x^2*sinh(d*x+c)/d^3+a^2
*x^2*sinh(d*x+c)/d+2*a*b*x^3*sinh(d*x+c)/d+b^2*x^4*sinh(d*x+c)/d
```

Rubi [A]
time = 0.26, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {6874, 3377, 2717, 2718}

$$\frac{2a^2\sinh(c+dx)}{d^3}-\frac{2a^2x\cosh(c+dx)}{d^2}+\frac{a^2x^2\sinh(c+dx)}{d}-\frac{12ab\cosh(c+dx)}{d^4}+\frac{12abx\sinh(c+dx)}{d^3}-\frac{6abx^2\cosh(c+dx)}{d^2}+\frac{2abx^3\sinh(c+dx)}{d}+\frac{24b^2\sinh(c+dx)}{d^5}-\frac{24b^2x\cosh(c+dx)}{d^4}+\frac{12b^2x^2\sinh(c+dx)}{d^3}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+\frac{b^2x^4\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^2*Cosh[c + d*x],x]
```

```
[Out] (-12*a*b*Cosh[c + d*x])/d^4 - (24*b^2*x*Cosh[c + d*x])/d^4 - (2*a^2*x*Cosh[
c + d*x])/d^2 - (6*a*b*x^2*Cosh[c + d*x])/d^2 - (4*b^2*x^3*Cosh[c + d*x])/d
^2 + (24*b^2*Sinh[c + d*x])/d^5 + (2*a^2*Sinh[c + d*x])/d^3 + (12*a*b*x*Sin
h[c + d*x])/d^3 + (12*b^2*x^2*Sinh[c + d*x])/d^3 + (a^2*x^2*Sinh[c + d*x])/
d + (2*a*b*x^3*Sinh[c + d*x])/d + (b^2*x^4*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx = \int \left(a^2x^2\cosh(c+dx)+2abx^3\cosh(c+dx)+b^2x^4\cosh(c+dx)\right)\,dx$$
$$= a^2\int x^2\cosh(c+dx)\,dx+(2ab)\int x^3\cosh(c+dx)\,dx+b^2\int x^4\cosh(c+$$
$$= \frac{a^2x^2\sinh(c+dx)}{d}+\frac{2abx^3\sinh(c+dx)}{d}+\frac{b^2x^4\sinh(c+dx)}{d}-\frac{(2a^2)\int x\,\mathrm{s}}{}$$
$$= -\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+\frac{a^2x^2\,\mathrm{s}}{}$$
$$= -\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+\frac{2a^2\,\mathrm{sir}}{}$$
$$= -\frac{12ab\cosh(c+dx)}{d^4}-\frac{24b^2x\cosh(c+dx)}{d^4}-\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{6abx^2\,\mathrm{c}}{}$$
$$= -\frac{12ab\cosh(c+dx)}{d^4}-\frac{24b^2x\cosh(c+dx)}{d^4}-\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{6abx^2\,\mathrm{c}}{}$$

Mathematica [A]
time = 0.16, size = 100, normalized size = 0.54

$$\frac{-2d(a+2bx)\left(ad^2x+b(6+d^2x^2)\right)\cosh(c+dx)+\left(a^2d^2(2+d^2x^2)+2abd^2x(6+d^2x^2)+b^2(24+12d^2x^2+d^4x^4)\right)\sinh(c+dx)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^2*Cosh[c + d*x],x]
```

```
[Out] (-2*d*(a + 2*b*x)*(a*d^2*x + b*(6 + d^2*x^2))*Cosh[c + d*x] + (a^2*d^2*(2 +
 d^2*x^2) + 2*a*b*d^2*x*(6 + d^2*x^2) + b^2*(24 + 12*d^2*x^2 + d^4*x^4))*Si
nh[c + d*x])/d^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(184) = 368$.
time = 0.62, size = 463, normalized size = 2.52

method	result
risch	$\frac{\left(x^4b^2d^4+2ab\,d^4x^3+d^4a^2x^2-4b^2d^3x^3-6ab\,d^3x^2-2a^2d^3x+12b^2d^2x^2+12ab\,d^2x+2a^2d^2-24b^2dx-12bda+24b^2\right)e^{dx+c}}{2d^5}$ −
derivativedivides	$\frac{6b^2c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}-\frac{4b^2c\left((dx+c)^3\sinh(dx+c)-3(dx+c)^2\cosh(dx+c)+6(dx+c)\sinh(d\right.}{d^2}$

default	$\frac{6b^2c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}-\frac{4b^2c\left((dx+c)^3\sinh(dx+c)-3(dx+c)^2\cosh(dx+c)+6(dx+c)\sinh(dx+}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(6*b^2/d^2*c^2*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*
x+c))-4*b^2/d^2*c*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*
sinh(d*x+c)-6*cosh(d*x+c))+b^2/d^2*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(
d*x+c)+12*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))-4*b^
2/d^2*c^3*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+6*b/d*a*c^2*((d*x+c)*sinh(d*x+c
)-cosh(d*x+c))-6*b/d*c*a*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sin
h(d*x+c))+2*b/d*a*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*
sinh(d*x+c)-6*cosh(d*x+c))+b^2/d^2*c^4*sinh(d*x+c)-2*b/d*c^3*a*sinh(d*x+c)+
a^2*c^2*sinh(d*x+c)-2*a^2*c*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+a^2*((d*x+c)^
2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c)))
```

Maxima [A]
time = 0.29, size = 329, normalized size = 1.79

$$-\frac{1}{60}\,d\left(\frac{10\,(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c)a^2e^{(dx)}}{d^4}+\frac{10\,(d^3x^3+3\,d^2x^2+6\,dx+6)a^2e^{(-dx-c)}}{d^4}+\frac{15\,(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2e^c-24\,dxe^c+24\,e^c)abe^{(dx)}}{d^5}+\frac{15\,(d^4x^4+4\,d^3x^3+12\,d^2x^2+24\,dx+24)abe^{(-dx-c)}}{d^5}+\frac{6\,(d^5x^5e^c-5\,d^4x^4e^c+20\,d^3x^3e^c-60\,d^2x^2e^c+120\,dxe^c-120\,e^c)b^2e^{(dx)}}{d^6}+\frac{6\,(d^5x^5+5\,d^4x^4+20\,d^3x^3+60\,d^2x^2+120\,dx+120)b^2e^{(-dx-c)}}{d^6}\right)+\frac{1}{30}\,(6\,b^2x^5+15\,abx^4+10\,a^2x^3)\cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/60*d*(10*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a^2*e^(d*x)/d
^4 + 10*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a^2*e^(-d*x - c)/d^4 + 15*(d^4*x^
4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*a*b*e^(d*x)/d
^5 + 15*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a*b*e^(-d*x - c)/d
^5 + 6*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120
*d*x*e^c - 120*e^c)*b^2*e^(d*x)/d^6 + 6*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 +
 60*d^2*x^2 + 120*d*x + 120)*b^2*e^(-d*x - c)/d^6) + 1/30*(6*b^2*x^5 + 15*a
*b*x^4 + 10*a^2*x^3)*cosh(d*x + c)
```

Fricas [A]
time = 0.40, size = 127, normalized size = 0.69

$$-\frac{2\,(2\,b^2d^3x^3+3\,abd^3x^2+6\,abd+(a^2d^3+12\,b^2d)x)\cosh(dx+c)-(b^2d^4x^4+2\,abd^4x^3+12\,abd^2x+2\,a^2d^2+(a^2d^4+12\,b^2d^2)x^2+24\,b^2)\sinh(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(2*(2*b^2*d^3*x^3 + 3*a*b*d^3*x^2 + 6*a*b*d + (a^2*d^3 + 12*b^2*d)*x)*cosh
(d*x + c) - (b^2*d^4*x^4 + 2*a*b*d^4*x^3 + 12*a*b*d^2*x + 2*a^2*d^2 + (a^2*
d^4 + 12*b^2*d^2)*x^2 + 24*b^2)*sinh(d*x + c))/d^5
```

Sympy [A]
time = 0.37, size = 228, normalized size = 1.24

$$\begin{cases} \frac{a^2x^2\sinh(c+dx)}{d} - \frac{2a^2x\cosh(c+dx)}{d^2} + \frac{2a^2\sinh(c+dx)}{d^3} + \frac{2abx^3\sinh(c+dx)}{d} - \frac{6abx^2\cosh(c+dx)}{d^2} + \frac{12abx\sinh(c+dx)}{d^3} - \frac{12ab\cosh(c+dx)}{d^4} + \frac{b^2x^4\sinh(c+dx)}{d} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + \frac{12b^2x^2\sinh(c+dx)}{d^3} - \frac{24b^2x\cosh(c+dx)}{d^4} + \frac{24b^2\sinh(c+dx)}{d^5} & \text{for } d \neq 0 \\ \left(\frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{b^2x^5}{5}\right)\cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*x**2*sinh(c + d*x)/d - 2*a**2*x*cosh(c + d*x)/d**2 + 2*a**2
*sinh(c + d*x)/d**3 + 2*a*b*x**3*sinh(c + d*x)/d - 6*a*b*x**2*cosh(c + d*x)
/d**2 + 12*a*b*x*sinh(c + d*x)/d**3 - 12*a*b*cosh(c + d*x)/d**4 + b**2*x**4
*sinh(c + d*x)/d - 4*b**2*x**3*cosh(c + d*x)/d**2 + 12*b**2*x**2*sinh(c + d
*x)/d**3 - 24*b**2*x*cosh(c + d*x)/d**4 + 24*b**2*sinh(c + d*x)/d**5, Ne(d,
 0)), ((a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5)*cosh(c), True))
```

Giac [A]
time = 0.42, size = 236, normalized size = 1.28

$$\frac{(b^2d^4x^4 + 2\,abd^4x^3 + a^2d^4x^2 - 4\,b^2d^3x^3 - 6\,abd^3x^2 - 2\,a^2d^3x + 12\,b^2d^2x^2 + 12\,abd^2x + 2\,a^2d^2 - 24\,b^2dx - 12\,abd + 24\,b^2)e^{(dx+c)}}{2\,d^5} - \frac{(b^2d^4x^4 + 2\,abd^4x^3 + a^2d^4x^2 + 4\,b^2d^3x^3 + 6\,abd^3x^2 + 2\,a^2d^3x + 12\,b^2d^2x^2 + 12\,abd^2x + 2\,a^2d^2 + 24\,b^2dx + 12\,abd + 24\,b^2)e^{(-dx-c)}}{2\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^4*x^4 + 2*a*b*d^4*x^3 + a^2*d^4*x^2 - 4*b^2*d^3*x^3 - 6*a*b*d^3*
x^2 - 2*a^2*d^3*x + 12*b^2*d^2*x^2 + 12*a*b*d^2*x + 2*a^2*d^2 - 24*b^2*d*x
- 12*a*b*d + 24*b^2)*e^(d*x + c)/d^5 - 1/2*(b^2*d^4*x^4 + 2*a*b*d^4*x^3 + a
^2*d^4*x^2 + 4*b^2*d^3*x^3 + 6*a*b*d^3*x^2 + 2*a^2*d^3*x + 12*b^2*d^2*x^2 +
 12*a*b*d^2*x + 2*a^2*d^2 + 24*b^2*d*x + 12*a*b*d + 24*b^2)*e^(-d*x - c)/d^
5
```

Mupad [B]
time = 0.16, size = 168, normalized size = 0.91

$$\frac{2\sinh(c+d\,x)\,(a^2\,d^2+12\,b^2)}{d^5} - \frac{4\,b^2\,x^3\cosh(c+d\,x)}{d^2} + \frac{b^2\,x^4\sinh(c+d\,x)}{d} - \frac{12\,a\,b\cosh(c+d\,x)}{d^4} - \frac{2\,x\cosh(c+d\,x)\,(a^2\,d^2+12\,b^2)}{d^4} + \frac{x^2\sinh(c+d\,x)\,(a^2\,d^2+12\,b^2)}{d^3} - \frac{6\,a\,b\,x^2\cosh(c+d\,x)}{d^2} + \frac{2\,a\,b\,x^3\sinh(c+d\,x)}{d} + \frac{12\,a\,b\,x\sinh(c+d\,x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(c + d*x)*(a + b*x)^2,x)
```

```
[Out] (2*sinh(c + d*x)*(12*b^2 + a^2*d^2))/d^5 - (4*b^2*x^3*cosh(c + d*x))/d^2 +
(b^2*x^4*sinh(c + d*x))/d - (12*a*b*cosh(c + d*x))/d^4 - (2*x*cosh(c + d*x)
*(12*b^2 + a^2*d^2))/d^4 + (x^2*sinh(c + d*x)*(12*b^2 + a^2*d^2))/d^3 - (6*
a*b*x^2*cosh(c + d*x))/d^2 + (2*a*b*x^3*sinh(c + d*x))/d + (12*a*b*x*sinh(c
 + d*x))/d^3
```

3.11 $\int x(a+bx)^2 \cosh(c+dx)\,dx$

Optimal. Leaf size=134

$$-\frac{6b^2\cosh(c+dx)}{d^4}-\frac{a^2\cosh(c+dx)}{d^2}-\frac{4abx\cosh(c+dx)}{d^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{4ab\sinh(c+dx)}{d^3}+\frac{6b^2x\sinh(c+dx)}{d^3}$$

```
[Out] -6*b^2*cosh(d*x+c)/d^4-a^2*cosh(d*x+c)/d^2-4*a*b*x*cosh(d*x+c)/d^2-3*b^2*x^
2*cosh(d*x+c)/d^2+4*a*b*sinh(d*x+c)/d^3+6*b^2*x*sinh(d*x+c)/d^3+a^2*x*sinh(
d*x+c)/d+2*a*b*x^2*sinh(d*x+c)/d+b^2*x^3*sinh(d*x+c)/d
```

Rubi [A]
time = 0.15, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6874, 3377, 2718, 2717}

$$-\frac{a^2\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(c+dx)}{d}+\frac{4ab\sinh(c+dx)}{d^3}-\frac{4abx\cosh(c+dx)}{d^2}+\frac{2abx^2\sinh(c+dx)}{d}-\frac{6b^2\cosh(c+dx)}{d^4}+\frac{6b^2x\sinh(c+dx)}{d^3}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{b^2x^3\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^2*Cosh[c + d*x],x]
```

```
[Out] (-6*b^2*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/d^2 - (4*a*b*x*Cosh[c + d*
x])/d^2 - (3*b^2*x^2*Cosh[c + d*x])/d^2 + (4*a*b*Sinh[c + d*x])/d^3 + (6*b^
2*x*Sinh[c + d*x])/d^3 + (a^2*x*Sinh[c + d*x])/d + (2*a*b*x^2*Sinh[c + d*x]
)/d + (b^2*x^3*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x(a+bx)^2\cosh(c+dx)\,dx=\int\left(a^2x\cosh(c+dx)+2abx^2\cosh(c+dx)+b^2x^3\cosh(c+dx)\right)\,dx$$
$$=a^2\int x\cosh(c+dx)\,dx+(2ab)\int x^2\cosh(c+dx)\,dx+b^2\int x^3\cosh(c+d$$
$$=\frac{a^2x\sinh(c+dx)}{d}+\frac{2abx^2\sinh(c+dx)}{d}+\frac{b^2x^3\sinh(c+dx)}{d}-\frac{a^2\int\sinh(c-}{d}$$
$$=-\frac{a^2\cosh(c+dx)}{d^2}-\frac{4abx\cosh(c+dx)}{d^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(c}{d}$$
$$=-\frac{a^2\cosh(c+dx)}{d^2}-\frac{4abx\cosh(c+dx)}{d^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{4ab\sinh(c}{d^3}$$
$$=-\frac{6b^2\cosh(c+dx)}{d^4}-\frac{a^2\cosh(c+dx)}{d^2}-\frac{4abx\cosh(c+dx)}{d^2}-\frac{3b^2x^2\cosh(c}{d^2}$$

Mathematica [A]
time = 0.12, size = 87, normalized size = 0.65

$$\frac{-((a^2d^2+4abd^2x+3b^2(2+d^2x^2))\cosh(c+dx))+d(a^2d^2x+2ab(2+d^2x^2)+b^2x(6+d^2x^2))\sinh(c+dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^2*Cosh[c + d*x],x]
```

```
[Out] (-((a^2*d^2 + 4*a*b*d^2*x + 3*b^2*(2 + d^2*x^2))*Cosh[c + d*x]) + d*(a^2*d^
2*x + 2*a*b*(2 + d^2*x^2) + b^2*x*(6 + d^2*x^2))*Sinh[c + d*x])/d^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(134) = 268.
time = 0.63, size = 283, normalized size = 2.11

method	result
risch	$\frac{(b^2d^3x^3+2ab\,d^3x^2+a^2d^3x-3b^2d^2x^2-4ab\,d^2x-a^2d^2+6b^2dx+4bda-6b^2)\,\mathrm{e}^{dx+c}}{2d^4}-\frac{(b^2d^3x^3+2ab\,d^3x^2+a^2d^3x+3b^2d^2x}{}$
derivativedivides	$\frac{b^2\left((dx+c)^3\sinh(dx+c)-3(dx+c)^2\cosh(dx+c)+6(dx+c)\sinh(dx+c)-6\cosh(dx+c)\right)}{d^2}-\frac{3b^2c\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c}{d^2}$
default	$\frac{b^2\left((dx+c)^3\sinh(dx+c)-3(dx+c)^2\cosh(dx+c)+6(dx+c)\sinh(dx+c)-6\cosh(dx+c)\right)}{d^2}-\frac{3b^2c\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(b^2/d^2*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sin
h(d*x+c)-6*cosh(d*x+c))-3*b^2/d^2*c*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d
```

```
*x+c)+2*sinh(d*x+c))+2*b/d*a*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2
*sinh(d*x+c))+3*b^2/d^2*c^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-4*b/d*c*a*((d
*x+c)*sinh(d*x+c)-cosh(d*x+c))+a^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-b^2/d^
2*c^3*sinh(d*x+c)+2*b/d*c^2*a*sinh(d*x+c)-c*a^2*sinh(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(134) = 268$.
time = 0.28, size = 275, normalized size = 2.05

$$-\frac{1}{24}\,d\left(\frac{6\,(d^2x^2e^c - 2\,dxe^c + 2\,e^c)a^2e^{(dx)}}{d^3} + \frac{6\,(d^2x^2 + 2\,dx + 2)a^2e^{(-dx-c)}}{d^3} + \frac{8\,(d^3x^3e^c - 3\,d^2x^2e^c + 6\,dxe^c - 6\,e^c)abe^{(dx)}}{d^4} + \frac{8\,(d^3x^3 + 3\,d^2x^2 + 6\,dx + 6)abe^{(-dx-c)}}{d^4} + \frac{3\,(d^4x^4e^c - 4\,d^3x^3e^c + 12\,d^2x^2e^c - 24\,dxe^c + 24\,e^c)b^2e^{(dx)}}{d^5} + \frac{3\,(d^4x^4 + 4\,d^3x^3 + 12\,d^2x^2 + 24\,dx + 24)b^2e^{(-dx-c)}}{d^5}\right) + \frac{1}{12}\,(3\,b^2x^4 + 8\,abx^3 + 6\,a^2x^2)\cosh\left(dx + c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/24*d*(6*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a^2*e^(d*x)/d^3 + 6*(d^2*x^2 +
 2*d*x + 2)*a^2*e^(-d*x - c)/d^3 + 8*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e
^c - 6*e^c)*a*b*e^(d*x)/d^4 + 8*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*b*e^(-d
*x - c)/d^4 + 3*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c
+ 24*e^c)*b^2*e^(d*x)/d^5 + 3*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x +
24)*b^2*e^(-d*x - c)/d^5) + 1/12*(3*b^2*x^4 + 8*a*b*x^3 + 6*a^2*x^2)*cosh(d
*x + c)
```

Fricas [A]
time = 0.36, size = 95, normalized size = 0.71

$$-\frac{(3\,b^2d^2x^2 + 4\,abd^2x + a^2d^2 + 6\,b^2)\cosh\left(dx + c\right) - (b^2d^3x^3 + 2\,abd^3x^2 + 4\,abd + (a^2d^3 + 6\,b^2d)x)\sinh\left(dx + c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 + 6*b^2)*cosh(d*x + c) - (b^2*d^3*
x^3 + 2*a*b*d^3*x^2 + 4*a*b*d + (a^2*d^3 + 6*b^2*d)*x)*sinh(d*x + c))/d^4
```

Sympy [A]
time = 0.23, size = 172, normalized size = 1.28

$$\begin{cases} \frac{a^2x\sinh\left(c+dx\right)}{d} - \frac{a^2\cosh\left(c+dx\right)}{d^2} + \frac{2abx^2\sinh\left(c+dx\right)}{d} - \frac{4abx\cosh\left(c+dx\right)}{d^2} + \frac{4ab\sinh\left(c+dx\right)}{d^3} + \frac{b^2x^3\sinh\left(c+dx\right)}{d} - \frac{3b^2x^2\cosh\left(c+dx\right)}{d^2} + \frac{6b^2x\sinh\left(c+dx\right)}{d^3} - \frac{6b^2\cosh\left(c+dx\right)}{d^4} & \text{for}\: d \neq 0 \\ \left(\frac{a^2x^2}{2} + \frac{2abx^3}{3} + \frac{b^2x^4}{4}\right)\cosh\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*x*sinh(c + d*x)/d - a**2*cosh(c + d*x)/d**2 + 2*a*b*x**2*si
nh(c + d*x)/d - 4*a*b*x*cosh(c + d*x)/d**2 + 4*a*b*sinh(c + d*x)/d**3 + b**
2*x**3*sinh(c + d*x)/d - 3*b**2*x**2*cosh(c + d*x)/d**2 + 6*b**2*x*sinh(c +
```

```
 d*x)/d**3 - 6*b**2*cosh(c + d*x)/d**4, Ne(d, 0)), ((a**2*x**2/2 + 2*a*b*x*
*3/3 + b**2*x**4/4)*cosh(c), True))
```

Giac [A]
time = 0.41, size = 171, normalized size = 1.28

$$\frac{(b^2d^3x^3 + 2\,abd^3x^2 + a^2d^3x - 3\,b^2d^2x^2 - 4\,abd^2x - a^2d^2 + 6\,b^2dx + 4\,abd - 6\,b^2)e^{(dx+c)}}{2\,d^4} - \frac{(b^2d^3x^3 + 2\,abd^3x^2 + a^2d^3x + 3\,b^2d^2x^2 + 4\,abd^2x + a^2d^2 + 6\,b^2dx + 4\,abd + 6\,b^2)e^{(-dx-c)}}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^3*x^3 + 2*a*b*d^3*x^2 + a^2*d^3*x - 3*b^2*d^2*x^2 - 4*a*b*d^2*x
- a^2*d^2 + 6*b^2*d*x + 4*a*b*d - 6*b^2)*e^(d*x + c)/d^4 - 1/2*(b^2*d^3*x^3
 + 2*a*b*d^3*x^2 + a^2*d^3*x + 3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 + 6*b^
2*d*x + 4*a*b*d + 6*b^2)*e^(-d*x - c)/d^4
```

Mupad [B]
time = 0.92, size = 125, normalized size = 0.93

$$\frac{b^2\,x^3\,\mathrm{sinh}(c+d\,x)}{d} - \frac{3\,b^2\,x^2\,\mathrm{cosh}(c+d\,x)}{d^2} - \frac{\mathrm{cosh}(c+d\,x)\,\left(a^2\,d^2+6\,b^2\right)}{d^4} + \frac{4\,a\,b\,\mathrm{sinh}(c+d\,x)}{d^3} + \frac{x\,\mathrm{sinh}(c+d\,x)\,\left(a^2\,d^2+6\,b^2\right)}{d^3} + \frac{2\,a\,b\,x^2\,\mathrm{sinh}(c+d\,x)}{d} - \frac{4\,a\,b\,x\,\mathrm{cosh}(c+d\,x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(c + d*x)*(a + b*x)^2,x)
```

```
[Out] (b^2*x^3*sinh(c + d*x))/d - (3*b^2*x^2*cosh(c + d*x))/d^2 - (cosh(c + d*x)*
(6*b^2 + a^2*d^2))/d^4 + (4*a*b*sinh(c + d*x))/d^3 + (x*sinh(c + d*x)*(6*b^
2 + a^2*d^2))/d^3 + (2*a*b*x^2*sinh(c + d*x))/d - (4*a*b*x*cosh(c + d*x))/d
^2
```

3.12 $\int (a+bx)^2 \cosh(c+dx)\,dx$

Optimal. Leaf size=49

$$-\frac{2b(a+bx)\cosh(c+dx)}{d^2}+\frac{2b^2\sinh(c+dx)}{d^3}+\frac{(a+bx)^2\sinh(c+dx)}{d}$$

```
[Out] -2*b*(b*x+a)*cosh(d*x+c)/d^2+2*b^2*sinh(d*x+c)/d^3+(b*x+a)^2*sinh(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3377, 2717}

$$-\frac{2b(a+bx)\cosh(c+dx)}{d^2}+\frac{(a+bx)^2\sinh(c+dx)}{d}+\frac{2b^2\sinh(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Cosh[c + d*x],x]
```

```
[Out] (-2*b*(a + b*x)*Cosh[c + d*x])/d^2 + (2*b^2*Sinh[c + d*x])/d^3 + ((a + b*x)
^2*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2\cosh(c+dx)\,dx &= \frac{(a+bx)^2\sinh(c+dx)}{d}-\frac{(2b)\int (a+bx)\sinh(c+dx)\,dx}{d}\\ &= -\frac{2b(a+bx)\cosh(c+dx)}{d^2}+\frac{(a+bx)^2\sinh(c+dx)}{d}+\frac{(2b^2)\int \cosh(c+dx)\,dx}{d^2}\\ &= -\frac{2b(a+bx)\cosh(c+dx)}{d^2}+\frac{2b^2\sinh(c+dx)}{d^3}+\frac{(a+bx)^2\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [**A**]
time = 0.09, size = 56, normalized size = 1.14

$$\frac{-2bd(a+bx)\cosh(c+dx)+\left(a^2d^2+2abd^2x+b^2(2+d^2x^2)\right)\sinh(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Cosh[c + d*x],x]
```

```
[Out] (-2*b*d*(a + b*x)*Cosh[c + d*x] + (a^2*d^2 + 2*a*b*d^2*x + b^2*(2 + d^2*x^2
))*Sinh[c + d*x])/d^3
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(49) = 98$.
time = 0.60, size = 147, normalized size = 3.00

method	result
risch	$\frac{\left(b^2d^2x^2+2ab\,d^2x+a^2d^2-2b^2dx-2bda+2b^2\right)\mathrm{e}^{dx+c}}{2d^3}-\frac{\left(b^2d^2x^2+2ab\,d^2x+a^2d^2+2b^2dx+2bda+2b^2\right)\mathrm{e}^{-dx-c}}{2d^3}$
derivativedivides	$\frac{\frac{b^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}-\frac{2b^2c((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^2}+\frac{2ba((dx+c)\sinh(dx+c)-\cosh}{d}}{d}$
default	$\frac{\frac{b^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}-\frac{2b^2c((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^2}+\frac{2ba((dx+c)\sinh(dx+c)-\cosh}{d}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2/d^2*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-2*
b^2/d^2*c*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+2*b/d*a*((d*x+c)*sinh(d*x+c)-co
sh(d*x+c))+b^2/d^2*c^2*sinh(d*x+c)-2*b/d*c*a*sinh(d*x+c)+a^2*sinh(d*x+c))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(49) = 98$.
time = 0.28, size = 135, normalized size = 2.76

$$\frac{a^2e^{(dx+c)}}{2\,d}+\frac{(dxe^c-e^c)abe^{(dx)}}{d^2}-\frac{(dx+1)abe^{(-dx-c)}}{d^2}-\frac{a^2e^{(-dx-c)}}{2\,d}+\frac{(d^2x^2e^c-2\,dxe^c+2\,e^c)b^2e^{(dx)}}{2\,d^3}-\frac{(d^2x^2+2\,dx+2)b^2e^{(-dx-c)}}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*e^(d*x + c)/d + (d*x*e^c - e^c)*a*b*e^(d*x)/d^2 - (d*x + 1)*a*b*e^(
-d*x - c)/d^2 - 1/2*a^2*e^(-d*x - c)/d + 1/2*(d^2*x^2*e^c - 2*d*x*e^c + 2*e
^c)*b^2*e^(d*x)/d^3 - 1/2*(d^2*x^2 + 2*d*x + 2)*b^2*e^(-d*x - c)/d^3
```

Fricas [**A**]
time = 0.41, size = 64, normalized size = 1.31

$$-\frac{2\,(b^2dx+abd)\cosh\left(dx+c\right)-(b^2d^2x^2+2\,abd^2x+a^2d^2+2\,b^2)\sinh\left(dx+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(2*(b^2*d*x + a*b*d)*cosh(d*x + c) - (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2
+ 2*b^2)*sinh(d*x + c))/d^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(48) = 96$.
time = 0.14, size = 112, normalized size = 2.29

$$\begin{cases} \frac{a^2 \sinh(c+dx)}{d} + \frac{2abx \sinh(c+dx)}{d} - \frac{2ab \cosh(c+dx)}{d^2} + \frac{b^2x^2 \sinh(c+dx)}{d} - \frac{2b^2x \cosh(c+dx)}{d^2} + \frac{2b^2 \sinh(c+dx)}{d^3} & \text{for } d \neq 0 \\ \left(a^2x + abx^2 + \frac{b^2x^3}{3}\right) \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*x*sinh(c + d*x)/d - 2*a*b*cosh(c +
d*x)/d**2 + b**2*x**2*sinh(c + d*x)/d - 2*b**2*x*cosh(c + d*x)/d**2 + 2*b**
2*sinh(c + d*x)/d**3, Ne(d, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)*cosh(c)
, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(49) = 98$.
time = 0.39, size = 112, normalized size = 2.29

$$\frac{(b^2d^2x^2 + 2\,abd^2x + a^2d^2 - 2\,b^2dx - 2\,abd + 2\,b^2)e^{(dx+c)}}{2\,d^3} - \frac{(b^2d^2x^2 + 2\,abd^2x + a^2d^2 + 2\,b^2dx + 2\,abd + 2\,b^2)e^{(-dx-c)}}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 - 2*b^2*d*x - 2*a*b*d + 2*b^2)*e^(
d*x + c)/d^3 - 1/2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 + 2*b^2*d*x + 2*a*b
*d + 2*b^2)*e^(-d*x - c)/d^3
```

Mupad [B]
time = 0.90, size = 82, normalized size = 1.67

$$\frac{\sinh(c+d\,x)\,\left(a^2\,d^2+2\,b^2\right)}{d^3} + \frac{b^2\,x^2\,\sinh(c+d\,x)}{d} - \frac{2\,a\,b\cosh(c+d\,x)}{d^2} - \frac{2\,b^2\,x\cosh(c+d\,x)}{d^2} + \frac{2\,a\,b\,x\sinh(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b*x)^2,x)
```

```
[Out] (sinh(c + d*x)*(2*b^2 + a^2*d^2))/d^3 + (b^2*x^2*sinh(c + d*x))/d - (2*a*b*
cosh(c + d*x))/d^2 - (2*b^2*x*cosh(c + d*x))/d^2 + (2*a*b*x*sinh(c + d*x))/
d
```

3.13 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x} \, dx$

Optimal. Leaf size=62

$$-\frac{b^2 \cosh(c+dx)}{d^2}+a^2 \cosh(c)\text{Chi}(dx)+\frac{2ab \sinh(c+dx)}{d}+\frac{b^2 x \sinh(c+dx)}{d}+a^2 \sinh(c)\text{Shi}(dx)$$

```
[Out] a^2*Chi(d*x)*cosh(c)-b^2*cosh(d*x+c)/d^2+a^2*Shi(d*x)*sinh(c)+2*a*b*sinh(d*
x+c)/d+b^2*x*sinh(d*x+c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6874, 2717, 3384, 3379, 3382, 3377, 2718}

$$a^2 \cosh(c)\text{Chi}(dx) + a^2 \sinh(c)\text{Shi}(dx) + \frac{2ab \sinh(c+dx)}{d} - \frac{b^2 \cosh(c+dx)}{d^2} + \frac{b^2 x \sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Cosh[c + d*x])/x,x]
```

```
[Out] -((b^2*Cosh[c + d*x])/d^2) + a^2*Cosh[c]*CoshIntegral[d*x] + (2*a*b*Sinh[c
+ d*x])/d + (b^2*x*Sinh[c + d*x])/d + a^2*Sinh[c]*SinhIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^2\cosh(c+dx)}{x}\,dx &= \int \left(2ab\cosh(c+dx)+\frac{a^2\cosh(c+dx)}{x}+b^2x\cosh(c+dx)\right)dx\\
&= a^2\int \frac{\cosh(c+dx)}{x}\,dx+(2ab)\int \cosh(c+dx)\,dx+b^2\int x\cosh(c+dx)\,dx\\
&= \frac{2ab\sinh(c+dx)}{d}+\frac{b^2x\sinh(c+dx)}{d}-\frac{b^2\int\sinh(c+dx)\,dx}{d}+\left(a^2\cosh(c)\right)\int\\
&= -\frac{b^2\cosh(c+dx)}{d^2}+a^2\cosh(c)\mathrm{Chi}(dx)+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2x\sinh(c+dx)}{d}
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 51, normalized size = 0.82

$$
a^2\cosh(c)\mathrm{Chi}(dx)+\frac{b(-b\cosh(c+dx)+d(2a+bx)\sinh(c+dx))}{d^2}+a^2\sinh(c)\mathrm{Shi}(dx)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x,x]
```

```
[Out] a^2*Cosh[c]*CoshIntegral[d*x] + (b*(-(b*Cosh[c + d*x]) + d*(2*a + b*x)*Sinh
[c + d*x]))/d^2 + a^2*Sinh[c]*SinhIntegral[d*x]
```

Maple [A]
time = 0.80, size = 121, normalized size = 1.95

method	result
risch	$-\frac{a^2 e^{-c} \operatorname{expIntegral}(1,dx)}{2}-\frac{b^2 e^{-dx-c} x}{2d}-\frac{b^2 e^{-dx-c}}{2d^2}-\frac{ab\, e^{-dx-c}}{d}-\frac{a^2 e^{c} \operatorname{expIntegral}(1,-dx)}{2}+\frac{b^2 e^{dx+c} x}{2d}-\frac{b^2 e^{dx+c}}{2d^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*cosh(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2*exp(-c)*Ei(1,d*x)-1/2/d*b^2*exp(-d*x-c)*x-1/2/d^2*b^2*exp(-d*x-c)-
1/d*a*b*exp(-d*x-c)-1/2*a^2*exp(c)*Ei(1,-d*x)+1/2/d*b^2*exp(d*x+c)*x-1/2/d^
2*b^2*exp(d*x+c)+a*b/d*exp(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(62) = 124$.
time = 0.35, size = 175, normalized size = 2.82

$$-\frac{1}{4}\left(4\,ab\left(\frac{(dxe^c-e^c)e^{(dx)}}{d^2}+\frac{(dx+1)e^{(-dx-c)}}{d^2}\right)+b^2\left(\frac{(d^2x^2e^c-2\,dxe^c+2\,e^c)e^{(dx)}}{d^3}+\frac{(d^2x^2+2\,dx+2)e^{(-dx-c)}}{d^3}\right)+\frac{4\,a^2\cosh(dx+c)\log(x)}{d}-\frac{2\left(\mathrm{Ei}(-dx)\,e^{(-c)}+\mathrm{Ei}(dx)\,e^c\right)a^2}{d}\right)d+\frac{1}{2}\left(b^2x^2+4\,abx+2\,a^2\log(x)\right)\cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(4*a*b*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2) + b^
2*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^
(-d*x - c)/d^3) + 4*a^2*cosh(d*x + c)*log(x)/d - 2*(Ei(-d*x)*e^(-c) + Ei(d*
x)*e^c)*a^2/d)*d + 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*cosh(d*x + c)
```

Fricas [A]
time = 0.35, size = 94, normalized size = 1.52

$$-\frac{2\,b^2\cosh(dx+c)-\left(a^2d^2\mathrm{Ei}(dx)+a^2d^2\mathrm{Ei}(-dx)\right)\cosh(c)-2\left(b^2dx+2\,abd\right)\sinh(dx+c)-\left(a^2d^2\mathrm{Ei}(dx)-a^2d^2\mathrm{Ei}(-dx)\right)\sinh(c)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*cosh(d*x + c) - (a^2*d^2*Ei(d*x) + a^2*d^2*Ei(-d*x))*cosh(c) -
2*(b^2*d*x + 2*a*b*d)*sinh(d*x + c) - (a^2*d^2*Ei(d*x) - a^2*d^2*Ei(-d*x))*
sinh(c))/d^2
```

Sympy [A]
time = 2.23, size = 73, normalized size = 1.18

$$a^2\sinh(c)\,\mathrm{Shi}(dx)+a^2\cosh(c)\,\mathrm{Chi}(dx)+2ab\left(\begin{cases}\frac{\sinh(c+dx)}{d} & \text{for } d\neq 0\\ x\cosh(c) & \text{otherwise}\end{cases}\right)+b^2\left(\begin{cases}\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2} & \text{for } d\neq 0\\ \frac{x^2\cosh(c)}{2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*cosh(d*x+c)/x,x)
```

```
[Out] a**2*sinh(c)*Shi(d*x) + a**2*cosh(c)*Chi(d*x) + 2*a*b*Piecewise((sinh(c + d
*x)/d, Ne(d, 0)), (x*cosh(c), True)) + b**2*Piecewise((x*sinh(c + d*x)/d -
cosh(c + d*x)/d**2, Ne(d, 0)), (x**2*cosh(c)/2, True))
```

Giac [A]
time = 0.42, size = 113, normalized size = 1.82

$$\frac{a^2d^2\mathrm{Ei}(-dx)\,e^{(-c)} + a^2d^2\mathrm{Ei}(dx)\,e^c + b^2dxe^{(dx+c)} - b^2dxe^{(-dx-c)} + 2\,abde^{(dx+c)} - 2\,abde^{(-dx-c)} - b^2e^{(dx+c)} - b^2e^{(-dx-c)}}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*d^2*Ei(-d*x)*e^(-c) + a^2*d^2*Ei(d*x)*e^c + b^2*d*x*e^(d*x + c) -
b^2*d*x*e^(-d*x - c) + 2*a*b*d*e^(d*x + c) - 2*a*b*d*e^(-d*x - c) - b^2*e^(
d*x + c) - b^2*e^(-d*x - c))/d^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x)^2)/x,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x)^2)/x, x)
```

3.14 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x^2} \, dx$

Optimal. Leaf size=70

$$-\frac{a^2\cosh(c+dx)}{x}+2ab\cosh(c)\text{Chi}(dx)+a^2d\text{Chi}(dx)\sinh(c)+\frac{b^2\sinh(c+dx)}{d}+a^2d\cosh(c)\text{Shi}(dx)+2ab\sinh$$

```
[Out] 2*a*b*Chi(d*x)*cosh(c)-a^2*cosh(d*x+c)/x+a^2*d*cosh(c)*Shi(d*x)+a^2*d*Chi(d
*x)*sinh(c)+2*a*b*Shi(d*x)*sinh(c)+b^2*sinh(d*x+c)/d
```

Rubi [A]
time = 0.19, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6874, 2717, 3378, 3384, 3379, 3382}

$$a^2d\sinh(c)\text{Chi}(dx)+a^2d\cosh(c)\text{Shi}(dx)-\frac{a^2\cosh(c+dx)}{x}+2ab\cosh(c)\text{Chi}(dx)+2ab\sinh(c)\text{Shi}(dx)+\frac{b^2\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a^2*Cosh[c + d*x])/x) + 2*a*b*Cosh[c]*CoshIntegral[d*x] + a^2*d*CoshInte
gral[d*x]*Sinh[c] + (b^2*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
 + 2*a*b*Sinh[c]*SinhIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2\cosh(c+dx)}{x^2}\,dx &= \int \left(b^2\cosh(c+dx)+\frac{a^2\cosh(c+dx)}{x^2}+\frac{2ab\cosh(c+dx)}{x}\right)dx\\ &= a^2\int \frac{\cosh(c+dx)}{x^2}\,dx+(2ab)\int \frac{\cosh(c+dx)}{x}\,dx+b^2\int \cosh(c+dx)\,dx\\ &= -\frac{a^2\cosh(c+dx)}{x}+\frac{b^2\sinh(c+dx)}{d}+\left(a^2d\right)\int \frac{\sinh(c+dx)}{x}\,dx+(2ab\cosh(\\ &= -\frac{a^2\cosh(c+dx)}{x}+2ab\cosh(c)\text{Chi}(dx)+\frac{b^2\sinh(c+dx)}{d}+2ab\sinh(c)\text{Shi}(\\ &= -\frac{a^2\cosh(c+dx)}{x}+2ab\cosh(c)\text{Chi}(dx)+a^2d\text{Chi}(dx)\sinh(c)+\frac{b^2\sinh(c+\\ d}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 62, normalized size = 0.89

$$-\frac{a^2\cosh(c+dx)}{x}+a\text{Chi}(dx)(2b\cosh(c)+ad\sinh(c))+\frac{b^2\sinh(c+dx)}{d}+a(ad\cosh(c)+2b\sinh(c))\text{Shi}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a^2*Cosh[c + d*x])/x) + a*CoshIntegral[d*x]*(2*b*Cosh[c] + a*d*Sinh[c])
+ (b^2*Sinh[c + d*x])/d + a*(a*d*Cosh[c] + 2*b*Sinh[c])*SinhIntegral[d*x]
```

Maple [A]
time = 0.76, size = 118, normalized size = 1.69

method	result
risch	$-\frac{a^2 \mathrm{e}^{-dx-c}}{2x} + \frac{d\,a^2 \mathrm{e}^{-c}\,\mathrm{expIntegral}(1,dx)}{2} - \frac{b^2 \mathrm{e}^{-dx-c}}{2d} - ab\,\mathrm{e}^{-c}\,\mathrm{expIntegral}\left(1,dx\right) - \frac{a^2 \mathrm{e}^{dx+c}}{2x} - \frac{d\,a^2 \mathrm{e}^{c}\,\mathrm{expIntegral}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*cosh(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2*exp(-d*x-c)/x+1/2*d*a^2*exp(-c)*Ei(1,d*x)-1/2/d*b^2*exp(-d*x-c)-a*
b*exp(-c)*Ei(1,d*x)-1/2*a^2/x*exp(d*x+c)-1/2*d*a^2*exp(c)*Ei(1,-d*x)+1/2/d*
exp(d*x+c)*b^2-a*b*exp(c)*Ei(1,-d*x)
```

Maxima [A]
time = 0.33, size = 136, normalized size = 1.94

$$-\frac{1}{2}\left(\left(\mathrm{Ei}(-dx)\,e^{(-c)} - \mathrm{Ei}(dx)\,e^{c}\right)a^2 + b^2\left(\frac{(dxe^c - e^c)e^{(dx)}}{d^2} + \frac{(dx+1)e^{(-dx-c)}}{d^2}\right) + \frac{4\,ab\cosh\left(dx+c\right)\log\left(x\right)}{d} - \frac{2\left(\mathrm{Ei}(-dx)\,e^{(-c)} + \mathrm{Ei}(dx)\,e^{c}\right)ab}{d}\right)d + \left(b^2x + 2\,ab\log\left(x\right) - \frac{a^2}{x}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*((Ei(-d*x)*e^(-c) - Ei(d*x)*e^c)*a^2 + b^2*((d*x*e^c - e^c)*e^(d*x)/d^
2 + (d*x + 1)*e^(-d*x - c)/d^2) + 4*a*b*cosh(d*x + c)*log(x)/d - 2*(Ei(-d*x
)*e^(-c) + Ei(d*x)*e^c)*a*b/d)*d + (b^2*x + 2*a*b*log(x) - a^2/x)*cosh(d*x
+ c)
```

Fricas [A]
time = 0.34, size = 122, normalized size = 1.74

$$-\frac{2\,a^2d\cosh\left(dx+c\right) - 2\,b^2x\sinh\left(dx+c\right) - \left((a^2d^2+2\,abd)x\mathrm{Ei}(dx) - (a^2d^2-2\,abd)x\mathrm{Ei}(-dx)\right)\cosh\left(c\right) - \left((a^2d^2+2\,abd)x\mathrm{Ei}(dx) + (a^2d^2-2\,abd)x\mathrm{Ei}(-dx)\right)\sinh\left(c\right)}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d*cosh(d*x + c) - 2*b^2*x*sinh(d*x + c) - ((a^2*d^2 + 2*a*b*d)*
x*Ei(d*x) - (a^2*d^2 - 2*a*b*d)*x*Ei(-d*x))*cosh(c) - ((a^2*d^2 + 2*a*b*d)*
x*Ei(d*x) + (a^2*d^2 - 2*a*b*d)*x*Ei(-d*x))*sinh(c))/(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx\right)^2\cosh\left(c+dx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*cosh(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x)**2*cosh(c + d*x)/x**2, x)
```

Giac [A]
time = 0.43, size = 119, normalized size = 1.70

$$-\frac{a^2d^2x\text{Ei}(-dx)\,e^{(-c)} - a^2d^2x\text{Ei}(dx)\,e^c - 2\,abdx\text{Ei}(-dx)\,e^{(-c)} - 2\,abdx\text{Ei}(dx)\,e^c + a^2de^{(dx+c)} - b^2xe^{(dx+c)} + a^2de^{(-dx-c)} + b^2xe^{(-dx-c)}}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*d^2*x*Ei(-d*x)*e^(-c) - a^2*d^2*x*Ei(d*x)*e^c - 2*a*b*d*x*Ei(-d*x
)*e^(-c) - 2*a*b*d*x*Ei(d*x)*e^c + a^2*d*e^(d*x + c) - b^2*x*e^(d*x + c) +
a^2*d*e^(-d*x - c) + b^2*x*e^(-d*x - c))/(d*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x)^2)/x^2,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x)^2)/x^2, x)
```

3.15 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x^3} \, dx$

Optimal. Leaf size=121

$$-\frac{a^2\cosh(c+dx)}{2x^2}-\frac{2ab\cosh(c+dx)}{x}+b^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}a^2d^2\cosh(c)\text{Chi}(dx)+2abd\text{Chi}(dx)\sinh(c)-\frac{a^2d}{}$$

```
[Out] b^2*Chi(d*x)*cosh(c)+1/2*a^2*d^2*Chi(d*x)*cosh(c)-1/2*a^2*cosh(d*x+c)/x^2-2
*a*b*cosh(d*x+c)/x+2*a*b*d*cosh(c)*Shi(d*x)+2*a*b*d*Chi(d*x)*sinh(c)+b^2*Sh
i(d*x)*sinh(c)+1/2*a^2*d^2*Shi(d*x)*sinh(c)-1/2*a^2*d*sinh(d*x+c)/x
```

Rubi [A]

time = 0.24, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\frac{1}{2}a^2d^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}a^2d^2\sinh(c)\text{Shi}(dx)-\frac{a^2\cosh(c+dx)}{2x^2}-\frac{a^2d\sinh(c+dx)}{2x}+2abd\sinh(c)\text{Chi}(dx)+2abd\cosh(c)\text{Shi}(dx)-\frac{2ab\cosh(c+dx)}{x}+b^2\cosh(c)\text{Chi}(dx)+b^2\sinh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Cosh[c + d*x])/x^3,x]
```

```
[Out] -1/2*(a^2*Cosh[c + d*x])/x^2 - (2*a*b*Cosh[c + d*x])/x + b^2*Cosh[c]*CoshIn
tegral[d*x] + (a^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 + 2*a*b*d*CoshIntegral[
d*x]*Sinh[c] - (a^2*d*Sinh[c + d*x])/(2*x) + 2*a*b*d*Cosh[c]*SinhIntegral[d
*x] + b^2*Sinh[c]*SinhIntegral[d*x] + (a^2*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx &= \int \left(\frac{a^2\cosh(c+dx)}{x^3}+\frac{2ab\cosh(c+dx)}{x^2}+\frac{b^2\cosh(c+dx)}{x}\right)dx\\
&= a^2\int \frac{\cosh(c+dx)}{x^3}\,dx+(2ab)\int \frac{\cosh(c+dx)}{x^2}\,dx+b^2\int \frac{\cosh(c+dx)}{x}\,dx\\
&= -\frac{a^2\cosh(c+dx)}{2x^2}-\frac{2ab\cosh(c+dx)}{x}+\frac{1}{2}\left(a^2d\right)\int \frac{\sinh(c+dx)}{x^2}\,dx+(2abd)\\
&= -\frac{a^2\cosh(c+dx)}{2x^2}-\frac{2ab\cosh(c+dx)}{x}+b^2\cosh(c)\text{Chi}(dx)-\frac{a^2d\sinh(c+dx}{2x}\\
&= -\frac{a^2\cosh(c+dx)}{2x^2}-\frac{2ab\cosh(c+dx)}{x}+b^2\cosh(c)\text{Chi}(dx)+2abd\text{Chi}(dx)\sin\\
&= -\frac{a^2\cosh(c+dx)}{2x^2}-\frac{2ab\cosh(c+dx)}{x}+b^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}a^2d^2\cosh(c)\text{C}
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 93, normalized size = 0.77

$$\frac{1}{2}\left(\text{Chi}(dx)\left(\left(2b^2+a^2d^2\right)\cosh(c)+4abd\sinh(c)\right)-\frac{a((a+4bx)\cosh(c+dx)+adx\sinh(c+dx))}{x^2}+\left(4abd\cosh(c)+\left(2b^2+a^2d^2\right)\sinh(c)\right)\text{Shi}(dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x^3,x]
```

```
[Out] (CoshIntegral[d*x]*((2*b^2 + a^2*d^2)*Cosh[c] + 4*a*b*d*Sinh[c]) - (a*((a +
 4*b*x)*Cosh[c + d*x] + a*d*x*Sinh[c + d*x]))/x^2 + (4*a*b*d*Cosh[c] + (2*b
^2 + a^2*d^2)*Sinh[c])*SinhIntegral[d*x])/2
```

Maple [A]
time = 0.69, size = 181, normalized size = 1.50

method	result

risch	$\frac{d\,a^2\mathrm{e}^{-dx-c}}{4x}-\frac{a^2\mathrm{e}^{-dx-c}}{4x^2}-\frac{d^2a^2\mathrm{e}^{-c}\,\mathrm{expIntegral}(1,dx)}{4}-\frac{b^2\mathrm{e}^{-c}\,\mathrm{expIntegral}(1,dx)}{2}-\frac{ab\,\mathrm{e}^{-dx-c}}{x}+dab\,\mathrm{e}^{-c}\,\mathrm{expIntegral}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*cosh(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d*a^2*exp(-d*x-c)/x-1/4*a^2*exp(-d*x-c)/x^2-1/4*d^2*a^2*exp(-c)*Ei(1,d*
x)-1/2*b^2*exp(-c)*Ei(1,d*x)-a*b*exp(-d*x-c)/x+d*a*b*exp(-c)*Ei(1,d*x)-a*b/
x*exp(d*x+c)-d*a*b*exp(c)*Ei(1,-d*x)-1/4*d^2*a^2*exp(c)*Ei(1,-d*x)-1/4*a^2/
x^2*exp(d*x+c)-1/4*d*a^2/x*exp(d*x+c)-1/2*b^2*exp(c)*Ei(1,-d*x)
```

Maxima [A]
time = 0.34, size = 126, normalized size = 1.04

$$\frac{1}{4}\left(\left(de^{(-c)}\Gamma(-1,dx)+de^{c}\Gamma(-1,-dx)\right)a^2-4\left(\mathrm{Ei}(-dx)\,e^{(-c)}-\mathrm{Ei}(dx)\,e^{c}\right)ab-\frac{4\,b^2\cosh\left(dx+c\right)\log\left(x\right)}{d}+\frac{2\left(\mathrm{Ei}(-dx)\,e^{(-c)}+\mathrm{Ei}(dx)\,e^{c}\right)b^2}{d}\right)d+\frac{1}{2}\left(2\,b^2\log\left(x\right)-\frac{4\,abx+a^2}{x^2}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*((d*e^(-c)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))*a^2 - 4*(Ei(-d*x)*e^
(-c) - Ei(d*x)*e^c)*a*b - 4*b^2*cosh(d*x + c)*log(x)/d + 2*(Ei(-d*x)*e^(-c)
 + Ei(d*x)*e^c)*b^2/d)*d + 1/2*(2*b^2*log(x) - (4*a*b*x + a^2)/x^2)*cosh(d*
x + c)
```

Fricas [A]
time = 0.37, size = 153, normalized size = 1.26

$$-\frac{2\,a^2dx\sinh\left(dx+c\right)+2\left(4\,abx+a^2\right)\cosh\left(dx+c\right)-\left(\left(a^2d^2+4\,abd+2\,b^2\right)x^2\mathrm{Ei}(dx)+\left(a^2d^2-4\,abd+2\,b^2\right)x^2\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-\left(\left(a^2d^2+4\,abd+2\,b^2\right)x^2\mathrm{Ei}(dx)-\left(a^2d^2-4\,abd+2\,b^2\right)x^2\mathrm{Ei}(-dx)\right)\sinh\left(c\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*d*x*sinh(d*x + c) + 2*(4*a*b*x + a^2)*cosh(d*x + c) - ((a^2*d^2
 + 4*a*b*d + 2*b^2)*x^2*Ei(d*x) + (a^2*d^2 - 4*a*b*d + 2*b^2)*x^2*Ei(-d*x))
*cosh(c) - ((a^2*d^2 + 4*a*b*d + 2*b^2)*x^2*Ei(d*x) - (a^2*d^2 - 4*a*b*d +
2*b^2)*x^2*Ei(-d*x))*sinh(c))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx\right)^2\cosh\left(c+dx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*cosh(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x)**2*cosh(c + d*x)/x**3, x)
```

Giac [A]
time = 0.40, size = 181, normalized size = 1.50

$$\frac{a^2d^2x^2\mathrm{Ei}(-dx)\,e^{(-c)}+a^2d^2x^2\mathrm{Ei}(dx)\,e^{c}-4\,abdx^2\mathrm{Ei}(-dx)\,e^{(-c)}+4\,abdx^2\mathrm{Ei}(dx)\,e^{c}+2\,b^2x^2\mathrm{Ei}(-dx)\,e^{(-c)}+2\,b^2x^2\mathrm{Ei}(dx)\,e^{c}-a^2dxe^{(dx+c)}+a^2dxe^{(-dx-c)}-4\,abxe^{(dx+c)}-4\,abxe^{(-dx-c)}-a^2e^{(dx+c)}-a^2e^{(-dx-c)}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^2*x^2*Ei(-d*x)*e^(-c) + a^2*d^2*x^2*Ei(d*x)*e^c - 4*a*b*d*x^2*Ei
(-d*x)*e^(-c) + 4*a*b*d*x^2*Ei(d*x)*e^c + 2*b^2*x^2*Ei(-d*x)*e^(-c) + 2*b^2
*x^2*Ei(d*x)*e^c - a^2*d*x*e^(d*x + c) + a^2*d*x*e^(-d*x - c) - 4*a*b*x*e^(
d*x + c) - 4*a*b*x*e^(-d*x - c) - a^2*e^(d*x + c) - a^2*e^(-d*x - c))/x^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x)^2)/x^3,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x)^2)/x^3, x)
```

3.16 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x^4} \, dx$

Optimal. Leaf size=172

$$-\frac{a^2 \cosh(c+dx)}{3x^3}-\frac{ab \cosh(c+dx)}{x^2}-\frac{b^2 \cosh(c+dx)}{x}-\frac{a^2d^2 \cosh(c+dx)}{6x}+abd^2 \cosh(c)\text{Chi}(dx)+b^2d\text{Chi}(dx$$

```
[Out] a*b*d^2*Chi(d*x)*cosh(c)-1/3*a^2*cosh(d*x+c)/x^3-a*b*cosh(d*x+c)/x^2-b^2*co
sh(d*x+c)/x-1/6*a^2*d^2*cosh(d*x+c)/x+b^2*d*cosh(c)*Shi(d*x)+1/6*a^2*d^3*co
sh(c)*Shi(d*x)+b^2*d*Chi(d*x)*sinh(c)+1/6*a^2*d^3*Chi(d*x)*sinh(c)+a*b*d^2*
Shi(d*x)*sinh(c)-1/6*a^2*d*sinh(d*x+c)/x^2-a*b*d*sinh(d*x+c)/x
```

Rubi [A]
time = 0.30, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\frac{1}{6}a^2d^3\sinh(c)\text{Chi}(dx)+\frac{1}{6}a^2d^3\cosh(c)\text{Shi}(dx)-\frac{a^2d^2\cosh(c+dx)}{6x}-\frac{a^2\cosh(c+dx)}{3x^3}-\frac{a^2d\sinh(c+dx)}{6x^2}+abd^2\cosh(c)\text{Chi}(dx)+abd^2\sinh(c)\text{Shi}(dx)-\frac{ab\cosh(c+dx)}{x^2}-\frac{abd\sinh(c+dx)}{x}+b^2d\sinh(c)\text{Chi}(dx)+b^2d\cosh(c)\text{Shi}(dx)-\frac{b^2\cosh(c+dx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Cosh[c + d*x])/x^4,x]
```

```
[Out] -1/3*(a^2*Cosh[c + d*x])/x^3 - (a*b*Cosh[c + d*x])/x^2 - (b^2*Cosh[c + d*x]
)/x - (a^2*d^2*Cosh[c + d*x])/(6*x) + a*b*d^2*Cosh[c]*CoshIntegral[d*x] + b
^2*d*CoshIntegral[d*x]*Sinh[c] + (a^2*d^3*CoshIntegral[d*x]*Sinh[c])/6 - (a
^2*d*Sinh[c + d*x])/(6*x^2) - (a*b*d*Sinh[c + d*x])/x + b^2*d*Cosh[c]*SinhI
ntegral[d*x] + (a^2*d^3*Cosh[c]*SinhIntegral[d*x])/6 + a*b*d^2*Sinh[c]*Sinh
Integral[d*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx &= \int \left(\frac{a^2\cosh(c+dx)}{x^4}+\frac{2ab\cosh(c+dx)}{x^3}+\frac{b^2\cosh(c+dx)}{x^2}\right)dx\\
&= a^2\int \frac{\cosh(c+dx)}{x^4}\,dx+(2ab)\int \frac{\cosh(c+dx)}{x^3}\,dx+b^2\int \frac{\cosh(c+dx)}{x^2}\,dx\\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{ab\cosh(c+dx)}{x^2}-\frac{b^2\cosh(c+dx)}{x}+\frac{1}{3}\left(a^2d\right)\int \frac{\sinh(c+}{x^3}\\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{ab\cosh(c+dx)}{x^2}-\frac{b^2\cosh(c+dx)}{x}-\frac{a^2d\sinh(c+dx)}{6x^2}-\\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{ab\cosh(c+dx)}{x^2}-\frac{b^2\cosh(c+dx)}{x}-\frac{a^2d^2\cosh(c+dx)}{6x}\\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{ab\cosh(c+dx)}{x^2}-\frac{b^2\cosh(c+dx)}{x}-\frac{a^2d^2\cosh(c+dx)}{6x}\\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{ab\cosh(c+dx)}{x^2}-\frac{b^2\cosh(c+dx)}{x}-\frac{a^2d^2\cosh(c+dx)}{6x}
\end{aligned}$$

Mathematica [A]
time = 0.28, size = 154, normalized size = 0.90

$$-\frac{2a^2\cosh(c+dx)+6abx\cosh(c+dx)+6b^2x^2\cosh(c+dx)+a^2d^2x^2\cosh(c+dx)-dx^3\mathrm{Chi}(dx)\left(6abd\cosh(c)+\left(6b^2+a^2d^2\right)\sinh(c)\right)+a^2dx\sinh(c+dx)+6abdx^2\sinh(c+dx)-dx^3\left(6b^2\cosh(c)+a^2d^2\cosh(c)+6abd\sinh(c)\right)\mathrm{Shi}(dx)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x^4,x]
```

```
[Out] -1/6*(2*a^2*Cosh[c + d*x] + 6*a*b*x*Cosh[c + d*x] + 6*b^2*x^2*Cosh[c + d*x]
 + a^2*d^2*x^2*Cosh[c + d*x] - d*x^3*CoshIntegral[d*x]*(6*a*b*d*Cosh[c] + (
```

```
6*b^2 + a^2*d^2)*Sinh[c]) + a^2*d*x*Sinh[c + d*x] + 6*a*b*d*x^2*Sinh[c + d*
x] - d*x^3*(6*b^2*Cosh[c] + a^2*d^2*Cosh[c] + 6*a*b*d*Sinh[c])*SinhIntegral
[d*x])/x^3
```

Maple [A]
time = 0.70, size = 287, normalized size = 1.67

method	result
risch	$\frac{d^{3} a^{2} {\mathrm e}^{-c} \operatorname{expIntegral}(1,d x)}{12}+\frac{d a b \,{\mathrm e}^{-d x -c}}{2 x}-\frac{a b \,{\mathrm e}^{-d x -c}}{2 x^{2}}-\frac{d^{2} a b \,{\mathrm e}^{-c} \operatorname{expIntegral}(1,d x)}{2}-\frac{d^{2} a^{2} {\mathrm e}^{-d x -c}}{12 x}-\frac{a^{2} {\mathrm e}^{-d x -c}}{6 x^{3}}-\frac{b^{2} {\mathrm e}^{-d}}{2 x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*cosh(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*d^3*a^2*exp(-c)*Ei(1,d*x)+1/2*d*a*b*exp(-d*x-c)/x-1/2*a*b*exp(-d*x-c)/
x^2-1/2*d^2*a*b*exp(-c)*Ei(1,d*x)-1/12*d^2*a^2*exp(-d*x-c)/x-1/6*a^2*exp(-d
*x-c)/x^3-1/2*b^2*exp(-d*x-c)/x+1/2*d*b^2*exp(-c)*Ei(1,d*x)+1/12*d*a^2*exp(
-d*x-c)/x^2-1/2*a*b/x^2*exp(d*x+c)-1/2*d*a*b/x*exp(d*x+c)-1/2*d^2*a*b*exp(c
)*Ei(1,-d*x)-1/12*d^3*a^2*exp(c)*Ei(1,-d*x)-1/6*a^2/x^3*exp(d*x+c)-1/12*d*a
^2/x^2*exp(d*x+c)-1/2*b^2/x*exp(d*x+c)-1/2*d*b^2*exp(c)*Ei(1,-d*x)-1/12*d^2
*a^2/x*exp(d*x+c)
```

Maxima [A]
time = 0.34, size = 117, normalized size = 0.68

$$\frac{1}{6} \, {\left(a^{2} d^{2} e^{\left(-c\right)} \Gamma\left(-2, d x\right) - a^{2} d^{2} e^{c} \Gamma\left(-2, -d x\right) + 3 \, a b d e^{\left(-c\right)} \Gamma\left(-1, d x\right) + 3 \, a b d e^{c} \Gamma\left(-1, -d x\right) - 3 \, b^{2} {\rm Ei}\left(-d x\right) e^{\left(-c\right)} + 3 \, b^{2} {\rm Ei}\left(d x\right) e^{c}\right)} d - \frac{{\left(3 \, b^{2} x^{2} + 3 \, a b x + a^{2}\right)} \cosh\left(d x + c\right)}{3 \, x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*d^2*e^(-c)*gamma(-2, d*x) - a^2*d^2*e^c*gamma(-2, -d*x) + 3*a*b*d*
e^(-c)*gamma(-1, d*x) + 3*a*b*d*e^c*gamma(-1, -d*x) - 3*b^2*Ei(-d*x)*e^(-c)
 + 3*b^2*Ei(d*x)*e^c)*d - 1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*cosh(d*x + c)/x^3
```

Fricas [A]
time = 0.34, size = 194, normalized size = 1.13

$$-\frac{2 \, {\left(6 \, a b x + {\left(a^{2} d^{2} + 6 \, b^{2}\right)} x^{2} + 2 \, a^{2}\right)} \cosh\left(d x + c\right) - {\left({\left(a^{2} d^{3} + 6 \, a b d^{2} + 6 \, b^{2} d\right)} x^{3} {\rm Ei}\left(d x\right) - {\left(a^{2} d^{3} - 6 \, a b d^{2} + 6 \, b^{2} d\right)} x^{3} {\rm Ei}\left(-d x\right)\right)} \cosh\left(c\right) + 2 \, {\left(6 \, a b d x^{2} + a^{2} d x\right)} \sinh\left(d x + c\right) - {\left({\left(a^{2} d^{3} + 6 \, a b d^{2} + 6 \, b^{2} d\right)} x^{3} {\rm Ei}\left(d x\right) + {\left(a^{2} d^{3} - 6 \, a b d^{2} + 6 \, b^{2} d\right)} x^{3} {\rm Ei}\left(-d x\right)\right)} \sinh\left(c\right)}{12 \, x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(6*a*b*x + (a^2*d^2 + 6*b^2)*x^2 + 2*a^2)*cosh(d*x + c) - ((a^2*d^
3 + 6*a*b*d^2 + 6*b^2*d)*x^3*Ei(d*x) - (a^2*d^3 - 6*a*b*d^2 + 6*b^2*d)*x^3*
Ei(-d*x))*cosh(c) + 2*(6*a*b*d*x^2 + a^2*d*x)*sinh(d*x + c) - ((a^2*d^3 + 6
*a*b*d^2 + 6*b^2*d)*x^3*Ei(d*x) + (a^2*d^3 - 6*a*b*d^2 + 6*b^2*d)*x^3*Ei(-d
*x))*sinh(c))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2 \cosh(c+dx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*cosh(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x)**2*cosh(c + d*x)/x**4, x)
```

Giac [A]
time = 0.42, size = 285, normalized size = 1.66

$$-\frac{a^2 d^3 x^3 \mathrm{Ei}(-dx) e^{(-c)} - a^2 d^3 x^3 \mathrm{Ei}(dx) e^c - 6abd^2x^3\mathrm{Ei}(-dx)e^{(-c)} - 6abd^2x^3\mathrm{Ei}(dx)e^c + 6b^2dx^3\mathrm{Ei}(-dx)e^{(-c)} - 6b^2dx^3\mathrm{Ei}(dx)e^c + a^2d^2x^2e^{(dx+c)} + a^2d^2x^2e^{(-dx-c)} + 6abdx^2e^{(dx+c)} - 6abdx^2e^{(-dx-c)} + a^2dxe^{(dx+c)} + 6b^2x^2e^{(dx+c)} - a^2dxe^{(-dx-c)} + 6b^2x^2e^{(-dx-c)} + 6abxe^{(dx+c)} + 6abxe^{(-dx-c)} + 2a^2e^{(dx+c)} + 2a^2e^{(-dx-c)}}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(a^2*d^3*x^3*Ei(-d*x)*e^(-c) - a^2*d^3*x^3*Ei(d*x)*e^c - 6*a*b*d^2*x^
3*Ei(-d*x)*e^(-c) - 6*a*b*d^2*x^3*Ei(d*x)*e^c + 6*b^2*d*x^3*Ei(-d*x)*e^(-c)
 - 6*b^2*d*x^3*Ei(d*x)*e^c + a^2*d^2*x^2*e^(d*x + c) + a^2*d^2*x^2*e^(-d*x
- c) + 6*a*b*d*x^2*e^(d*x + c) - 6*a*b*d*x^2*e^(-d*x - c) + a^2*d*x*e^(d*x
+ c) + 6*b^2*x^2*e^(d*x + c) - a^2*d*x*e^(-d*x - c) + 6*b^2*x^2*e^(-d*x - c
) + 6*a*b*x*e^(d*x + c) + 6*a*b*x*e^(-d*x - c) + 2*a^2*e^(d*x + c) + 2*a^2*
e^(-d*x - c))/x^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,{(a+b\,x)}^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x)^2)/x^4,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x)^2)/x^4, x)
```

3.17 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x^5} \, dx$

Optimal. Leaf size=248

$$-\frac{a^2 \cosh(c+dx)}{4x^4}-\frac{2ab \cosh(c+dx)}{3x^3}-\frac{b^2 \cosh(c+dx)}{2x^2}-\frac{a^2d^2 \cosh(c+dx)}{24x^2}-\frac{abd^2 \cosh(c+dx)}{3x}+\frac{1}{2}b^2d^2 \cosh($$

```
[Out] 1/2*b^2*d^2*Chi(d*x)*cosh(c)+1/24*a^2*d^4*Chi(d*x)*cosh(c)-1/4*a^2*cosh(d*x
+c)/x^4-2/3*a*b*cosh(d*x+c)/x^3-1/2*b^2*cosh(d*x+c)/x^2-1/24*a^2*d^2*cosh(d
*x+c)/x^2-1/3*a*b*d^2*cosh(d*x+c)/x+1/3*a*b*d^3*cosh(c)*Shi(d*x)+1/3*a*b*d^
3*Chi(d*x)*sinh(c)+1/2*b^2*d^2*Shi(d*x)*sinh(c)+1/24*a^2*d^4*Shi(d*x)*sinh(
c)-1/12*a^2*d*sinh(d*x+c)/x^3-1/3*a*b*d*sinh(d*x+c)/x^2-1/2*b^2*d*sinh(d*x+
c)/x-1/24*a^2*d^3*sinh(d*x+c)/x
```

Rubi [A]
time = 0.36, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\frac{1}{24}a^2d^4\cosh(c)\text{Chi}(dx)+\frac{1}{24}a^2d^4\sinh(c)\text{Shi}(dx)-\frac{a^2d^3\sinh(c+dx)}{24x}-\frac{a^2d^2\cosh(c+dx)}{24x^2}-\frac{a^2\cosh(c+dx)}{4x^4}-\frac{a^2d\sinh(c+dx)}{12x^3}+\frac{1}{3}abd^3\sinh(c)\text{Chi}(dx)+\frac{1}{3}abd^3\cosh(c)\text{Shi}(dx)-\frac{abd^2\cosh(c+dx)}{3x}-\frac{2ab\cosh(c+dx)}{3x^3}-\frac{abd\sinh(c+dx)}{3x^2}+\frac{1}{2}b^2d^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}b^2d^2\sinh(c)\text{Shi}(dx)-\frac{b^2\cosh(c+dx)}{2x^2}-\frac{b^2d\sinh(c+dx)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Cosh[c + d*x])/x^5,x]
```

```
[Out] -1/4*(a^2*Cosh[c + d*x])/x^4 - (2*a*b*Cosh[c + d*x])/(3*x^3) - (b^2*Cosh[c
 + d*x])/(2*x^2) - (a^2*d^2*Cosh[c + d*x])/(24*x^2) - (a*b*d^2*Cosh[c + d*x]
)/(3*x) + (b^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (a^2*d^4*Cosh[c]*CoshInte
gral[d*x])/24 + (a*b*d^3*CoshIntegral[d*x]*Sinh[c])/3 - (a^2*d*Sinh[c + d*x
])/(12*x^3) - (a*b*d*Sinh[c + d*x])/(3*x^2) - (b^2*d*Sinh[c + d*x])/(2*x) -
 (a^2*d^3*Sinh[c + d*x])/(24*x) + (a*b*d^3*Cosh[c]*SinhIntegral[d*x])/3 + (
b^2*d^2*Sinh[c]*SinhIntegral[d*x])/2 + (a^2*d^4*Sinh[c]*SinhIntegral[d*x])/
24
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2\cosh(c+dx)}{x^5}\,dx &= \int \left(\frac{a^2\cosh(c+dx)}{x^5}+\frac{2ab\cosh(c+dx)}{x^4}+\frac{b^2\cosh(c+dx)}{x^3}\right)dx \\
&= a^2\int \frac{\cosh(c+dx)}{x^5}\,dx+(2ab)\int \frac{\cosh(c+dx)}{x^4}\,dx+b^2\int \frac{\cosh(c+dx)}{x^3}\,dx \\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{2ab\cosh(c+dx)}{3x^3}-\frac{b^2\cosh(c+dx)}{2x^2}+\frac{1}{4}\left(a^2d\right)\int \frac{\sinh(c
\\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{2ab\cosh(c+dx)}{3x^3}-\frac{b^2\cosh(c+dx)}{2x^2}-\frac{a^2d\sinh(c+dx)}{12x^3}
\\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{2ab\cosh(c+dx)}{3x^3}-\frac{b^2\cosh(c+dx)}{2x^2}-\frac{a^2d^2\cosh(c+dx)}{24x^2}
\\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{2ab\cosh(c+dx)}{3x^3}-\frac{b^2\cosh(c+dx)}{2x^2}-\frac{a^2d^2\cosh(c+dx)}{24x^2}
\\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{2ab\cosh(c+dx)}{3x^3}-\frac{b^2\cosh(c+dx)}{2x^2}-\frac{a^2d^2\cosh(c+dx)}{24x^2}
\\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{2ab\cosh(c+dx)}{3x^3}-\frac{b^2\cosh(c+dx)}{2x^2}-\frac{a^2d^2\cosh(c+dx)}{24x^2}
\end{aligned}$$

Mathematica [A]
time = 0.36, size = 206, normalized size = 0.83

$$-\frac{6a^2\cosh(c+dx)+16abx\cosh(c+dx)+12b^2x^2\cosh(c+dx)+a^2d^2x^2\cosh(c+dx)+8abd^2x^3\cosh(c+dx)-d^2x^4\text{Chi}(dx)\left(\left(12b^2+a^2d^2\right)\cosh(c)+8abd\sinh(c)\right)+2a^2dx\sinh(c+dx)+8abdx^2\sinh(c+dx)+12b^2dx^3\sinh(c+dx)+a^2d^3x^3\sinh(c+dx)-d^2x^4\left(8abd\cosh(c)+12b^2\sinh(c)+a^2d^2\sinh(c)\right)\text{Shi}(dx)}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x^5,x]
```

```
[Out] -1/24*(6*a^2*Cosh[c + d*x] + 16*a*b*x*Cosh[c + d*x] + 12*b^2*x^2*Cosh[c + d
*x] + a^2*d^2*x^2*Cosh[c + d*x] + 8*a*b*d^2*x^3*Cosh[c + d*x] - d^2*x^4*Cos
hIntegral[d*x]*((12*b^2 + a^2*d^2)*Cosh[c] + 8*a*b*d*Sinh[c]) + 2*a^2*d*x*S
inh[c + d*x] + 8*a*b*d*x^2*Sinh[c + d*x] + 12*b^2*d*x^3*Sinh[c + d*x] + a^2
*d^3*x^3*Sinh[c + d*x] - d^2*x^4*(8*a*b*d*Cosh[c] + 12*b^2*Sinh[c] + a^2*d^
2*Sinh[c])*SinhIntegral[d*x])/x^4
```

Maple [A]
time = 0.70, size = 396, normalized size = 1.60

method	result
risch	$-\frac{d^4 a^2 \mathrm{e}^{-c} \operatorname{expIntegral}(1,dx)}{48}+\frac{d^3 a^2 \mathrm{e}^{-dx-c}}{48x}-\frac{d^2 a^2 \mathrm{e}^{-dx-c}}{48x^2}+\frac{d\,a^2 \mathrm{e}^{-dx-c}}{24x^3}-\frac{a^2 \mathrm{e}^{-dx-c}}{8x^4}-\frac{d^2 ab\,\mathrm{e}^{-dx-c}}{6x}+\frac{dab\,\mathrm{e}^{-dx-c}}{6x^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*cosh(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*d^4*a^2*exp(-c)*Ei(1,d*x)+1/48*d^3*a^2*exp(-d*x-c)/x-1/48*d^2*a^2*exp
(-d*x-c)/x^2+1/24*d*a^2*exp(-d*x-c)/x^3-1/8*a^2*exp(-d*x-c)/x^4-1/6*d^2*a*b
*exp(-d*x-c)/x+1/6*d*a*b*exp(-d*x-c)/x^2-1/3*a*b*exp(-d*x-c)/x^3+1/4*d*b^2*
exp(-d*x-c)/x-1/4*b^2*exp(-d*x-c)/x^2+1/6*d^3*a*b*exp(-c)*Ei(1,d*x)-1/4*d^2
*b^2*exp(-c)*Ei(1,d*x)-1/4*d*b^2/x*exp(d*x+c)-1/4*d^2*b^2*exp(c)*Ei(1,-d*x)
-1/3*a*b/x^3*exp(d*x+c)-1/6*d*a*b/x^2*exp(d*x+c)-1/6*d^2*a*b/x*exp(d*x+c)-1
/6*d^3*a*b*exp(c)*Ei(1,-d*x)-1/48*d^4*a^2*exp(c)*Ei(1,-d*x)-1/4*b^2/x^2*exp
(d*x+c)-1/8*a^2/x^4*exp(d*x+c)-1/24*d*a^2/x^3*exp(d*x+c)-1/48*d^2*a^2/x^2*e
xp(d*x+c)-1/48*d^3*a^2/x*exp(d*x+c)
```

Maxima [A]
time = 0.35, size = 128, normalized size = 0.52

$$\frac{1}{24}\left(3\,a^2 d^3 e^{(-c)}\Gamma(-3,dx)+3\,a^2 d^3 e^c\Gamma(-3,-dx)+8\,abd^2 e^{(-c)}\Gamma(-2,dx)-8\,abd^2 e^c\Gamma(-2,-dx)+6\,b^2 de^{(-c)}\Gamma(-1,dx)+6\,b^2 de^c\Gamma(-1,-dx)\right)d-\frac{(6\,b^2x^2+8\,abx+3\,a^2)\cosh(dx+c)}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] 1/24*(3*a^2*d^3*e^(-c)*gamma(-3, d*x) + 3*a^2*d^3*e^c*gamma(-3, -d*x) + 8*a
*b*d^2*e^(-c)*gamma(-2, d*x) - 8*a*b*d^2*e^c*gamma(-2, -d*x) + 6*b^2*d*e^(-
c)*gamma(-1, d*x) + 6*b^2*d*e^c*gamma(-1, -d*x))*d - 1/12*(6*b^2*x^2 + 8*a*
b*x + 3*a^2)*cosh(d*x + c)/x^4
```

Fricas [A]
time = 0.36, size = 231, normalized size = 0.93

$$-\frac{2\,(8\,abd^2x^3+16\,abx+(a^2d^2+12\,b^2)x^2+6\,a^2)\cosh(dx+c)-((a^2d^4+8\,abd^3+12\,b^2d^2)x^4\operatorname{Ei}(dx)+(a^2d^4-8\,abd^3+12\,b^2d^2)x^4\operatorname{Ei}(-dx))\cosh(c)+2\,(8\,abdx^2+2\,a^2dx+(a^2d^3+12\,b^2d)x^3)\sinh(dx+c)-((a^2d^4+8\,abd^3+12\,b^2d^2)x^4\operatorname{Ei}(dx)-(a^2d^4-8\,abd^3+12\,b^2d^2)x^4\operatorname{Ei}(-dx))\sinh(c)}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(2*(8*a*b*d^2*x^3 + 16*a*b*x + (a^2*d^2 + 12*b^2)*x^2 + 6*a^2)*cosh(d
*x + c) - ((a^2*d^4 + 8*a*b*d^3 + 12*b^2*d^2)*x^4*Ei(d*x) + (a^2*d^4 - 8*a*
b*d^3 + 12*b^2*d^2)*x^4*Ei(-d*x))*cosh(c) + 2*(8*a*b*d*x^2 + 2*a^2*d*x + (a
^2*d^3 + 12*b^2*d)*x^3)*sinh(d*x + c) - ((a^2*d^4 + 8*a*b*d^3 + 12*b^2*d^2)
*x^4*Ei(d*x) - (a^2*d^4 - 8*a*b*d^3 + 12*b^2*d^2)*x^4*Ei(-d*x))*sinh(c))/x^
4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*cosh(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x)**2*cosh(c + d*x)/x**5, x)
```

Giac [A]
time = 0.41, size = 395, normalized size = 1.59

$$\frac{a^2d^4x^4\mathrm{Ei}(-dx)\,e^{(-c)} + a^2d^4x^4\mathrm{Ei}(dx)\,e^{c} - 8\,abd^3x^4\mathrm{Ei}(-dx)\,e^{(-c)} + 8\,abd^3x^4\mathrm{Ei}(dx)\,e^{c} + 12\,b^2d^2x^4\mathrm{Ei}(-dx)\,e^{(-c)} + 12\,b^2d^2x^4\mathrm{Ei}(dx)\,e^{c} - a^2d^3x^3e^{(dx+c)} + a^2d^3x^3e^{(-dx-c)} - 8\,abd^2x^3e^{(dx+c)} - 8\,abd^2x^3e^{(-dx-c)} - a^2d^2x^2e^{(dx+c)} - 12\,b^2dx^3e^{(dx+c)} - a^2d^2x^2e^{(-dx-c)} + 12\,b^2dx^3e^{(-dx-c)} - 8\,abdx^2e^{(dx+c)} + 8\,abdx^2e^{(-dx-c)} - 2\,a^2dxe^{(dx+c)} - 12\,b^2x^2e^{(dx+c)} + 2\,a^2dxe^{(-dx-c)} - 12\,b^2x^2e^{(-dx-c)} - 16\,abxe^{(dx+c)} - 16\,abxe^{(-dx-c)} - 6\,a^2e^{(dx+c)} - 6\,a^2e^{(-dx-c)}}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] 1/48*(a^2*d^4*x^4*Ei(-d*x)*e^(-c) + a^2*d^4*x^4*Ei(d*x)*e^c - 8*a*b*d^3*x^4
*Ei(-d*x)*e^(-c) + 8*a*b*d^3*x^4*Ei(d*x)*e^c + 12*b^2*d^2*x^4*Ei(-d*x)*e^(-
c) + 12*b^2*d^2*x^4*Ei(d*x)*e^c - a^2*d^3*x^3*e^(d*x + c) + a^2*d^3*x^3*e^(
-d*x - c) - 8*a*b*d^2*x^3*e^(d*x + c) - 8*a*b*d^2*x^3*e^(-d*x - c) - a^2*d^
2*x^2*e^(d*x + c) - 12*b^2*d*x^3*e^(d*x + c) - a^2*d^2*x^2*e^(-d*x - c) + 1
2*b^2*d*x^3*e^(-d*x - c) - 8*a*b*d*x^2*e^(d*x + c) + 8*a*b*d*x^2*e^(-d*x -
c) - 2*a^2*d*x*e^(d*x + c) - 12*b^2*x^2*e^(d*x + c) + 2*a^2*d*x*e^(-d*x - c
) - 12*b^2*x^2*e^(-d*x - c) - 16*a*b*x*e^(d*x + c) - 16*a*b*x*e^(-d*x - c)
- 6*a^2*e^(d*x + c) - 6*a^2*e^(-d*x - c))/x^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,{(a+b\,x)}^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x)^2)/x^5,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x)^2)/x^5, x)
```

3.18 $\int \frac{x^4 \cosh(c+dx)}{a+bx} \, dx$

Optimal. Leaf size=219

$$-\frac{6\cosh(c+dx)}{bd^4}-\frac{a^2\cosh(c+dx)}{b^3d^2}+\frac{2ax\cosh(c+dx)}{b^2d^2}-\frac{3x^2\cosh(c+dx)}{bd^2}+\frac{a^4\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^5}-$$

```
[Out] a^4*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^5-6*cosh(d*x+c)/b/d^4-a^2*cosh(d*x+c)/b
^3/d^2+2*a*x*cosh(d*x+c)/b^2/d^2-3*x^2*cosh(d*x+c)/b/d^2-a^4*Shi(a*d/b+d*x)
*sinh(-c+a*d/b)/b^5-2*a*sinh(d*x+c)/b^2/d^3-a^3*sinh(d*x+c)/b^4/d+6*x*sinh(
d*x+c)/b/d^3+a^2*x*sinh(d*x+c)/b^3/d-a*x^2*sinh(d*x+c)/b^2/d+x^3*sinh(d*x+c
)/b/d
```

Rubi [A]
time = 0.34, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6874, 2717, 3377, 2718, 3384, 3379, 3382}

$$\frac{a^4\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{b^5}+\frac{a^4\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{b^5}-\frac{a^3\sinh(c+dx)}{b^4d}-\frac{a^2\cosh(c+dx)}{b^3d^2}+\frac{a^2x\sinh(c+dx)}{b^3d}-\frac{2a\sinh(c+dx)}{b^2d^3}+\frac{2ax\cosh(c+dx)}{b^2d^2}-\frac{ax^2\sinh(c+dx)}{b^2d}-\frac{6\cosh(c+dx)}{bd^4}+\frac{6x\sinh(c+dx)}{bd^3}-\frac{3x^2\cosh(c+dx)}{bd^2}+\frac{x^3\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] (-6*Cosh[c + d*x])/(b*d^4) - (a^2*Cosh[c + d*x])/(b^3*d^2) + (2*a*x*Cosh[c
+ d*x])/(b^2*d^2) - (3*x^2*Cosh[c + d*x])/(b*d^2) + (a^4*Cosh[c - (a*d)/b]*
CoshIntegral[(a*d)/b + d*x])/b^5 - (2*a*Sinh[c + d*x])/(b^2*d^3) - (a^3*Sin
h[c + d*x])/(b^4*d) + (6*x*Sinh[c + d*x])/(b*d^3) + (a^2*x*Sinh[c + d*x])/(
b^3*d) - (a*x^2*Sinh[c + d*x])/(b^2*d) + (x^3*Sinh[c + d*x])/(b*d) + (a^4*S
inh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^5
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^4\cosh(c+dx)}{a+bx}\,dx = \int \left(-\frac{a^3\cosh(c+dx)}{b^4} + \frac{a^2x\cosh(c+dx)}{b^3} - \frac{ax^2\cosh(c+dx)}{b^2} + \frac{x^3\cosh(c+dx)}{b} + \cdots\right.$$

$$= -\frac{a^3\int\cosh(c+dx)\,dx}{b^4} + \frac{a^4\int\frac{\cosh(c+dx)}{a+bx}\,dx}{b^4} + \frac{a^2\int x\cosh(c+dx)\,dx}{b^3} - \frac{a\int x^2\cosh(c\cdots}{b^2}$$

$$= -\frac{a^3\sinh(c+dx)}{b^4d} + \frac{a^2x\sinh(c+dx)}{b^3d} - \frac{ax^2\sinh(c+dx)}{b^2d} + \frac{x^3\sinh(c+dx)}{bd} - \frac{a^2\int\cdots}{\cdots}$$

$$= -\frac{a^2\cosh(c+dx)}{b^3d^2} + \frac{2ax\cosh(c+dx)}{b^2d^2} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{a^4\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}\cdots\right.}{b^5}$$

$$= -\frac{a^2\cosh(c+dx)}{b^3d^2} + \frac{2ax\cosh(c+dx)}{b^2d^2} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{a^4\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}\cdots\right.}{b^5}$$

$$= -\frac{6\cosh(c+dx)}{bd^4} - \frac{a^2\cosh(c+dx)}{b^3d^2} + \frac{2ax\cosh(c+dx)}{b^2d^2} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{a^4\text{co}\cdots}{\cdots}$$

Mathematica [A]
time = 0.40, size = 159, normalized size = 0.73

$$\frac{a^4d^4\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right) - b\left(b\left(a^2d^2-2abd^2x+3b^2(2+d^2x^2)\right)\cosh(c+dx) + d\left(a^3d^2-a^2bd^2x+ab^2(2+d^2x^2)-b^3x(6+d^2x^2)\right)\sinh(c+dx)\right) + a^4d^4\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{b^5d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] (a^4*d^4*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] - b*(b*(a^2*d^2 - 2*a*
b*d^2*x + 3*b^2*(2 + d^2*x^2))*Cosh[c + d*x] + d*(a^3*d^2 - a^2*b*d^2*x + a
*b^2*(2 + d^2*x^2) - b^3*x*(6 + d^2*x^2))*Sinh[c + d*x]) + a^4*d^4*Sinh[c -
 (a*d)/b]*SinhIntegral[d*(a/b + x)])/(b^5*d^4)
```

Maple [A]
time = 0.97, size = 442, normalized size = 2.02

method	result
risch	$-\frac{e^{\frac{ad-bc}{b}} \operatorname{expIntegral}\left(1, dx+c+\frac{ad-bc}{b}\right) a^{4}}{2b^{5}}-\frac{3\,e^{-dx-c}x}{d^{3}b}+\frac{e^{-dx-c}a}{d^{3}b^{2}}+\frac{e^{-dx-c}ax}{d^{2}b^{2}}-\frac{3\,e^{-dx-c}}{d^{4}b}-\frac{e^{-dx-c}x^{3}}{2db}+\frac{e^{-dx-c}a^{3}}{2d\,b^{4}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cosh(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b^5*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^4-3/d^3*exp(-d*x-c)/b*x
+1/d^3*exp(-d*x-c)/b^2*a+1/d^2*exp(-d*x-c)/b^2*a*x-3/d^4*exp(-d*x-c)/b-1/2/
d*exp(-d*x-c)/b*x^3+1/2/d*exp(-d*x-c)/b^4*a^3-1/2/d*exp(-d*x-c)/b^3*a^2*x+1
/2/d*exp(-d*x-c)/b^2*a*x^2-3/2/d^2*exp(-d*x-c)/b*x^2-1/2/d^2*exp(-d*x-c)/b^
3*a^2+3/d^3/b*exp(d*x+c)*x+1/2/d/b*exp(d*x+c)*x^3-3/2/d^2/b*exp(d*x+c)*x^2-
1/2/d/b^4*a^3*exp(d*x+c)-1/d^3/b^2*a*exp(d*x+c)-1/2/d^2/b^3*a^2*exp(d*x+c)+
1/2/d/b^3*a^2*exp(d*x+c)*x+1/d^2/b^2*a*exp(d*x+c)*x-1/2/b^5*exp(-(a*d-b*c)/
b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^4-3/d^4/b*exp(d*x+c)-1/2/d/b^2*a*exp(d*x+c)*x
^2
```

Maxima [A]
time = 0.31, size = 437, normalized size = 2.00

$$-\frac{1}{24}\,d\left(\frac{12\,a^{4}\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_{1}\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_{1}\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^{4}d}-\frac{12\,a^{3}\left(\frac{(dxe^{c}-e^{c})e^{(dx)}}{d^{2}}+\frac{(dx+1)e^{(-dx-c)}}{d^{2}}\right)}{b^{4}}+\frac{6\,a^{2}\left(\frac{(d^{2}x^{2}e^{c}-2\,dxe^{c}+2\,e^{c})e^{(dx)}}{d^{3}}+\frac{(d^{2}x^{2}+2\,dx+2)e^{(-dx-c)}}{d^{3}}\right)}{b^{3}}-\frac{4\,a\left(\frac{(d^{3}x^{3}e^{c}-3\,d^{2}x^{2}e^{c}+6\,dxe^{c}-6\,e^{c})e^{(dx)}}{d^{4}}+\frac{(d^{3}x^{3}+3\,d^{2}x^{2}+6\,dx+6)e^{(-dx-c)}}{d^{4}}\right)}{b^{2}}+\frac{3\left(\frac{(d^{4}x^{4}e^{c}-4\,d^{3}x^{3}e^{c}+12\,d^{2}x^{2}e^{c}-24\,dxe^{c}+24\,e^{c})e^{(dx)}}{d^{5}}+\frac{(d^{4}x^{4}+4\,d^{3}x^{3}+12\,d^{2}x^{2}+24\,dx+24)e^{(-dx-c)}}{d^{5}}\right)}{b}+\frac{24\,a^{4}\cosh(dx+c)\log(bx+a)}{b^{5}d}\right)+\frac{1}{12}\left(\frac{12\,a^{4}\log(bx+a)}{b^{5}}+\frac{3\,b^{3}x^{4}-4\,ab^{2}x^{3}+6\,a^{2}bx^{2}-12\,a^{3}x}{b^{4}}\right)\cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/24*d*(12*a^4*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c -
 a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b^4*d) - 12*a^3*((d*x*e^c - e
^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/b^4 + 6*a^2*((d^2*x^2*e^c - 2
*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^(-d*x - c)/d^3)/b^3
 - 4*a*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4 + (d^
3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4)/b^2 + 3*((d^4*x^4*e^c - 4*
d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*e^(d*x)/d^5 + (d^4*x^4
+ 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*e^(-d*x - c)/d^5)/b + 24*a^4*cosh(d
```

```
*x + c)*log(b*x + a)/(b^5*d)) + 1/12*(12*a^4*log(b*x + a)/b^5 + (3*b^3*x^4
- 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^3*x)/b^4)*cosh(d*x + c)
```

Fricas [A]
time = 0.37, size = 236, normalized size = 1.08

$$-\frac{2\left(3\,b^{4}d^{2}x^{2}-2\,ab^{3}d^{2}x+a^{2}b^{2}d^{2}+6\,b^{4}\right)\cosh\left(dx+c\right)-\left(a^{4}d^{4}\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+a^{4}d^{4}\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-2\left(b^{4}d^{3}x^{3}-ab^{3}d^{3}x^{2}-a^{3}bd^{3}-2\,ab^{3}d+\left(a^{2}b^{2}d^{3}+6\,b^{4}d\right)x\right)\sinh\left(dx+c\right)+\left(a^{4}d^{4}\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-a^{4}d^{4}\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2\,b^{5}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(3*b^4*d^2*x^2 - 2*a*b^3*d^2*x + a^2*b^2*d^2 + 6*b^4)*cosh(d*x + c)
 - (a^4*d^4*Ei((b*d*x + a*d)/b) + a^4*d^4*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c
- a*d)/b) - 2*(b^4*d^3*x^3 - a*b^3*d^3*x^2 - a^3*b*d^3 - 2*a*b^3*d + (a^2*b
^2*d^3 + 6*b^4*d)*x)*sinh(d*x + c) + (a^4*d^4*Ei((b*d*x + a*d)/b) - a^4*d^4
*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^5*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}\cosh\left(c+dx\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cosh(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x**4*cosh(c + d*x)/(a + b*x), x)
```

Giac [A]
time = 0.42, size = 407, normalized size = 1.86

$$\frac{b^{4}d^{3}x^{3}e^{(dx+c)}-b^{4}d^{3}x^{3}e^{(-dx-c)}-ab^{3}d^{3}x^{2}e^{(dx+c)}+ab^{3}d^{3}x^{2}e^{(-dx-c)}+a^{4}d^{4}\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+a^{4}d^{4}\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+a^{2}b^{2}d^{3}xe^{(dx+c)}-3\,b^{4}d^{2}x^{2}e^{(dx+c)}-a^{2}b^{2}d^{3}xe^{(-dx-c)}-3\,b^{4}d^{2}x^{2}e^{(-dx-c)}-a^{3}bd^{3}e^{(dx+c)}+2\,ab^{3}d^{2}xe^{(dx+c)}+a^{3}bd^{3}e^{(-dx-c)}+2\,ab^{3}d^{2}xe^{(-dx-c)}-a^{2}b^{2}d^{2}e^{(dx+c)}+6\,b^{4}dxe^{(dx+c)}-a^{2}b^{2}d^{2}e^{(-dx-c)}-6\,b^{4}dxe^{(-dx-c)}-2\,ab^{3}de^{(dx+c)}+2\,ab^{3}de^{(-dx-c)}-6\,b^{4}e^{(dx+c)}-6\,b^{4}e^{(-dx-c)}}{2\,b^{5}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^4*d^3*x^3*e^(d*x + c) - b^4*d^3*x^3*e^(-d*x - c) - a*b^3*d^3*x^2*e^(
d*x + c) + a*b^3*d^3*x^2*e^(-d*x - c) + a^4*d^4*Ei((b*d*x + a*d)/b)*e^(c -
a*d/b) + a^4*d^4*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + a^2*b^2*d^3*x*e^(d*x
 + c) - 3*b^4*d^2*x^2*e^(d*x + c) - a^2*b^2*d^3*x*e^(-d*x - c) - 3*b^4*d^2*
x^2*e^(-d*x - c) - a^3*b*d^3*e^(d*x + c) + 2*a*b^3*d^2*x*e^(d*x + c) + a^3*
b*d^3*e^(-d*x - c) + 2*a*b^3*d^2*x*e^(-d*x - c) - a^2*b^2*d^2*e^(d*x + c) +
 6*b^4*d*x*e^(d*x + c) - a^2*b^2*d^2*e^(-d*x - c) - 6*b^4*d*x*e^(-d*x - c)
- 2*a*b^3*d*e^(d*x + c) + 2*a*b^3*d*e^(-d*x - c) - 6*b^4*e^(d*x + c) - 6*b^
4*e^(-d*x - c))/(b^5*d^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \cosh(c + d\,x)}{a + b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*cosh(c + d*x))/(a + b*x),x)
[Out] int((x^4*cosh(c + d*x))/(a + b*x), x)
```

3.19 $\int \frac{x^3 \cosh(c+dx)}{a+bx} dx$

Optimal. Leaf size=150

$$\frac{a\cosh(c+dx)}{b^2d^2} - \frac{2x\cosh(c+dx)}{bd^2} - \frac{a^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{2\sinh(c+dx)}{bd^3} + \frac{a^2\sinh(c+dx)}{b^3d} - \frac{ax\sinh(}{b^2}$$

```
[Out] -a^3*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^4+a*cosh(d*x+c)/b^2/d^2-2*x*cosh(d*x+c
)/b/d^2+a^3*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^4+2*sinh(d*x+c)/b/d^3+a^2*sinh(
d*x+c)/b^3/d-a*x*sinh(d*x+c)/b^2/d+x^2*sinh(d*x+c)/b/d
```

Rubi [A]
time = 0.25, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6874, 2717, 3377, 2718, 3384, 3379, 3382}

$$-\frac{a^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^4} - \frac{a^3\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^4} + \frac{a^2\sinh(c+dx)}{b^3d} + \frac{a\cosh(c+dx)}{b^2d^2} - \frac{ax\sinh(c+dx)}{b^2d} + \frac{2\sinh(c+dx)}{bd^3} - \frac{2x\cosh(c+dx)}{bd^2} + \frac{x^2\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] (a*Cosh[c + d*x])/(b^2*d^2) - (2*x*Cosh[c + d*x])/(b*d^2) - (a^3*Cosh[c - (
a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^4 + (2*Sinh[c + d*x])/(b*d^3) + (a^2
*Sinh[c + d*x])/(b^3*d) - (a*x*Sinh[c + d*x])/(b^2*d) + (x^2*Sinh[c + d*x])
/(b*d) - (a^3*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^4
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^3\cosh(c+dx)}{a+bx}\,dx = \int \left(\frac{a^2\cosh(c+dx)}{b^3} - \frac{ax\cosh(c+dx)}{b^2} + \frac{x^2\cosh(c+dx)}{b} - \frac{a^3\cosh(c+dx)}{b^3(a+bx)}\right) dx$$

$$= \frac{a^2\int \cosh(c+dx)\,dx}{b^3} - \frac{a^3\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b^3} - \frac{a\int x\cosh(c+dx)\,dx}{b^2} + \frac{\int x^2\cosh(c+ \cdots}{b}$$

$$= \frac{a^2\sinh(c+dx)}{b^3d} - \frac{ax\sinh(c+dx)}{b^2d} + \frac{x^2\sinh(c+dx)}{bd} + \frac{a\int \sinh(c+dx)\,dx}{b^2d} - \frac{2\int \cdots}{\cdots}$$

$$= \frac{a\cosh(c+dx)}{b^2d^2} - \frac{2x\cosh(c+dx)}{bd^2} - \frac{a^3\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{a^2\sinh(c+ \cdots}{b^3d}$$

$$= \frac{a\cosh(c+dx)}{b^2d^2} - \frac{2x\cosh(c+dx)}{bd^2} - \frac{a^3\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{2\sinh(c+d \cdots}{bd^3}$$

Mathematica [A]
time = 0.31, size = 118, normalized size = 0.79

$$\frac{-a^3d^3\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right) + b\left(bd(a-2bx)\cosh(c+dx) + \left(a^2d^2 - abd^2x + b^2(2+d^2x^2)\right)\sinh(c+dx)\right) - a^3d^3\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{b^4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] (-(a^3*d^3*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)]) + b*(b*d*(a - 2*b*x
)*Cosh[c + d*x] + (a^2*d^2 - a*b*d^2*x + b^2*(2 + d^2*x^2))*Sinh[c + d*x])
- a^3*d^3*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/(b^4*d^3)
```

Maple [A]
time = 0.91, size = 292, normalized size = 1.95

method	result
risch	$-\frac{\mathrm{e}^{-dx-c}x^{2}}{2db}+\frac{\mathrm{e}^{-dx-c}ax}{2d\,b^{2}}-\frac{\mathrm{e}^{-dx-c}a^{2}}{2b^{3}d}-\frac{\mathrm{e}^{-dx-c}x}{d^{2}b}+\frac{\mathrm{e}^{-dx-c}a}{2d^{2}b^{2}}-\frac{\mathrm{e}^{-dx-c}}{d^{3}b}+\frac{\mathrm{e}^{\frac{ad-bc}{b}}\,\mathrm{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)a^{3}}{2b^{4}}+\mathrm{e}^{dx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*exp(-d*x-c)/b*x^2+1/2/d*exp(-d*x-c)/b^2*a*x-1/2/b^3/d*exp(-d*x-c)*a^
2-1/d^2*exp(-d*x-c)/b*x+1/2/d^2*exp(-d*x-c)/b^2*a-1/d^3*exp(-d*x-c)/b+1/2/b
^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3+1/2/d/b*exp(d*x+c)*x^2+1/2/
b^4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3+1/2/d^2/b^2*a*exp(d*x+c)
+1/2/b^3/d*exp(d*x+c)*a^2-1/d^2/b*exp(d*x+c)*x+1/d^3/b*exp(d*x+c)-1/2/d/b^2
*a*exp(d*x+c)*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(151) = 302$.
time = 0.32, size = 328, normalized size = 2.19

$$\frac{1}{12}\,d\left(\frac{6\,a^{3}\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_{1}\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_{1}\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^{3}d}-\frac{6\,a^{2}\left(\frac{(dxe^{c}-e^{c})e^{(dx)}}{d^{2}}+\frac{(dx+1)e^{(-dx-c)}}{d^{2}}\right)}{b^{3}}+\frac{3\,a\left(\frac{(d^{2}x^{2}e^{c}-2\,dxe^{c}+2\,e^{c})e^{(dx)}}{d^{3}}+\frac{(d^{2}x^{2}+2\,dx+2)e^{(-dx-c)}}{d^{3}}\right)}{b^{2}}-\frac{2\left(\frac{(d^{3}x^{3}e^{c}-3\,d^{2}x^{2}e^{c}+6\,dxe^{c}-6\,e^{c})e^{(dx)}}{d^{4}}+\frac{(d^{3}x^{3}+3\,d^{2}x^{2}+6\,dx+6)e^{(-dx-c)}}{d^{4}}\right)}{b}+\frac{12\,a^{3}\cosh\left(dx+c\right)\log\left(bx+a\right)}{b^{4}d}\right)-\frac{1}{6}\left(\frac{6\,a^{3}\log\left(bx+a\right)}{b^{4}}-\frac{2\,b^{2}x^{3}-3\,abx^{2}+6\,a^{2}x}{b^{3}}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/12*d*(6*a^3*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c - a
*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b^3*d) - 6*a^2*((d*x*e^c - e^c)
*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/b^3 + 3*a*((d^2*x^2*e^c - 2*d*x*
e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^(-d*x - c)/d^3)/b^2 - 2*
((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4 + (d^3*x^3 +
 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4)/b + 12*a^3*cosh(d*x + c)*log(b*x
+ a)/(b^4*d)) - 1/6*(6*a^3*log(b*x + a)/b^4 - (2*b^2*x^3 - 3*a*b*x^2 + 6*a^
2*x)/b^3)*cosh(d*x + c)
```

Fricas [A]
time = 0.37, size = 190, normalized size = 1.27

$$-\frac{2\left(2\,b^{3}dx-ab^{2}d\right)\cosh\left(dx+c\right)+\left(a^{3}d^{3}\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+a^{3}d^{3}\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-2\left(b^{3}d^{2}x^{2}-ab^{2}d^{2}x+a^{2}bd^{2}+2\,b^{3}\right)\sinh\left(dx+c\right)-\left(a^{3}d^{3}\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-a^{3}d^{3}\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2\,b^{4}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(2*b^3*d*x - a*b^2*d)*cosh(d*x + c) + (a^3*d^3*Ei((b*d*x + a*d)/b)
+ a^3*d^3*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(b^3*d^2*x^2 - a*b
^2*d^2*x + a^2*b*d^2 + 2*b^3)*sinh(d*x + c) - (a^3*d^3*Ei((b*d*x + a*d)/b)
- a^3*d^3*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^4*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \cosh(c + dx)}{a + bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x), x)
```

Giac [A]
time = 0.41, size = 256, normalized size = 1.71

$$\frac{b^3d^2x^2e^{(dx+c)} - b^3d^2x^2e^{(-dx-c)} - a^3d^3\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} - a^3d^3\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} - ab^2d^2xe^{(dx+c)} + ab^2d^2xe^{(-dx-c)} + a^2bd^2e^{(dx+c)} - 2\,b^3dxe^{(dx+c)} - a^2bd^2e^{(-dx-c)} - 2\,b^3dxe^{(-dx-c)} + ab^2de^{(dx+c)} + ab^2de^{(-dx-c)} + 2\,b^3e^{(dx+c)} - 2\,b^3e^{(-dx-c)}}{2\,b^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^3*d^2*x^2*e^(d*x + c) - b^3*d^2*x^2*e^(-d*x - c) - a^3*d^3*Ei((b*d*x
 + a*d)/b)*e^(c - a*d/b) - a^3*d^3*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a*
b^2*d^2*x*e^(d*x + c) + a*b^2*d^2*x*e^(-d*x - c) + a^2*b*d^2*e^(d*x + c) -
2*b^3*d*x*e^(d*x + c) - a^2*b*d^2*e^(-d*x - c) - 2*b^3*d*x*e^(-d*x - c) + a
*b^2*d*e^(d*x + c) + a*b^2*d*e^(-d*x - c) + 2*b^3*e^(d*x + c) - 2*b^3*e^(-d
*x - c))/(b^4*d^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \cosh(c + d\,x)}{a + b\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*cosh(c + d*x))/(a + b*x),x)
```

```
[Out] int((x^3*cosh(c + d*x))/(a + b*x), x)
```

3.20 $\int \frac{x^2 \cosh(c+dx)}{a+bx} \, dx$

Optimal. Leaf size=100

$$-\frac{\cosh(c+dx)}{bd^2}+\frac{a^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^3}-\frac{a\sinh(c+dx)}{b^2d}+\frac{x\sinh(c+dx)}{bd}+\frac{a^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+c\right.}{b^3}$$

```
[Out] a^2*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^3-cosh(d*x+c)/b/d^2-a^2*Shi(a*d/b+d*x)*
sinh(-c+a*d/b)/b^3-a*sinh(d*x+c)/b^2/d+x*sinh(d*x+c)/b/d
```

Rubi [A]
time = 0.19, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6874, 2717, 3377, 2718, 3384, 3379, 3382}

$$\frac{a^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{a^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^3}-\frac{a\sinh(c+dx)}{b^2d}-\frac{\cosh(c+dx)}{bd^2}+\frac{x\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] -(Cosh[c + d*x]/(b*d^2)) + (a^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*
x])/b^3 - (a*Sinh[c + d*x])/(b^2*d) + (x*Sinh[c + d*x])/(b*d) + (a^2*Sinh[c
 - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2\cosh(c+dx)}{a+bx}\,dx = \int \left(-\frac{a\cosh(c+dx)}{b^2}+\frac{x\cosh(c+dx)}{b}+\frac{a^2\cosh(c+dx)}{b^2(a+bx)}\right)dx$$

$$= -\frac{a\int\cosh(c+dx)\,dx}{b^2}+\frac{a^2\int\frac{\cosh(c+dx)}{a+bx}\,dx}{b^2}+\frac{\int x\cosh(c+dx)\,dx}{b}$$

$$= -\frac{a\sinh(c+dx)}{b^2d}+\frac{x\sinh(c+dx)}{bd}-\frac{\int\sinh(c+dx)\,dx}{bd}+\frac{\left(a^2\cosh\left(c-\frac{ad}{b}\right)\right)\int\frac{\cosh}{}}{b^2}$$

$$= -\frac{\cosh(c+dx)}{bd^2}+\frac{a^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^3}-\frac{a\sinh(c+dx)}{b^2d}+\frac{x\sinh(c+dx}{bd}$$

Mathematica [A]
time = 0.19, size = 89, normalized size = 0.89

$$\frac{a^2d^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right)+b(-b\cosh(c+dx)+d(-a+bx)\sinh(c+dx))+a^2d^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{b^3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] (a^2*d^2*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] + b*(-(b*Cosh[c + d*x]
) + d*(-a + b*x)*Sinh[c + d*x]) + a^2*d^2*Sinh[c - (a*d)/b]*SinhIntegral[d*
(a/b + x)])/(b^3*d^2)
```

Maple [A]
time = 0.88, size = 184, normalized size = 1.84

method	result
risch	$-\frac{\mathrm{e}^{-dx-c}x}{2db}+\frac{\mathrm{e}^{-dx-c}a}{2b^2d}-\frac{\mathrm{e}^{-dx-c}}{2d^2b}-\frac{\mathrm{e}^{\frac{ad-bc}{b}}\,\mathrm{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)a^2}{2b^3}+\frac{\mathrm{e}^{dx+c}x}{2db}-\frac{\mathrm{e}^{dx+c}}{2d^2b}-\frac{a\,\mathrm{e}^{dx+c}}{2b^2d}-\frac{\mathrm{e}^{-\frac{ad-bc}{b}}\,\mathrm{exp}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*exp(-d*x-c)/b*x+1/2/b^2/d*exp(-d*x-c)*a-1/2/d^2*exp(-d*x-c)/b-1/2/b^
3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2+1/2/d/b*exp(d*x+c)*x-1/2/d^2
/b*exp(d*x+c)-1/2*a/b^2/d*exp(d*x+c)-1/2/b^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-
(a*d-b*c)/b)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(103) = 206$.
time = 0.31, size = 233, normalized size = 2.33

$$-\frac{1}{4}\,d\left(\frac{2\,a^2\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^2d}-\frac{2\,a\left(\frac{(dxe^c-e^c)e^{(dx)}}{d^2}+\frac{(dx+1)e^{(-dx-c)}}{d^2}\right)}{b^2}+\frac{\frac{(d^2x^2e^c-2\,dxe^c+2\,e^c)e^{(dx)}}{d^3}+\frac{(d^2x^2+2\,dx+2)e^{(-dx-c)}}{d^3}}{b}+\frac{4\,a^2\cosh\left(dx+c\right)\log\left(bx+a\right)}{b^3d}\right)+\frac{1}{2}\left(\frac{2\,a^2\log\left(bx+a\right)}{b^3}+\frac{bx^2-2\,ax}{b^2}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*d*(2*a^2*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c - a
*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b^2*d) - 2*a*((d*x*e^c - e^c)*e
^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/b^2 + ((d^2*x^2*e^c - 2*d*x*e^c +
2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^(-d*x - c)/d^3)/b + 4*a^2*cosh
(d*x + c)*log(b*x + a)/(b^3*d)) + 1/2*(2*a^2*log(b*x + a)/b^3 + (b*x^2 - 2*
a*x)/b^2)*cosh(d*x + c)
```

Fricas [A]
time = 0.38, size = 156, normalized size = 1.56

$$-\frac{2\,b^2\cosh\left(dx+c\right)-\left(a^2d^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+a^2d^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-2\left(b^2dx-abd\right)\sinh\left(dx+c\right)+\left(a^2d^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-a^2d^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2\,b^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*cosh(d*x + c) - (a^2*d^2*Ei((b*d*x + a*d)/b) + a^2*d^2*Ei(-(b*d
*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(b^2*d*x - a*b*d)*sinh(d*x + c) + (a
^2*d^2*Ei((b*d*x + a*d)/b) - a^2*d^2*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d
)/b))/(b^3*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\cosh\left(c+dx\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x), x)
```

Giac [A]
time = 0.41, size = 148, normalized size = 1.48

$$\frac{a^2d^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+a^2d^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+b^2dxe^{(dx+c)}-b^2dxe^{(-dx-c)}-abde^{(dx+c)}+abde^{(-dx-c)}-b^2e^{(dx+c)}-b^2e^{(-dx-c)}}{2\,b^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(a^2*d^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*d^2*Ei(-(b*d*x + a*d)/
b)*e^(-c + a*d/b) + b^2*d*x*e^(d*x + c) - b^2*d*x*e^(-d*x - c) - a*b*d*e^(d
*x + c) + a*b*d*e^(-d*x - c) - b^2*e^(d*x + c) - b^2*e^(-d*x - c))/(b^3*d^2
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\cosh(c+d\,x)}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(c + d*x))/(a + b*x),x)
```

```
[Out] int((x^2*cosh(c + d*x))/(a + b*x), x)
```

3.21 $\int \frac{x\cosh(c+dx)}{a+bx}\,dx$

Optimal. Leaf size=68

$$-\frac{a\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^2}+\frac{\sinh(c+dx)}{bd}-\frac{a\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^2}$$

```
[Out] -a*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^2+a*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^2+si
nh(d*x+c)/b/d
```

Rubi [A]
time = 0.12, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6874, 2717, 3384, 3379, 3382}

$$-\frac{a\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^2}-\frac{a\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^2}+\frac{\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] -((a*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^2) + Sinh[c + d*x]/(b
*d) - (a*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cosh(c+dx)}{a+bx}\,dx &= \int \left(\frac{\cosh(c+dx)}{b}-\frac{a\cosh(c+dx)}{b(a+bx)}\right)dx\\ &= \frac{\int \cosh(c+dx)\,dx}{b}-\frac{a\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b}\\ &= \frac{\sinh(c+dx)}{bd}-\frac{\left(a\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}-\frac{\left(a\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}}{b}\\ &= -\frac{a\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^2}+\frac{\sinh(c+dx)}{bd}-\frac{a\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 64, normalized size = 0.94

$$\frac{-ad\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right)+b\sinh(c+dx)-ad\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] (-(a*d*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)]) + b*Sinh[c + d*x] - a*d
*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/(b^2*d)
```

Maple [A]
time = 0.82, size = 114, normalized size = 1.68

method	result	size
risch	$-\frac{e^{-dx-c}}{2bd}+\frac{e^{\frac{ad-bc}{b}}\,\mathrm{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)a}{2b^2}+\frac{e^{dx+c}}{2bd}+\frac{e^{-\frac{ad-bc}{b}}\,\mathrm{expIntegral}\left(1,-dx-c-\frac{ad-bc}{b}\right)a}{2b^2}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/d*exp(-d*x-c)+1/2/b^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a+1/2
/b/d*exp(d*x+c)+1/2/b^2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(69) = 138$.
time = 0.31, size = 156, normalized size = 2.29

$$\frac{1}{2}\,d\left(\frac{a\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{bd}-\frac{\frac{(dxe^c-e^c)e^{(dx)}}{d^2}+\frac{(dx+1)e^{(-dx-c)}}{d^2}}{b}+\frac{2\,a\cosh\left(dx+c\right)\log\left(bx+a\right)}{b^2d}\right)+\left(\frac{x}{b}-\frac{a\log\left(bx+a\right)}{b^2}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*d*(a*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)
*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b*d) - ((d*x*e^c - e^c)*e^(d*x)/d^2
+ (d*x + 1)*e^(-d*x - c)/d^2)/b + 2*a*cosh(d*x + c)*log(b*x + a)/(b^2*d)) +
 (x/b - a*log(b*x + a)/b^2)*cosh(d*x + c)
```

Fricas [A]
time = 0.39, size = 118, normalized size = 1.74

$$-\frac{\left(ad\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+ad\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-2\,b\sinh\left(dx+c\right)-\left(ad\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-ad\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*((a*d*Ei((b*d*x + a*d)/b) + a*d*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d
)/b) - 2*b*sinh(d*x + c) - (a*d*Ei((b*d*x + a*d)/b) - a*d*Ei(-(b*d*x + a*d)
/b))*sinh(-(b*c - a*d)/b))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh\left(c+dx\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x*cosh(c + d*x)/(a + b*x), x)
```

Giac [A]
time = 0.42, size = 83, normalized size = 1.22

$$-\frac{ad\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+ad\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}-be^{(dx+c)}+be^{(-dx-c)}}{2\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(a*d*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a*d*Ei(-(b*d*x + a*d)/b)*e^(-
c + a*d/b) - b*e^(d*x + c) + b*e^(-d*x - c))/(b^2*d)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \cosh(c + d\,x)}{a + b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(c + d*x))/(a + b*x),x)
```

```
[Out] int((x*cosh(c + d*x))/(a + b*x), x)
```

3.22 $\int \frac{\cosh(c+dx)}{a+bx}\,dx$

Optimal. Leaf size=51

$$\frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b}+\frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b}$$

```
[Out] Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b-Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b
```

Rubi [A]
time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3384, 3379, 3382}

$$\frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{b}+\frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*x),x]
```

```
[Out] (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b + (Sinh[c - (a*d)/b]*Sinh
Integral[(a*d)/b + d*x])/b
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{a+bx}\,dx = \cosh\left(c-\frac{ad}{b}\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx + \sinh\left(c-\frac{ad}{b}\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx$$
$$= \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b} + \frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b}$$

Mathematica [A]
time = 0.06, size = 49, normalized size = 0.96

$$\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)+\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x),x]
```

```
[Out] (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x] + Sinh[c - (a*d)/b]*SinhInte
gral[(a*d)/b + d*x])/b
```

Maple [A]
time = 0.74, size = 81, normalized size = 1.59

method	result	size
risch	$-\frac{e^{\frac{ad-bc}{b}}\,\text{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)}{2b}-\frac{e^{-\frac{ad-bc}{b}}\,\text{expIntegral}\left(1,-dx-c-\frac{ad-bc}{b}\right)}{2b}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2/b*exp(-(a*d-b*c)/b)*Ei(
1,-d*x-c-(a*d-b*c)/b)
```

Maxima [A]
time = 0.30, size = 57, normalized size = 1.12

$$-\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{2\,b}-\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b - 1/2*e^(c - a*d/b)*
exp_integral_e(1, -(b*x + a)*d/b)/b
```

Fricas [A]
time = 0.41, size = 95, normalized size = 1.86

$$\frac{\left(\operatorname{Ei}\left(\frac{bdx+ad}{b}\right)+\operatorname{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-\left(\operatorname{Ei}\left(\frac{bdx+ad}{b}\right)-\operatorname{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*((Ei((b*d*x + a*d)/b) + Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - (E
i((b*d*x + a*d)/b) - Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh\left(c+dx\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(a + b*x), x)
```

Giac [A]
time = 0.42, size = 56, normalized size = 1.10

$$\frac{\operatorname{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+\operatorname{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b
))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\cosh(c+d\,x)}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*x),x)
```

```
[Out] int(cosh(c + d*x)/(a + b*x), x)
```

3.23 $\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx$

Optimal. Leaf size=73

$$\frac{\cosh(c)\mathrm{Chi}(dx)}{a}-\frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a}+\frac{\sinh(c)\mathrm{Shi}(dx)}{a}-\frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a}$$

```
[Out] Chi(d*x)*cosh(c)/a-Chi(a*d/b+d*x)*cosh(-c+a*d/b)/a+Shi(d*x)*sinh(c)/a+Shi(a
*d/b+d*x)*sinh(-c+a*d/b)/a
```

Rubi [A]
time = 0.18, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {6874, 3384, 3379, 3382}

$$-\frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{a}-\frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{a}+\frac{\cosh(c)\mathrm{Chi}(dx)}{a}+\frac{\sinh(c)\mathrm{Shi}(dx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x)),x]
```

```
[Out] (Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d
*x])/a + (Sinh[c]*SinhIntegral[d*x])/a - (Sinh[c - (a*d)/b]*SinhIntegral[(a
*d)/b + d*x])/a
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx = \int \left(\frac{\cosh(c+dx)}{ax} - \frac{b\cosh(c+dx)}{a(a+bx)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a} - \frac{b\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a}$$

$$= \frac{\cosh(c)\int \frac{\cosh(dx)}{x}\,dx}{a} - \frac{\left(b\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a} + \frac{\sinh(c)\int \frac{\sinh(dx)}{x}\,dx}{a} - \frac{(b\,\mathrm{si}}{}$$

$$= \frac{\cosh(c)\mathrm{Chi}(dx)}{a} - \frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a} + \frac{\sinh(c)\mathrm{Shi}(dx)}{a} - \frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}(}{a}$$

Mathematica [**A**]
time = 0.09, size = 63, normalized size = 0.86

$$\frac{\cosh(c)\mathrm{Chi}(dx) - \cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right) + \sinh(c)\mathrm{Shi}(dx) - \sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x)),x]
```

```
[Out] (Cosh[c]*CoshIntegral[d*x] - Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] +
Sinh[c]*SinhIntegral[d*x] - Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/a
```

Maple [**A**]
time = 0.72, size = 108, normalized size = 1.48

method	result
risch	$-\frac{e^{-c}\,\mathrm{expIntegral}(1,dx)}{2a} + \frac{e^{\frac{ad-bc}{b}}\,\mathrm{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)}{2a} - \frac{e^{c}\,\mathrm{expIntegral}(1,-dx)}{2a} + \frac{e^{-\frac{ad-bc}{b}}\,\mathrm{expIntegral}\left(1,-dx-c-\frac{ad}{}\right.}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*exp(-c)*Ei(1,d*x)+1/2/a*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2
/a*exp(c)*Ei(1,-d*x)+1/2/a*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(74) = 148.

time = 0.33, size = 155, normalized size = 2.12

$$\frac{1}{2}\,d\left(\frac{b\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{ad}+\frac{2\cosh\left(dx+c\right)\log\left(bx+a\right)}{ad}-\frac{2\cosh\left(dx+c\right)\log\left(x\right)}{ad}+\frac{\mathrm{Ei}(-dx)\,e^{(-c)}+\mathrm{Ei}(dx)\,e^{c}}{ad}\right)-\left(\frac{\log\left(bx+a\right)}{a}-\frac{\log\left(x\right)}{a}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*d*(b*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)
*exp_integral_e(1, -(b*x + a)*d/b)/b)/(a*d) + 2*cosh(d*x + c)*log(b*x + a)/
(a*d) - 2*cosh(d*x + c)*log(x)/(a*d) + (Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)/(a*d
)) - (log(b*x + a)/a - log(x)/a)*cosh(d*x + c)
```

Fricas [A]
time = 0.37, size = 123, normalized size = 1.68

$$\frac{\left(\mathrm{Ei}(dx)+\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-\left(\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)+\left(\mathrm{Ei}(dx)-\mathrm{Ei}(-dx)\right)\sinh\left(c\right)+\left(\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*((Ei(d*x) + Ei(-d*x))*cosh(c) - (Ei((b*d*x + a*d)/b) + Ei(-(b*d*x + a*d
)/b))*cosh(-(b*c - a*d)/b) + (Ei(d*x) - Ei(-d*x))*sinh(c) + (Ei((b*d*x + a*
d)/b) - Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh\left(c+dx\right)}{x\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(x*(a + b*x)), x)
```

Giac [A]
time = 0.41, size = 75, normalized size = 1.03

$$\frac{\mathrm{Ei}(-dx)\,e^{(-c)}-\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+\mathrm{Ei}(dx)\,e^{c}-\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(Ei(-d*x)*e^(-c) - Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + Ei(d*x)*e^c - Ei
(-(b*d*x + a*d)/b)*e^(-c + a*d/b))/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + d\,x)}{x\,(a + b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x*(a + b*x)),x)
```

```
[Out] int(cosh(c + d*x)/(x*(a + b*x)), x)
```

3.24 $\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx$

Optimal. Leaf size=113

$$-\frac{\cosh(c+dx)}{ax}-\frac{b\cosh(c)\text{Chi}(dx)}{a^2}+\frac{b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^2}+\frac{d\text{Chi}(dx)\sinh(c)}{a}+\frac{d\cosh(c)\text{Shi}(dx)}{a}-\frac{b\,\text{s}}{}$$

```
[Out] -b*Chi(d*x)*cosh(c)/a^2+b*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/a^2-cosh(d*x+c)/a/x
+d*cosh(c)*Shi(d*x)/a+d*Chi(d*x)*sinh(c)/a-b*Shi(d*x)*sinh(c)/a^2-b*Shi(a*d
/b+d*x)*sinh(-c+a*d/b)/a^2
```

Rubi [A]
time = 0.27, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3379, 3382}

$$-\frac{b\cosh(c)\text{Chi}(dx)}{a^2}+\frac{b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^2}-\frac{b\sinh(c)\text{Shi}(dx)}{a^2}+\frac{b\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{d\sinh(c)\text{Chi}(dx)}{a}+\frac{d\cosh(c)\text{Shi}(dx)}{a}-\frac{\cosh(c+dx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x)),x]
```

```
[Out] -(Cosh[c + d*x]/(a*x)) - (b*Cosh[c]*CoshIntegral[d*x])/a^2 + (b*Cosh[c - (a
*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^2 + (d*CoshIntegral[d*x]*Sinh[c])/a +
 (d*Cosh[c]*SinhIntegral[d*x])/a - (b*Sinh[c]*SinhIntegral[d*x])/a^2 + (b*S
inh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx &= \int \left(\frac{\cosh(c+dx)}{ax^2}-\frac{b\cosh(c+dx)}{a^2x}+\frac{b^2\cosh(c+dx)}{a^2(a+bx)}\right)dx\\ &= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a}-\frac{b\int \frac{\cosh(c+dx)}{x}\,dx}{a^2}+\frac{b^2\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^2}\\ &= -\frac{\cosh(c+dx)}{ax}+\frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{a}-\frac{(b\cosh(c))\int \frac{\cosh(dx)}{x}\,dx}{a^2}+\frac{\left(b^2\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh}{}}{a^2}\\ &= -\frac{\cosh(c+dx)}{ax}-\frac{b\cosh(c)\mathrm{Chi}(dx)}{a^2}+\frac{b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^2}-\frac{b\sinh(c)\mathrm{Shi}(dx)}{a^2}\\ &= -\frac{\cosh(c+dx)}{ax}-\frac{b\cosh(c)\mathrm{Chi}(dx)}{a^2}+\frac{b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^2}+\frac{d\mathrm{Chi}(dx)\sinh(c}{a}\end{aligned}$$

Mathematica [A]
time = 0.24, size = 101, normalized size = 0.89

$$\frac{-a\cosh(c+dx)+bx\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right)+\mathrm{Chi}(dx)(-bx\cosh(c)+adx\sinh(c))+adx\cosh(c)\mathrm{Shi}(dx)-bx\sinh(c)\mathrm{Shi}(dx)+bx\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x)),x]
```

```
[Out] (-(a*Cosh[c + d*x]) + b*x*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] + Cos
hIntegral[d*x]*(-(b*x*Cosh[c]) + a*d*x*Sinh[c]) + a*d*x*Cosh[c]*SinhIntegra
l[d*x] - b*x*Sinh[c]*SinhIntegral[d*x] + b*x*Sinh[c - (a*d)/b]*SinhIntegral
[d*(a/b + x)])/(a^2*x)
```

Maple [A]
time = 0.72, size = 172, normalized size = 1.52

method	result

risch	$-\frac{e^{-dx-c}}{2ax}+\frac{d\,e^{-c}\,\text{expIntegral}(1,dx)}{2a}+\frac{e^{-c}\,\text{expIntegral}(1,dx)b}{2a^2}-\frac{b\,e^{\frac{ad-bc}{b}}\,\text{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)}{2a^2}+\frac{b\,e^{c}\,\text{expIntegral}(1$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*exp(-d*x-c)/a/x+1/2*d/a*exp(-c)*Ei(1,d*x)+1/2/a^2*exp(-c)*Ei(1,d*x)*b-
1/2*b/a^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+1/2*b/a^2*exp(c)*Ei(1,-d
*x)-1/2/a^2*b*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-1/2/a/x*exp(d*x+c)
-1/2*d/a*exp(c)*Ei(1,-d*x)
```

Maxima [A]
time = 0.36, size = 192, normalized size = 1.70

$$-\frac{1}{2}\,d\left(\frac{\text{Ei}(-dx)\,e^{(-c)}-\text{Ei}(dx)\,e^{c}}{a}+\frac{b^2\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{a^2d}+\frac{2\,b\cosh\left(dx+c\right)\log\left(bx+a\right)}{a^2d}-\frac{2\,b\cosh\left(dx+c\right)\log\left(x\right)}{a^2d}+\frac{\left(\text{Ei}(-dx)\,e^{(-c)}+\text{Ei}(dx)\,e^{c}\right)b}{a^2d}\right)+\left(\frac{b\log\left(bx+a\right)}{a^2}-\frac{b\log\left(x\right)}{a^2}-\frac{1}{ax}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*d*((Ei(-d*x)*e^(-c) - Ei(d*x)*e^c)/a + b^2*(e^(-c + a*d/b)*exp_integra
l_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b
)/(a^2*d) + 2*b*cosh(d*x + c)*log(b*x + a)/(a^2*d) - 2*b*cosh(d*x + c)*log(
x)/(a^2*d) + (Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b/(a^2*d)) + (b*log(b*x + a)/a
^2 - b*log(x)/a^2 - 1/(a*x))*cosh(d*x + c)
```

Fricas [A]
time = 0.44, size = 179, normalized size = 1.58

$$-\frac{2\,a\cosh\left(dx+c\right)-\left((ad-b)x\text{Ei}(dx)-(ad+b)x\text{Ei}(-dx)\right)\cosh\left(c\right)-\left(bx\text{Ei}\left(\frac{bdx+ad}{b}\right)+bx\text{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-\left((ad-b)x\text{Ei}(dx)+(ad+b)x\text{Ei}(-dx)\right)\sinh\left(c\right)+\left(bx\text{Ei}\left(\frac{bdx+ad}{b}\right)-bx\text{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2\,a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*cosh(d*x + c) - ((a*d - b)*x*Ei(d*x) - (a*d + b)*x*Ei(-d*x))*cosh
(c) - (b*x*Ei((b*d*x + a*d)/b) + b*x*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d
)/b) - ((a*d - b)*x*Ei(d*x) + (a*d + b)*x*Ei(-d*x))*sinh(c) + (b*x*Ei((b*d*
x + a*d)/b) - b*x*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh\left(c+dx\right)}{x^2\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**2/(b*x+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(x**2*(a + b*x)), x)
```

Giac [A]
time = 0.41, size = 129, normalized size = 1.14

$$-\frac{adx\mathrm{Ei}(-dx)\,e^{(-c)} - adx\mathrm{Ei}(dx)\,e^{c} + bx\mathrm{Ei}(-dx)\,e^{(-c)} - bx\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)\,e^{\left(c-\frac{ad}{b}\right)} + bx\mathrm{Ei}(dx)\,e^{c} - bx\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\,e^{\left(-c+\frac{ad}{b}\right)} + ae^{(dx+c)} + ae^{(-dx-c)}}{2\,a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(a*d*x*Ei(-d*x)*e^(-c) - a*d*x*Ei(d*x)*e^c + b*x*Ei(-d*x)*e^(-c) - b*x
*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + b*x*Ei(d*x)*e^c - b*x*Ei(-(b*d*x + a*d
)/b)*e^(-c + a*d/b) + a*e^(d*x + c) + a*e^(-d*x - c))/(a^2*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)}{x^2\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x^2*(a + b*x)),x)
```

```
[Out] int(cosh(c + d*x)/(x^2*(a + b*x)), x)
```

3.25 $\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx$

Optimal. Leaf size=190

$$-\frac{\cosh(c+dx)}{2ax^2}+\frac{b\cosh(c+dx)}{a^2x}+\frac{b^2\cosh(c)\text{Chi}(dx)}{a^3}+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a}-\frac{b^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^3}-b$$

```
[Out] b^2*Chi(d*x)*cosh(c)/a^3+1/2*d^2*Chi(d*x)*cosh(c)/a-b^2*Chi(a*d/b+d*x)*cosh
(-c+a*d/b)/a^3-1/2*cosh(d*x+c)/a/x^2+b*cosh(d*x+c)/a^2/x-b*d*cosh(c)*Shi(d*
x)/a^2-b*d*Chi(d*x)*sinh(c)/a^2+b^2*Shi(d*x)*sinh(c)/a^3+1/2*d^2*Shi(d*x)*s
inh(c)/a+b^2*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/a^3-1/2*d*sinh(d*x+c)/a/x
```

Rubi [A]
time = 0.35, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\frac{b^2\cosh(c)\text{Chi}(dx)}{a^3}-\frac{b^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^3}+\frac{b^2\sinh(c)\text{Shi}(dx)}{a^3}-\frac{b^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{bd\sinh(c)\text{Chi}(dx)}{a^2}-\frac{bd\cosh(c)\text{Shi}(dx)}{a^2}+\frac{b\cosh(c+dx)}{a^2x}+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a}+\frac{d^2\sinh(c)\text{Shi}(dx)}{2a}-\frac{\cosh(c+dx)}{2ax^2}-\frac{d\sinh(c+dx)}{2ax}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^3*(a + b*x)),x]
```

```
[Out] -1/2*Cosh[c + d*x]/(a*x^2) + (b*Cosh[c + d*x])/(a^2*x) + (b^2*Cosh[c]*CoshI
ntegral[d*x])/a^3 + (d^2*Cosh[c]*CoshIntegral[d*x])/(2*a) - (b^2*Cosh[c - (
a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^3 - (b*d*CoshIntegral[d*x]*Sinh[c])/
a^2 - (d*Sinh[c + d*x])/(2*a*x) - (b*d*Cosh[c]*SinhIntegral[d*x])/a^2 + (b^
2*Sinh[c]*SinhIntegral[d*x])/a^3 + (d^2*Sinh[c]*SinhIntegral[d*x])/(2*a) -
(b^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx = \int \left(\frac{\cosh(c+dx)}{ax^3} - \frac{b\cosh(c+dx)}{a^2x^2} + \frac{b^2\cosh(c+dx)}{a^3x} - \frac{b^3\cosh(c+dx)}{a^3(a+bx)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^3}\,dx}{a} - \frac{b\int \frac{\cosh(c+dx)}{x^2}\,dx}{a^2} + \frac{b^2\int \frac{\cosh(c+dx)}{x}\,dx}{a^3} - \frac{b^3\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^3}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b\cosh(c+dx)}{a^2x} + \frac{d\int \frac{\sinh(c+dx)}{x^2}\,dx}{2a} - \frac{(bd)\int \frac{\sinh(c+dx)}{x}\,dx}{a^2} + (b^2\cosh(c)$$

$$= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b\cosh(c+dx)}{a^2x} + \frac{b^2\cosh(c)\mathrm{Chi}(dx)}{a^3} - \frac{b^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^3}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b\cosh(c+dx)}{a^2x} + \frac{b^2\cosh(c)\mathrm{Chi}(dx)}{a^3} - \frac{b^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^3}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b\cosh(c+dx)}{a^2x} + \frac{b^2\cosh(c)\mathrm{Chi}(dx)}{a^3} + \frac{d^2\cosh(c)\mathrm{Chi}(dx)}{2a} - b^2\cosh($$

Mathematica [**A**]
time = 0.29, size = 178, normalized size = 0.94

$$\frac{-a^2\cosh(c+dx) + 2abx\cosh(c+dx) - 2b^2x^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right) + x^2\mathrm{Chi}(dx)\left((2b^2+a^2d^2)\cosh(c) - 2abd\sinh(c)\right) - a^2dx\sinh(c+dx) - 2abdx^2\cosh(c)\mathrm{Shi}(dx) + 2b^2x^2\sinh(c)\mathrm{Shi}(dx) + a^2d^2x^2\sinh(c)\mathrm{Shi}(dx) - 2b^2x^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x)),x]
```

```
[Out] (-(a^2*Cosh[c + d*x]) + 2*a*b*x*Cosh[c + d*x] - 2*b^2*x^2*Cosh[c - (a*d)/b]
*CoshIntegral[d*(a/b + x)] + x^2*CoshIntegral[d*x]*((2*b^2 + a^2*d^2)*Cosh[
c] - 2*a*b*d*Sinh[c]) - a^2*d*x*Sinh[c + d*x] - 2*a*b*d*x^2*Cosh[c]*SinhInt
```

```
egral[d*x] + 2*b^2*x^2*Sinh[c]*SinhIntegral[d*x] + a^2*d^2*x^2*Sinh[c]*Sinh
Integral[d*x] - 2*b^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/(2*a
^3*x^2)
```

Maple [A]
time = 0.73, size = 281, normalized size = 1.48

method	result
risch	$\frac{d\,\mathrm{e}^{-dx-c}}{4ax}+\frac{\mathrm{e}^{-dx-c}b}{2a^2x}-\frac{\mathrm{e}^{-dx-c}}{4a\,x^2}-\frac{d^2\mathrm{e}^{-c}\,\mathrm{expIntegral}(1,dx)}{4a}-\frac{d\,\mathrm{e}^{-c}\,\mathrm{expIntegral}(1,dx)b}{2a^2}-\frac{\mathrm{e}^{-c}\,\mathrm{expIntegral}(1,dx)b^2}{2a^3}+\frac{b^2\mathrm{e}^{\frac{ad}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^3/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d*exp(-d*x-c)/a/x+1/2*exp(-d*x-c)/a^2/x*b-1/4*exp(-d*x-c)/a/x^2-1/4*d^2
/a*exp(-c)*Ei(1,d*x)-1/2*d/a^2*exp(-c)*Ei(1,d*x)*b-1/2/a^3*exp(-c)*Ei(1,d*x
)*b^2+1/2*b^2/a^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/4/a/x^2*exp(d*
x+c)-1/4*d/a/x*exp(d*x+c)-1/4*d^2/a*exp(c)*Ei(1,-d*x)-1/2*b^2/a^3*exp(c)*Ei
(1,-d*x)+1/2/a^3*b^2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)+1/2*b/a^2/x
*exp(d*x+c)+1/2*d*b/a^2*exp(c)*Ei(1,-d*x)
```

Maxima [A]
time = 0.38, size = 242, normalized size = 1.27

$$\frac{1}{4}\,d\left(\frac{de^{(-c)}\Gamma(-1,dx)+de^{c}\Gamma(-1,-dx)}{a}+\frac{2\left(\mathrm{Ei}(-dx)\,e^{(-c)}-\mathrm{Ei}(dx)\,e^{c}\right)b}{a^2}+\frac{2\,b^3\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{a^3d}+\frac{4\,b^2\cosh\left(dx+c\right)\log\left(bx+a\right)}{a^3d}-\frac{4\,b^2\cosh\left(dx+c\right)\log\left(x\right)}{a^3d}+\frac{2\left(\mathrm{Ei}(-dx)\,e^{(-c)}+\mathrm{Ei}(dx)\,e^{c}\right)b^2}{a^3d}\right)-\frac{1}{2}\left(\frac{2\,b^2\log\left(bx+a\right)}{a^3}-\frac{2\,b^2\log\left(x\right)}{a^3}-\frac{2\,bx-a}{a^2x^2}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*d*((d*e^(-c)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))/a + 2*(Ei(-d*x)*e^
(-c) - Ei(d*x)*e^c)*b/a^2 + 2*b^3*(e^(-c + a*d/b)*exp_integral_e(1, (b*x +
a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(a^3*d) + 4*
b^2*cosh(d*x + c)*log(b*x + a)/(a^3*d) - 4*b^2*cosh(d*x + c)*log(x)/(a^3*d)
 + 2*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b^2/(a^3*d)) - 1/2*(2*b^2*log(b*x + a)
/a^3 - 2*b^2*log(x)/a^3 - (2*b*x - a)/(a^2*x^2))*cosh(d*x + c)
```

Fricas [A]
time = 0.37, size = 275, normalized size = 1.45

$$-\frac{2\,a^2dx\sinh\left(dx+c\right)-2\left(2\,abx-a^2\right)\cosh\left(dx+c\right)-\left(\left(a^2d^2-2\,abd+2\,b^2\right)x^2\mathrm{Ei}\left(dx\right)+\left(a^2d^2+2\,abd+2\,b^2\right)x^2\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)+2\left(b^2x^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+b^2x^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-\left(\left(a^2d^2-2\,abd+2\,b^2\right)x^2\mathrm{Ei}\left(dx\right)-\left(a^2d^2+2\,abd+2\,b^2\right)x^2\mathrm{Ei}\left(-dx\right)\right)\sinh\left(c\right)-2\left(b^2x^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-b^2x^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{4\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*d*x*sinh(d*x + c) - 2*(2*a*b*x - a^2)*cosh(d*x + c) - ((a^2*d^2
 - 2*a*b*d + 2*b^2)*x^2*Ei(d*x) + (a^2*d^2 + 2*a*b*d + 2*b^2)*x^2*Ei(-d*x))
```

```
*cosh(c) + 2*(b^2*x^2*Ei((b*d*x + a*d)/b) + b^2*x^2*Ei(-(b*d*x + a*d)/b))*c
osh(-(b*c - a*d)/b) - ((a^2*d^2 - 2*a*b*d + 2*b^2)*x^2*Ei(d*x) - (a^2*d^2 +
 2*a*b*d + 2*b^2)*x^2*Ei(-d*x))*sinh(c) - 2*(b^2*x^2*Ei((b*d*x + a*d)/b) -
b^2*x^2*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh (c + dx)}{x^3 (a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**3/(b*x+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(x**3*(a + b*x)), x)
```

Giac [A]
time = 0.40, size = 248, normalized size = 1.31

$$\frac{a^2d^2x^2\mathrm{Ei}(-dx)\, e^{(-c)} + a^2d^2x^2\mathrm{Ei}(dx)\, e^c + 2\, abdx^2\mathrm{Ei}(-dx)\, e^{(-c)} - 2\, abdx^2\mathrm{Ei}(dx)\, e^c + 2\, b^2x^2\mathrm{Ei}(-dx)\, e^{(-c)} - 2\, b^2x^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} + 2\, b^2x^2\mathrm{Ei}(dx)\, e^c - 2\, b^2x^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)} - a^2dxe^{(dx+c)} + a^2dxe^{(-dx-c)} + 2\, abxe^{(dx+c)} + 2\, abxe^{(-dx-c)} - a^2e^{(dx+c)} - a^2e^{(-dx-c)}}{4\, a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^2*x^2*Ei(-d*x)*e^(-c) + a^2*d^2*x^2*Ei(d*x)*e^c + 2*a*b*d*x^2*Ei
(-d*x)*e^(-c) - 2*a*b*d*x^2*Ei(d*x)*e^c + 2*b^2*x^2*Ei(-d*x)*e^(-c) - 2*b^2
*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*b^2*x^2*Ei(d*x)*e^c - 2*b^2*x^2*
Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^2*d*x*e^(d*x + c) + a^2*d*x*e^(-d*x
 - c) + 2*a*b*x*e^(d*x + c) + 2*a*b*x*e^(-d*x - c) - a^2*e^(d*x + c) - a^2*
e^(-d*x - c))/(a^3*x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + d\, x)}{x^3\, (a + b\, x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x^3*(a + b*x)),x)
```

```
[Out] int(cosh(c + d*x)/(x^3*(a + b*x)), x)
```

3.26 $\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2} \, dx$

Optimal. Leaf size=231

$$\frac{2a\cosh(c+dx)}{b^3d^2} - \frac{2x\cosh(c+dx)}{b^2d^2} - \frac{a^4\cosh(c+dx)}{b^5(a+bx)} - \frac{4a^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^5} + \frac{a^4d\text{Chi}\left(\frac{ad}{b}+dx\right)\sin}{b^6}$$

```
[Out] -4*a^3*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^5+2*a*cosh(d*x+c)/b^3/d^2-2*x*cosh(d
*x+c)/b^2/d^2-a^4*cosh(d*x+c)/b^5/(b*x+a)+a^4*d*cosh(-c+a*d/b)*Shi(a*d/b+d*
x)/b^6-a^4*d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/b^6+4*a^3*Shi(a*d/b+d*x)*sinh(-c
+a*d/b)/b^5+2*sinh(d*x+c)/b^2/d^3+3*a^2*sinh(d*x+c)/b^4/d-2*a*x*sinh(d*x+c)
/b^3/d+x^2*sinh(d*x+c)/b^2/d
```

Rubi [A]
time = 0.40, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {6874, 2717, 3377, 2718, 3378, 3384, 3379, 3382}

$$\frac{a^4d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^6} + \frac{a^4d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^6} - \frac{a^4\cosh(c+dx)}{b^5(a+bx)} - \frac{4a^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^5} - \frac{4a^3\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^5} + \frac{3a^2\sinh(c+dx)}{b^4d} + \frac{2a\cosh(c+dx)}{b^3d^2} - \frac{2ax\sinh(c+dx)}{b^3d} + \frac{2\sinh(c+dx)}{b^2d^3} - \frac{2x\cosh(c+dx)}{b^2d^2} + \frac{x^2\sinh(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (2*a*Cosh[c + d*x])/(b^3*d^2) - (2*x*Cosh[c + d*x])/(b^2*d^2) - (a^4*Cosh[c
 + d*x])/(b^5*(a + b*x)) - (4*a^3*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b +
d*x])/b^5 + (a^4*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^6 + (2*
Sinh[c + d*x])/(b^2*d^3) + (3*a^2*Sinh[c + d*x])/(b^4*d) - (2*a*x*Sinh[c +
d*x])/(b^3*d) + (x^2*Sinh[c + d*x])/(b^2*d) + (a^4*d*Cosh[c - (a*d)/b]*Sinh
Integral[(a*d)/b + d*x])/b^6 - (4*a^3*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/
b + d*x])/b^5
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2}\,dx = \int \left(\frac{3a^2\cosh(c+dx)}{b^4} - \frac{2ax\cosh(c+dx)}{b^3} + \frac{x^2\cosh(c+dx)}{b^2} + \frac{a^4\cosh(c+dx)}{b^4(a+bx)^2} - \right.$$

$$= \frac{(3a^2)\int \cosh(c+dx)\,dx}{b^4} - \frac{(4a^3)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b^4} + \frac{a^4\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b^4} - \frac{(2a)\int x\cosh}{b^3}$$

$$= -\frac{a^4\cosh(c+dx)}{b^5(a+bx)} + \frac{3a^2\sinh(c+dx)}{b^4 d} - \frac{2ax\sinh(c+dx)}{b^3 d} + \frac{x^2\sinh(c+dx)}{b^2 d} + \frac{(2a}{}$$

$$= \frac{2a\cosh(c+dx)}{b^3d^2} - \frac{2x\cosh(c+dx)}{b^2d^2} - \frac{a^4\cosh(c+dx)}{b^5(a+bx)} - \frac{4a^3\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}\right.}{b^5}$$

$$= \frac{2a\cosh(c+dx)}{b^3d^2} - \frac{2x\cosh(c+dx)}{b^2d^2} - \frac{a^4\cosh(c+dx)}{b^5(a+bx)} - \frac{4a^3\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}\right.}{b^5}$$

Mathematica [A]
time = 0.75, size = 173, normalized size = 0.75

$$\frac{-\frac{b(-2a^2b^2+a^4d^2+2b^4x^2)\cosh(c+dx)}{d^2(a+bx)} + a^3\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(-4b\cosh\left(c-\frac{ad}{b}\right)+ad\sinh\left(c-\frac{ad}{b}\right)\right) + \frac{b^2(3a^2d^2-2abd^2x+b^2(2+d^2x^2))\sinh(c+dx)}{d^3} + a^3\left(ad\cosh\left(c-\frac{ad}{b}\right)-4b\sinh\left(c-\frac{ad}{b}\right)\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (-((b*(-2*a^2*b^2 + a^4*d^2 + 2*b^4*x^2)*Cosh[c + d*x])/(d^2*(a + b*x))) +
a^3*CoshIntegral[d*(a/b + x)]*(-4*b*Cosh[c - (a*d)/b] + a*d*Sinh[c - (a*d)/
b]) + (b^2*(3*a^2*d^2 - 2*a*b*d^2*x + b^2*(2 + d^2*x^2))*Sinh[c + d*x])/d^3
 + a^3*(a*d*Cosh[c - (a*d)/b] - 4*b*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b
+ x)])/b^6
```

Maple [A]
time = 0.99, size = 431, normalized size = 1.87

method	result
risch	$-\frac{d\,\mathrm{e}^{-dx-c}a^4}{2b^5(bdx+ad)} + \frac{\mathrm{e}^{-dx-c}ax}{d\,b^3} + \frac{d\,\mathrm{e}^{\frac{ad-bc}{b}}\,\mathrm{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)a^4}{2b^6} + \frac{2\,\mathrm{e}^{\frac{ad-bc}{b}}\,\mathrm{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)a^3}{b^5} - \frac{\mathrm{e}^{-dx}}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cosh(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d*exp(-d*x-c)/b^5/(b*d*x+a*d)*a^4+1/d*exp(-d*x-c)/b^3*a*x+1/2*d/b^6*ex
p((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^4+2/b^5*exp((a*d-b*c)/b)*Ei(1,d*x+
c+(a*d-b*c)/b)*a^3-1/d^3*exp(-d*x-c)/b^2-1/2/d*exp(-d*x-c)/b^2*x^2-3/2/d*ex
```

```
p(-d*x-c)/b^4*a^2-1/d^2*exp(-d*x-c)/b^2*x+1/d^2*exp(-d*x-c)/b^3*a+3/2/d/b^4
*a^2*exp(d*x+c)+1/d^2/b^3*a*exp(d*x+c)+1/d^3/b^2*exp(d*x+c)-1/d/b^3*a*exp(d
*x+c)*x-1/2*d/b^6*exp(d*x+c)/(a*d/b+d*x)*a^4-1/2*d/b^6*exp(-(a*d-b*c)/b)*Ei
(1,-d*x-c-(a*d-b*c)/b)*a^4+2/b^5*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
*a^3+1/2/d/b^2*exp(d*x+c)*x^2-1/d^2/b^2*exp(d*x+c)*x
```

Maxima [A]
time = 0.34, size = 406, normalized size = 1.76

$$\frac{1}{6}\left(3a^4\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b^6}-\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b^6}\right)+\frac{12a^3\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^4d}-\frac{9a^2\left(\frac{(dxe^c-e^c)e^{(dx)}}{d^2}+\frac{(dx+1)e^{(-dx-c)}}{d^2}\right)}{b^4}+\frac{3a\left(\frac{(d^2x^2e^c-2dxe^c+2e^c)e^{(dx)}}{d^3}+\frac{(d^2x^2+2dx+2)e^{(-dx-c)}}{d^3}\right)}{b^3}-\frac{\frac{(d^3x^3e^c-3d^2x^2e^c+6dxe^c-6e^c)e^{(dx)}}{d^4}+\frac{(d^3x^3+3d^2x^2+6dx+6)e^{(-dx-c)}}{d^4}}{b^2}+\frac{24a^3\cosh(dx+c)\log(bx+a)}{b^5d}\right)d-\frac{1}{3}\left(\frac{3a^4}{b^6x+ab^5}+\frac{12a^3\log(bx+a)}{b^5}-\frac{b^2x^3-3abx^2+9a^2x}{b^4}\right)\cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^4*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b^6 - e^(c - a*
d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b^6) + 12*a^3*(e^(-c + a*d/b)*exp_in
tegral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d
/b)/b)/(b^4*d) - 9*a^2*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c
)/d^2)/b^4 + 3*a*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2
+ 2*d*x + 2)*e^(-d*x - c)/d^3)/b^3 - ((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*
e^c - 6*e^c)*e^(d*x)/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d
^4)/b^2 + 24*a^3*cosh(d*x + c)*log(b*x + a)/(b^5*d))*d - 1/3*(3*a^4/(b^6*x
+ a*b^5) + 12*a^3*log(b*x + a)/b^5 - (b^2*x^3 - 3*a*b*x^2 + 9*a^2*x)/b^4)*c
osh(d*x + c)
```

Fricas [A]
time = 0.38, size = 371, normalized size = 1.61

$$-\frac{2(a^4bd^3+2b^5dx^2-2a^2b^3d)\cosh(dx+c)-\left((a^5d^4-4a^4bd^3+(a^4bd^4-4a^3b^2d^3)x)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-(a^5d^4+4a^4bd^3+(a^4bd^4+4a^3b^2d^3)x)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-2(b^5d^2x^3-ab^4d^2x^2+3a^3b^2d^2+2ab^4+(a^2b^3d^2+2b^5)x)\sinh(dx+c)+\left((a^5d^4-4a^4bd^3+(a^4bd^4-4a^3b^2d^3)x)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+(a^5d^4+4a^4bd^3+(a^4bd^4+4a^3b^2d^3)x)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2(b^7d^3x+ab^6d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^4*b*d^3 + 2*b^5*d*x^2 - 2*a^2*b^3*d)*cosh(d*x + c) - ((a^5*d^4 -
 4*a^4*b*d^3 + (a^4*b*d^4 - 4*a^3*b^2*d^3)*x)*Ei((b*d*x + a*d)/b) - (a^5*d^
4 + 4*a^4*b*d^3 + (a^4*b*d^4 + 4*a^3*b^2*d^3)*x)*Ei(-(b*d*x + a*d)/b))*cosh
(-(b*c - a*d)/b) - 2*(b^5*d^2*x^3 - a*b^4*d^2*x^2 + 3*a^3*b^2*d^2 + 2*a*b^4
 + (a^2*b^3*d^2 + 2*b^5)*x)*sinh(d*x + c) + ((a^5*d^4 - 4*a^4*b*d^3 + (a^4*
b*d^4 - 4*a^3*b^2*d^3)*x)*Ei((b*d*x + a*d)/b) + (a^5*d^4 + 4*a^4*b*d^3 + (a
^4*b*d^4 + 4*a^3*b^2*d^3)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b
^7*d^3*x + a*b^6*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\cosh(c+dx)}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*cosh(d*x+c)/(b*x+a)**2,x)

[Out] Integral(x**4*cosh(c + d*x)/(a + b*x)**2, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2979 vs. $2(236) = 472$.

time = 0.46, size = 2979, normalized size = 12.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")

[Out] 1/2*((b*x + a)*a^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^4*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a^4*b*c*d^4*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + a^5*d^5*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^4*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + a^4*b*c*d^4*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^5*d^5*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - 4*(b*x + a)*a^3*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + 4*a^3*b^2*c*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - 4*a^4*b*d^4*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - 4*(b*x + a)*a^3*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + 4*a^3*b^2*c*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - 4*a^4*b*d^4*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^4*b*d^4*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - a^4*b*d^4*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a)^3*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^3*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 3*(b*x + a)^2*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*c*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 3*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c^2*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - b^5*c^3*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - (b*x + a)^2*a*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2*(b*x + a)*a*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - a*b^4*c^2*d*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a)*a^2*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - a^2*b^3*c*d^2*e^((b*x + a)*(b*c/(

```
b*x + a) - a*d/(b*x + a) + d)/b) + 3*a^3*b^2*d^3*e^((b*x + a)*(b*c/(b*x + a
) - a*d/(b*x + a) + d)/b) - (b*x + a)^3*b^2*(b*c/(b*x + a) - a*d/(b*x + a)
+ d)^3*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 3*(b*x + a)^2
*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*c*e^(-(b*x + a)*(b*c/(b*x + a) -
 a*d/(b*x + a) + d)/b) - 3*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d
)*c^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + b^5*c^3*e^(-(b
*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a)^2*a*b^2*(b*c/(b*
x + a) - a*d/(b*x + a) + d)^2*d*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a
) + d)/b) - 2*(b*x + a)*a*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d*e^(-(
b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + a*b^4*c^2*d*e^(-(b*x + a)
*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - (b*x + a)*a^2*b^2*(b*c/(b*x + a)
- a*d/(b*x + a) + d)*d^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/
b) + a^2*b^3*c*d^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 3
*a^3*b^2*d^3*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*(b*x
+ a)^2*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*e^((b*x + a)*(b*c/(b*x + a
) - a*d/(b*x + a) + d)/b) + 4*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a)
+ d)*c*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*b^5*c^2*e^((
b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2*a^2*b^3*d^2*e^((b*x + a
)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*(b*x + a)^2*b^3*(b*c/(b*x + a)
 - a*d/(b*x + a) + d)^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b
) + 4*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*e^(-(b*x + a)*(b*
c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*b^5*c^2*e^(-(b*x + a)*(b*c/(b*x + a
) - a*d/(b*x + a) + d)/b) + 2*a^2*b^3*d^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*
d/(b*x + a) + d)/b) + 2*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*e
^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*b^5*c*e^((b*x + a)*(
b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2*a*b^4*d*e^((b*x + a)*(b*c/(b*x +
a) - a*d/(b*x + a) + d)/b) - 2*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a)
 + d)*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2*b^5*c*e^(-(b
*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*a*b^4*d*e^(-(b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d)/b))*b^2/(((b*x + a)*b^8*(b*c/(b*x + a) -
a*d/(b*x + a) + d)*d^2 - b^9*c*d^2 + a*b^8*d^3)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \cosh(c + d\,x)}{(a + b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*cosh(c + d*x))/(a + b*x)^2,x)
```

```
[Out] int((x^4*cosh(c + d*x))/(a + b*x)^2, x)
```

3.27 $\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2}\,dx$

Optimal. Leaf size=182

$$-\frac{\cosh(c+dx)}{b^2d^2}+\frac{a^3\cosh(c+dx)}{b^4(a+bx)}+\frac{3a^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^4}-\frac{a^3d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^5}-\frac{2a\sinh}{}$$

```
[Out] 3*a^2*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^4-cosh(d*x+c)/b^2/d^2+a^3*cosh(d*x+c)
/b^4/(b*x+a)-a^3*d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/b^5+a^3*d*Chi(a*d/b+d*x)*s
inh(-c+a*d/b)/b^5-3*a^2*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^4-2*a*sinh(d*x+c)/b
^3/d+x*sinh(d*x+c)/b^2/d
```

Rubi [A]
time = 0.33, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {6874, 2717, 3377, 2718, 3378, 3384, 3379, 3382}

$$-\frac{a^3d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^5}-\frac{a^3d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^5}+\frac{a^3\cosh(c+dx)}{b^4(a+bx)}+\frac{3a^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{3a^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{2a\sinh(c+dx)}{b^3d}-\frac{\cosh(c+dx)}{b^2d^2}+\frac{x\sinh(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] -(Cosh[c + d*x]/(b^2*d^2)) + (a^3*Cosh[c + d*x])/(b^4*(a + b*x)) + (3*a^2*C
osh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^4 - (a^3*d*CoshIntegral[(a*
d)/b + d*x]*Sinh[c - (a*d)/b])/b^5 - (2*a*Sinh[c + d*x])/(b^3*d) + (x*Sinh[
c + d*x])/(b^2*d) - (a^3*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b
^5 + (3*a^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^4
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^3\cosh(c+dx)}{(a+bx)^2}\,dx = \int \left(-\frac{2a\cosh(c+dx)}{b^3}+\frac{x\cosh(c+dx)}{b^2}-\frac{a^3\cosh(c+dx)}{b^3(a+bx)^2}+\frac{3a^2\cosh(c+dx)}{b^3(a+bx)}\right)\,dx$$

$$= -\frac{(2a)\int\cosh(c+dx)\,dx}{b^3}+\frac{(3a^2)\int\frac{\cosh(c+dx)}{a+bx}\,dx}{b^3}-\frac{a^3\int\frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b^3}+\frac{\int x\cosh(c+ \ldots}{b^2}$$

$$= \frac{a^3\cosh(c+dx)}{b^4(a+bx)}-\frac{2a\sinh(c+dx)}{b^3d}+\frac{x\sinh(c+dx)}{b^2d}-\frac{\int\sinh(c+dx)\,dx}{b^2d}-\frac{(a^3d)\int \ldots}{\ldots}$$

$$= -\frac{\cosh(c+dx)}{b^2d^2}+\frac{a^3\cosh(c+dx)}{b^4(a+bx)}+\frac{3a^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^4}-\frac{2a\sinh(c+ \ldots}{b^3d}$$

$$= -\frac{\cosh(c+dx)}{b^2d^2}+\frac{a^3\cosh(c+dx)}{b^4(a+bx)}+\frac{3a^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^4}-\frac{a^3d\text{Chi}\left(\frac{ad}{b}+ \ldots\right.}{\ldots}$$

Mathematica [A]
time = 0.63, size = 156, normalized size = 0.86

$$\frac{a^2\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(3b\cosh\left(c-\frac{ad}{b}\right)-ad\sinh\left(c-\frac{ad}{b}\right)\right)+\frac{b\left(\left(-ab^2+a^3d^2-b^3x\right)\cosh(c+dx)+bd\left(-2a^2-abx+b^2x^2\right)\sinh(c+dx)\right)}{d^2(a+bx)}-a^2\left(ad\cosh\left(c-\frac{ad}{b}\right)-3b\sinh\left(c-\frac{ad}{b}\right)\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (a^2*CoshIntegral[d*(a/b + x)]*(3*b*Cosh[c - (a*d)/b] - a*d*Sinh[c - (a*d)/
b]) + (b*((-(a*b^2) + a^3*d^2 - b^3*x)*Cosh[c + d*x] + b*d*(-2*a^2 - a*b*x
+ b^2*x^2)*Sinh[c + d*x]))/(d^2*(a + b*x)) - a^2*(a*d*Cosh[c - (a*d)/b] - 3
*b*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)])/b^5
```

Maple [A]
time = 0.94, size = 325, normalized size = 1.79

method	result
risch	$\frac{d\,\mathrm{e}^{-dx-c}a^3}{2b^4(bdx+ad)}-\frac{3\,\mathrm{e}^{\frac{ad-bc}{b}}\,\mathrm{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)a^2}{2b^4}-\frac{d\,\mathrm{e}^{\frac{ad-bc}{b}}\,\mathrm{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)a^3}{2b^5}-\frac{\mathrm{e}^{-dx-c}x}{2d\,b^2}+\frac{\mathrm{e}^{-dx-c}}{d\,b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d*exp(-d*x-c)/b^4/(b*d*x+a*d)*a^3-3/2/b^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(
a*d-b*c)/b)*a^2-1/2*d/b^5*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3-1/2/
d*exp(-d*x-c)/b^2*x+1/d*exp(-d*x-c)/b^3*a-1/2/d^2*exp(-d*x-c)/b^2-3/2/b^4*e
xp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2+1/2/d/b^2*exp(d*x+c)*x-1/d/b^
3*a*exp(d*x+c)-1/2/d^2/b^2*exp(d*x+c)+1/2*d/b^5*exp(d*x+c)/(a*d/b+d*x)*a^3+
1/2*d/b^5*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3
```

Maxima [A]
time = 0.34, size = 312, normalized size = 1.71

$$-\frac{1}{4}\left(2\,a^3\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b^5}-\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b^5}\right)+\frac{6\,a^2\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^3d}-\frac{4\,a\left(\frac{(dxe^c-e^c)e^{(dx)}}{d^2}+\frac{(dx+1)e^{(-dx-c)}}{d^2}\right)}{b^3}+\frac{\frac{(d^2x^2e^c-2\,dxe^c+2\,e^c)e^{(dx)}}{d^3}+\frac{(d^2x^2+2\,dx+2)e^{(-dx-c)}}{d^3}}{b^2}+\frac{12\,a^2\cosh(dx+c)\log(bx+a)}{b^4d}\right)d+\frac{1}{2}\left(\frac{2\,a^3}{b^5x+ab^4}+\frac{6\,a^2\log(bx+a)}{b^4}+\frac{bx^2-4\,ax}{b^3}\right)\cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^3*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b^5 - e^(c - a
*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b^5) + 6*a^2*(e^(-c + a*d/b)*exp_in
tegral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d
/b)/b)/(b^3*d) - 4*a*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/
d^2)/b^3 + ((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*
x + 2)*e^(-d*x - c)/d^3)/b^2 + 12*a^2*cosh(d*x + c)*log(b*x + a)/(b^4*d))*d
```

```
 + 1/2*(2*a^3/(b^5*x + a*b^4) + 6*a^2*log(b*x + a)/b^4 + (b*x^2 - 4*a*x)/b^
3)*cosh(d*x + c)
```

Fricas [A]
time = 0.42, size = 333, normalized size = 1.83

$$\frac{2\left(a^{3}bd^{2}-b^{4}x-ab^{3}\right)\cosh\left(dx+c\right)-\left(\left(a^{4}d^{3}-3\,a^{3}bd^{2}+\left(a^{3}bd^{3}-3\,a^{2}b^{2}d^{2}\right)x\right){\rm Ei}\left(\frac{bdx+ad}{b}\right)-\left(a^{4}d^{3}+3\,a^{3}bd^{2}+\left(a^{3}bd^{3}+3\,a^{2}b^{2}d^{2}\right)x\right){\rm Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)+2\left(b^{4}dx^{2}-ab^{3}dx-2\,a^{2}b^{2}d\right)\sinh\left(dx+c\right)+\left(\left(a^{4}d^{3}-3\,a^{3}bd^{2}+\left(a^{3}bd^{3}-3\,a^{2}b^{2}d^{2}\right)x\right){\rm Ei}\left(\frac{bdx+ad}{b}\right)+\left(a^{4}d^{3}+3\,a^{3}bd^{2}+\left(a^{3}bd^{3}+3\,a^{2}b^{2}d^{2}\right)x\right){\rm Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2\left(b^{6}d^{2}x+ab^{5}d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a^3*b*d^2 - b^4*x - a*b^3)*cosh(d*x + c) - ((a^4*d^3 - 3*a^3*b*d^2
+ (a^3*b*d^3 - 3*a^2*b^2*d^2)*x)*Ei((b*d*x + a*d)/b) - (a^4*d^3 + 3*a^3*b*d
^2 + (a^3*b*d^3 + 3*a^2*b^2*d^2)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)
/b) + 2*(b^4*d*x^2 - a*b^3*d*x - 2*a^2*b^2*d)*sinh(d*x + c) + ((a^4*d^3 - 3
*a^3*b*d^2 + (a^3*b*d^3 - 3*a^2*b^2*d^2)*x)*Ei((b*d*x + a*d)/b) + (a^4*d^3
+ 3*a^3*b*d^2 + (a^3*b*d^3 + 3*a^2*b^2*d^2)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-
(b*c - a*d)/b))/(b^6*d^2*x + a*b^5*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\cosh\left(c+dx\right)}{\left(a+bx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1991 vs. $2(185) = 370$.
time = 0.46, size = 1991, normalized size = 10.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*((b*x + a)*a^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*Ei(((b*x + a)*(
b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a^3*
b*c*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e
^((b*c - a*d)/b) + a^4*d^4*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d
) - b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a^3*(b*c/(b*x + a) - a*d/(b
*x + a) + d)*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c +
 a*d)/b)*e^(-(b*c - a*d)/b) + a^3*b*c*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a
```

```
*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^4*d^4*Ei(-((b*x +
a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) -
 3*(b*x + a)*a^2*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + 3*a^2
*b^2*c*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b
)*e^((b*c - a*d)/b) - 3*a^3*b*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
 a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - 3*(b*x + a)*a^2*b*(b*c/(b*x +
a) - a*d/(b*x + a) + d)*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
 d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + 3*a^2*b^2*c*d^2*Ei(-((b*x + a)*(b*
c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - 3*a^3
*b*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e
^(-(b*c - a*d)/b) - a^3*b*d^3*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
 d)/b) - a^3*b*d^3*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - (
b*x + a)^2*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*e^((b*x + a)*(b*c/(b*x
 + a) - a*d/(b*x + a) + d)/b) + 2*(b*x + a)*b^3*(b*c/(b*x + a) - a*d/(b*x +
 a) + d)*c*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - b^4*c^2*e^
((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a)*a*b^2*(b*c/(b
*x + a) - a*d/(b*x + a) + d)*d*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
+ d)/b) - a*b^3*c*d*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2
*a^2*b^2*d^2*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a
)^2*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*e^(-(b*x + a)*(b*c/(b*x + a)
- a*d/(b*x + a) + d)/b) - 2*(b*x + a)*b^3*(b*c/(b*x + a) - a*d/(b*x + a) +
d)*c*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + b^4*c^2*e^(-(b*
x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - (b*x + a)*a*b^2*(b*c/(b*x +
 a) - a*d/(b*x + a) + d)*d*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d
)/b) + a*b^3*c*d*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*a
^2*b^2*d^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a)
*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*e^((b*x + a)*(b*c/(b*x + a) - a*d/
(b*x + a) + d)/b) - b^4*c*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/
b) + a*b^3*d*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a
)*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*e^(-(b*x + a)*(b*c/(b*x + a) - a*
d/(b*x + a) + d)/b) - b^4*c*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
d)/b) + a*b^3*d*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b))*b^2/(
((b*x + a)*b^7*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d - b^8*c*d + a*b^7*d^2)
*d)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \cosh(c + d\,x)}{(a + b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*cosh(c + d*x))/(a + b*x)^2,x)
[Out] int((x^3*cosh(c + d*x))/(a + b*x)^2, x)
```

3.28 $\int \frac{x^2 \cosh(c+dx)}{(a+bx)^2} \, dx$

Optimal. Leaf size=147

$$-\frac{a^2 \cosh(c+dx)}{b^3(a+bx)}-\frac{2a \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{a^2 d \text{Chi}\left(\frac{ad}{b}+dx\right) \sinh\left(c-\frac{ad}{b}\right)}{b^4}+\frac{\sinh(c+dx)}{b^2 d}+\frac{a^2 d \cosh}{}$$

```
[Out] -2*a*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^3-a^2*cosh(d*x+c)/b^3/(b*x+a)+a^2*d*co
sh(-c+a*d/b)*Shi(a*d/b+d*x)/b^4-a^2*d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/b^4+2*a
*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^3+sinh(d*x+c)/b^2/d
```

Rubi [A]
time = 0.29, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6874, 2717, 3378, 3384, 3379, 3382}

$$\frac{a^2 d \sinh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{a^2 d \cosh\left(c-\frac{ad}{b}\right) \text{Shi}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{a^2 \cosh(c+dx)}{b^3(a+bx)}-\frac{2a \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{ad}{b}\right)}{b^3}-\frac{2a \sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{\sinh(c+dx)}{b^2 d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] -((a^2*Cosh[c + d*x])/(b^3*(a + b*x))) - (2*a*Cosh[c - (a*d)/b]*CoshIntegra
l[(a*d)/b + d*x])/b^3 + (a^2*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b
])/b^4 + Sinh[c + d*x]/(b^2*d) + (a^2*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d
)/b + d*x])/b^4 - (2*a*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\cosh(c+dx)}{(a+bx)^2}\,dx &= \int \left(\frac{\cosh(c+dx)}{b^2}+\frac{a^2\cosh(c+dx)}{b^2(a+bx)^2}-\frac{2a\cosh(c+dx)}{b^2(a+bx)}\right)dx\\ &= \frac{\int \cosh(c+dx)\,dx}{b^2}-\frac{(2a)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b^2}+\frac{a^2\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b^2}\\ &= -\frac{a^2\cosh(c+dx)}{b^3(a+bx)}+\frac{\sinh(c+dx)}{b^2d}+\frac{(a^2d)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b^3}-\frac{\left(2a\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos}{}}{b^2}\\ &= -\frac{a^2\cosh(c+dx)}{b^3(a+bx)}-\frac{2a\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{\sinh(c+dx)}{b^2d}-\frac{2a\sinh\left(c-\right.}{}\\ &= -\frac{a^2\cosh(c+dx)}{b^3(a+bx)}-\frac{2a\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{a^2d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\right.}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.50, size = 115, normalized size = 0.78

$$\frac{a\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(-2b\cosh\left(c-\frac{ad}{b}\right)+ad\sinh\left(c-\frac{ad}{b}\right)\right)+b\left(-\frac{a^2\cosh(c+dx)}{a+bx}+\frac{b\sinh(c+dx)}{d}\right)+a\left(ad\cosh\left(c-\frac{ad}{b}\right)-2b\sinh\left(c-\frac{ad}{b}\right)\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (a*CoshIntegral[d*(a/b + x)]*(-2*b*Cosh[c - (a*d)/b] + a*d*Sinh[c - (a*d)/b
]) + b*(-((a^2*Cosh[c + d*x])/(a + b*x)) + (b*Sinh[c + d*x])/d) + a*(a*d*Co
sh[c - (a*d)/b] - 2*b*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)])/b^4
```

Maple [A]
time = 0.85, size = 254, normalized size = 1.73

method	result
risch	$-\frac{\mathrm{e}^{-dx-c}}{2d\,b^{2}}-\frac{d\,\mathrm{e}^{-dx-c}a^{2}}{2b^{3}(bdx+ad)}+\frac{d\,\mathrm{e}^{\frac{ad-bc}{b}}\,\mathrm{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)a^{2}}{2b^{4}}+\frac{\mathrm{e}^{\frac{ad-bc}{b}}\,\mathrm{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)a}{b^{3}}+\frac{\mathrm{e}^{dx+c}}{2b^{2}d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*exp(-d*x-c)/b^2-1/2*d*exp(-d*x-c)/b^3/(b*d*x+a*d)*a^2+1/2*d/b^4*exp(
(a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2+1/b^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+
(a*d-b*c)/b)*a+1/2/b^2/d*exp(d*x+c)-1/2*d/b^4*exp(d*x+c)/(a*d/b+d*x)*a^2-1/
2*d/b^4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2+1/b^3*exp(-(a*d-b*c)
/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a
```

Maxima [A]
time = 0.34, size = 236, normalized size = 1.61

$$\frac{1}{2}\left(a^{2}\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_{1}\left(\frac{(bx+a)d}{b}\right)}{b^{4}}-\frac{e^{\left(c-\frac{ad}{b}\right)}E_{1}\left(-\frac{(bx+a)d}{b}\right)}{b^{4}}\right)+\frac{2\,a\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_{1}\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_{1}\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^{2}d}-\frac{\frac{(dxe^{c}-e^{c})e^{(dx)}}{d^{2}}+\frac{(dx+1)e^{(-dx-c)}}{d^{2}}}{b^{2}}+\frac{4\,a\cosh\left(dx+c\right)\log\left(bx+a\right)}{b^{3}d}\right)d-\left(\frac{a^{2}}{b^{4}x+ab^{3}}-\frac{x}{b^{2}}+\frac{2\,a\log\left(bx+a\right)}{b^{3}}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b^4 - e^(c - a*d/
b)*exp_integral_e(1, -(b*x + a)*d/b)/b^4) + 2*a*(e^(-c + a*d/b)*exp_integra
l_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b
)/(b^2*d) - ((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/b^2
+ 4*a*cosh(d*x + c)*log(b*x + a)/(b^3*d))*d - (a^2/(b^4*x + a*b^3) - x/b^2
+ 2*a*log(b*x + a)/b^3)*cosh(d*x + c)
```

Fricas [A]
time = 0.36, size = 274, normalized size = 1.86

$$-\frac{2\,a^{2}bd\cosh\left(dx+c\right)-\left(\left(a^{3}d^{2}-2\,a^{2}bd+\left(a^{2}bd^{2}-2\,ab^{2}d\right)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-\left(a^{3}d^{2}+2\,a^{2}bd+\left(a^{2}bd^{2}+2\,ab^{2}d\right)x\right)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-2\left(b^{3}x+ab^{2}\right)\sinh\left(dx+c\right)+\left(\left(a^{3}d^{2}-2\,a^{2}bd+\left(a^{2}bd^{2}-2\,ab^{2}d\right)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+\left(a^{3}d^{2}+2\,a^{2}bd+\left(a^{2}bd^{2}+2\,ab^{2}d\right)x\right)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2\left(b^{5}dx+ab^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b*d*cosh(d*x + c) - ((a^3*d^2 - 2*a^2*b*d + (a^2*b*d^2 - 2*a*b^
2*d)*x)*Ei((b*d*x + a*d)/b) - (a^3*d^2 + 2*a^2*b*d + (a^2*b*d^2 + 2*a*b^2*d
)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(b^3*x + a*b^2)*sinh(d*
x + c) + ((a^3*d^2 - 2*a^2*b*d + (a^2*b*d^2 - 2*a*b^2*d)*x)*Ei((b*d*x + a*d
```

```
)/b) + (a^3*d^2 + 2*a^2*b*d + (a^2*b*d^2 + 2*a*b^2*d)*x)*Ei(-(b*d*x + a*d)/
b))*sinh(-(b*c - a*d)/b))/(b^5*d*x + a*b^4*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cosh(c + dx)}{(a + bx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1308 vs. $2(152) = 304$.
time = 0.46, size = 1308, normalized size = 8.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*((b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a^2*b
*c*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^
((b*c - a*d)/b) + a^3*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)
 - b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*
x + a) + d)*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c +
a*d)/b)*e^(-(b*c - a*d)/b) + a^2*b*c*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*
d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^3*d^3*Ei(-((b*x + a
)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) -
2*(b*x + a)*a*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(((b*x + a)*(b*c/(b
*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + 2*a*b^2*c*
d*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c
 - a*d)/b) - 2*a^2*b*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)
- b*c + a*d)/b)*e^((b*c - a*d)/b) - 2*(b*x + a)*a*b*(b*c/(b*x + a) - a*d/(b
*x + a) + d)*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a
*d)/b)*e^(-(b*c - a*d)/b) + 2*a*b^2*c*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d
/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - 2*a^2*b*d^2*Ei(-((b*x
+ a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b)
 - a^2*b*d^2*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - a^2*b*d^
2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a)*b^2*(b*c
/(b*x + a) - a*d/(b*x + a) + d)*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
 + d)/b) - b^3*c*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + a*b^
2*d*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - (b*x + a)*b^2*(b*
```

```
c/(b*x + a) - a*d/(b*x + a) + d)*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
a) + d)/b) + b^3*c*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - a
*b^2*d*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b))*b^2/(((b*x + a
)*b^6*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b^7*c + a*b^6*d)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \cosh(c + d\,x)}{(a + b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(c + d*x))/(a + b*x)^2,x)
```

```
[Out] int((x^2*cosh(c + d*x))/(a + b*x)^2, x)
```

3.29 $\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx$

Optimal. Leaf size=125

$$\frac{a\cosh(c+dx)}{b^2(a+bx)}+\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^2}-\frac{ad\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^3}-\frac{ad\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right.}{b^3}$$

```
[Out] Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^2+a*cosh(d*x+c)/b^2/(b*x+a)-a*d*cosh(-c+a*d
/b)*Shi(a*d/b+d*x)/b^3+a*d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/b^3-Shi(a*d/b+d*x)
*sinh(-c+a*d/b)/b^2
```

Rubi [A]
time = 0.22, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6874, 3378, 3384, 3379, 3382}

$$-\frac{ad\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^3}-\frac{ad\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^2}+\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^2}+\frac{a\cosh(c+dx)}{b^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (a*Cosh[c + d*x])/(b^2*(a + b*x)) + (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b
 + d*x])/b^2 - (a*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^3 - (a
*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3 + (Sinh[c - (a*d)/b]*
SinhIntegral[(a*d)/b + d*x])/b^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx &= \int \left(-\frac{a\cosh(c+dx)}{b(a+bx)^2}+\frac{\cosh(c+dx)}{b(a+bx)}\right)dx\\
&= \frac{\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b}-\frac{a\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b}\\
&= \frac{a\cosh(c+dx)}{b^2(a+bx)}-\frac{(ad)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b^2}+\frac{\cosh\left(c-\frac{ad}{b}\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}+\frac{\sinh\left(c-\frac{ad}{b}\right.}{}\\
&= \frac{a\cosh(c+dx)}{b^2(a+bx)}+\frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^2}+\frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^2}-\frac{(ad\,\mathrm{co}}{}\\
&= \frac{a\cosh(c+dx)}{b^2(a+bx)}+\frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^2}-\frac{ad\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^3}-\frac{ad}{}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 97, normalized size = 0.78

$$\frac{\frac{ab\cosh(c+dx)}{a+bx}+\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(b\cosh\left(c-\frac{ad}{b}\right)-ad\sinh\left(c-\frac{ad}{b}\right)\right)+\left(-ad\cosh\left(c-\frac{ad}{b}\right)+b\sinh\left(c-\frac{ad}{b}\right)\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] ((a*b*Cosh[c + d*x])/(a + b*x) + CoshIntegral[d*(a/b + x)]*(b*Cosh[c - (a*d
)/b] - a*d*Sinh[c - (a*d)/b]) + (-(a*d*Cosh[c - (a*d)/b]) + b*Sinh[c - (a*d
)/b])*SinhIntegral[d*(a/b + x)])/b^3
```

Maple [A]
time = 0.74, size = 215, normalized size = 1.72

method	result

risch	$\frac{d\,\mathrm{e}^{-dx-c}a}{2b^2(bdx+ad)} - \frac{d\,\mathrm{e}^{\frac{ad-bc}{b}}\,\mathrm{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)a}{2b^3} - \frac{\mathrm{e}^{\frac{ad-bc}{b}}\,\mathrm{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)}{2b^2} + \frac{d\,\mathrm{e}^{dx+c}a}{2b^3\left(\frac{ad}{b}+dx\right)} + d\,\mathrm{e}^{-\frac{ad-}{b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d*exp(-d*x-c)/b^2/(b*d*x+a*d)*a-1/2*d/b^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(
a*d-b*c)/b)*a-1/2/b^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+1/2*d/b^3*ex
p(d*x+c)/(a*d/b+d*x)*a+1/2*d/b^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
*a-1/2/b^2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Maxima [A]
time = 0.32, size = 178, normalized size = 1.42

$$-\frac{1}{2}\left(a\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b^3} - \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b^3}\right) + \frac{\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}}{bd} + \frac{2\cosh(dx+c)\log(bx+a)}{b^2d}\right)d + \left(\frac{a}{b^3x+ab^2} + \frac{\log(bx+a)}{b^2}\right)\cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b^3 - e^(c - a*d/b
)*exp_integral_e(1, -(b*x + a)*d/b)/b^3) + (e^(-c + a*d/b)*exp_integral_e(1
, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b*
d) + 2*cosh(d*x + c)*log(b*x + a)/(b^2*d))*d + (a/(b^3*x + a*b^2) + log(b*x
 + a)/b^2)*cosh(d*x + c)
```

Fricas [A]
time = 0.37, size = 200, normalized size = 1.60

$$\frac{2\,ab\cosh(dx+c) - \left(\left(a^2d - ab + (abd - b^2)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right) - \left(a^2d + ab + (abd + b^2)x\right)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right) + \left(\left(a^2d - ab + (abd - b^2)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right) + \left(a^2d + ab + (abd + b^2)x\right)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2\left(b^4x + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*cosh(d*x + c) - ((a^2*d - a*b + (a*b*d - b^2)*x)*Ei((b*d*x + a*d
)/b) - (a^2*d + a*b + (a*b*d + b^2)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a
*d)/b) + ((a^2*d - a*b + (a*b*d - b^2)*x)*Ei((b*d*x + a*d)/b) + (a^2*d + a*
b + (a*b*d + b^2)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^4*x + a
*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x+a)**2,x)

[Out] Integral(x*cosh(c + d*x)/(a + b*x)**2, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 994 vs. 2(129) = 258.
time = 0.46, size = 994, normalized size = 7.95

$$-\frac{\left((bx+a)a\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)d^2\operatorname{Ei}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(\frac{bc-ad}{b}\right)}-abcd^2\operatorname{Ei}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(\frac{bc-ad}{b}\right)}+a^2d^3\operatorname{Ei}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(\frac{bc-ad}{b}\right)}-(bx+a)a\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)d^2\operatorname{Ei}\left(-\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(-\frac{bc-ad}{b}\right)}+abcd^2\operatorname{Ei}\left(-\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(-\frac{bc-ad}{b}\right)}-a^2d^3\operatorname{Ei}\left(-\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(-\frac{bc-ad}{b}\right)}-(bx+a)b\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)d\operatorname{Ei}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(\frac{bc-ad}{b}\right)}+b^2cd\operatorname{Ei}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(\frac{bc-ad}{b}\right)}-abd^2\operatorname{Ei}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(\frac{bc-ad}{b}\right)}-(bx+a)b\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)d\operatorname{Ei}\left(-\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(-\frac{bc-ad}{b}\right)}+b^2cd\operatorname{Ei}\left(-\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(-\frac{bc-ad}{b}\right)}-abd^2\operatorname{Ei}\left(-\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(-\frac{bc-ad}{b}\right)}-abd^2e^{\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)}{b}\right)}-abd^2e^{\left(-\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)}{b}\right)}\right)b}{2\left((bx+a)b^4\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-b^5c+ab^4d\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")

[Out] -1/2*((b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a*b*c*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + a^2*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + a*b*c*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^2*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - (b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + b^2*c*d*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a*b*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + b^2*c*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a*b*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a*b*d^2*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - a*b*d^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b))*b/(((b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b^5*c + a*b^4*d)*d)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*cosh(c + d*x))/(a + b*x)^2,x)

[Out] int((x*cosh(c + d*x))/(a + b*x)^2, x)

3.30 $\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx$

Optimal. Leaf size=71

$$-\frac{\cosh(c+dx)}{b(a+bx)}+\frac{d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^2}+\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^2}$$

```
[Out] -cosh(d*x+c)/b/(b*x+a)+d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/b^2-d*Chi(a*d/b+d*x)
*sinh(-c+a*d/b)/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3378, 3384, 3379, 3382}

$$\frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^2}+\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^2}-\frac{\cosh(c+dx)}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*x)^2,x]
```

```
[Out] -(Cosh[c + d*x]/(b*(a + b*x))) + (d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a
*d)/b])/b^2 + (d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx &= -\frac{\cosh(c+dx)}{b(a+bx)}+\frac{d\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b}\\ &= -\frac{\cosh(c+dx)}{b(a+bx)}+\frac{\left(d\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}+\frac{\left(d\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,d\\ &= -\frac{\cosh(c+dx)}{b(a+bx)}+\frac{d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^2}+\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 65, normalized size = 0.92

$$\frac{-\frac{b\cosh(c+dx)}{a+bx}+d\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\sinh\left(c-\frac{ad}{b}\right)+d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x)^2,x]
```

```
[Out] (-((b*Cosh[c + d*x])/(a + b*x)) + d*CoshIntegral[d*(a/b + x)]*Sinh[c - (a*d
)/b] + d*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/b^2
```

Maple [A]
time = 0.71, size = 132, normalized size = 1.86

method	result	size
risch	$-\frac{d\,\mathrm{e}^{-dx-c}}{2b(bdx+ad)}+\frac{d\,\mathrm{e}^{\frac{ad-bc}{b}}\,\text{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)}{2b^2}-\frac{d\,\mathrm{e}^{dx+c}}{2b^2\left(\frac{ad}{b}+dx\right)}-\frac{d\,\mathrm{e}^{-\frac{ad-bc}{b}}\,\text{expIntegral}\left(1,-dx-c-\frac{ad-bc}{b}\right)}{2b^2}$	132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d*exp(-d*x-c)/b/(b*d*x+a*d)+1/2*d/b^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d
-b*c)/b)-1/2*d/b^2*exp(d*x+c)/(a*d/b+d*x)-1/2*d/b^2*exp(-(a*d-b*c)/b)*Ei(1,
-d*x-c-(a*d-b*c)/b)
```

Maxima [A]
time = 0.32, size = 81, normalized size = 1.14

$$\frac{d\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}-\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{2\,b}-\frac{\cosh\left(dx+c\right)}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b - e^(c - a*d/b)*ex
p_integral_e(1, -(b*x + a)*d/b)/b)/b - cosh(d*x + c)/((b*x + a)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(74) = 148.
time = 0.38, size = 149, normalized size = 2.10

$$-\frac{2\,b\cosh\left(dx+c\right)-\left((bdx+ad)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-(bdx+ad)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)+\left((bdx+ad)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+(bdx+ad)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2\,(b^3x+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*cosh(d*x + c) - ((b*d*x + a*d)*Ei((b*d*x + a*d)/b) - (b*d*x + a*d
)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + ((b*d*x + a*d)*Ei((b*d*x + a
*d)/b) + (b*d*x + a*d)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^3*x +
 a*b^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 615 vs. 2(74) = 148.
time = 0.44, size = 615, normalized size = 8.66

$$\frac{\left((bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)d^2\mathrm{Ei}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(\frac{bc-ad}{b}\right)}-bcd^2\mathrm{Ei}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(\frac{bc-ad}{b}\right)}+ad^3\mathrm{Ei}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(\frac{bc-ad}{b}\right)}-bd^2e^{\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)}{b}\right)}\right)b^2}{2\left((bx+a)b^4\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-b^5c+ab^4d\right)d}-\frac{\left((bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)d^2\mathrm{Ei}\left(-\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(-\frac{bc-ad}{b}\right)}-bcd^2\mathrm{Ei}\left(-\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(-\frac{bc-ad}{b}\right)}+ad^3\mathrm{Ei}\left(-\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(-\frac{bc-ad}{b}\right)}+bd^2e^{\left(-\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)}{b}\right)}\right)b^2}{2\left((bx+a)b^4\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-b^5c+ab^4d\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - b*c*d^2*E
i(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c -
a*d)/b) + a*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a
*d)/b)*e^((b*c - a*d)/b) - b*d^2*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a
) + d)/b))*b^2/(((b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b^5*c
+ a*b^4*d)*d) - 1/2*((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(-
((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a
*d)/b) - b*c*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c +
 a*d)/b)*e^(-(b*c - a*d)/b) + a*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*
x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + b*d^2*e^(-(b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d)/b))*b^2/(((b*x + a)*b^4*(b*c/(b*x + a) - a*d/
(b*x + a) + d) - b^5*c + a*b^4*d)*d)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + d\,x)}{{(a + b\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*x)^2,x)
```

```
[Out] int(cosh(c + d*x)/(a + b*x)^2, x)
```

3.31 $\int \frac{\cosh(c+dx)}{x(a+bx)^2} \, dx$

Optimal. Leaf size=150

$$\frac{\cosh(c+dx)}{a(a+bx)}+\frac{\cosh(c)\text{Chi}(dx)}{a^2}-\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^2}-\frac{d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{ab}+\frac{\sinh(c)\text{Shi}(dx)}{a^2}$$

```
[Out] Chi(d*x)*cosh(c)/a^2-Chi(a*d/b+d*x)*cosh(-c+a*d/b)/a^2+cosh(d*x+c)/a/(b*x+a
)-d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/a/b+Shi(d*x)*sinh(c)/a^2+d*Chi(a*d/b+d*x)
*sinh(-c+a*d/b)/a/b+Shi(a*d/b+d*x)*sinh(-c+a*d/b)/a^2
```

Rubi [A]
time = 0.31, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3384, 3379, 3382, 3378}

$$-\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^2}-\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{\cosh(c)\text{Chi}(dx)}{a^2}+\frac{\sinh(c)\text{Shi}(dx)}{a^2}-\frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{ab}-\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{ab}+\frac{\cosh(c+dx)}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x)^2),x]
```

```
[Out] Cosh[c + d*x]/(a*(a + b*x)) + (Cosh[c]*CoshIntegral[d*x])/a^2 - (Cosh[c - (
a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^2 - (d*CoshIntegral[(a*d)/b + d*x]*S
inh[c - (a*d)/b])/(a*b) + (Sinh[c]*SinhIntegral[d*x])/a^2 - (d*Cosh[c - (a*
d)/b]*SinhIntegral[(a*d)/b + d*x])/(a*b) - (Sinh[c - (a*d)/b]*SinhIntegral[
(a*d)/b + d*x])/a^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x(a+bx)^2}\,dx &= \int \left(\frac{\cosh(c+dx)}{a^2x}-\frac{b\cosh(c+dx)}{a(a+bx)^2}-\frac{b\cosh(c+dx)}{a^2(a+bx)}\right)dx\\ &= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a^2}-\frac{b\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^2}-\frac{b\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{a}\\ &= \frac{\cosh(c+dx)}{a(a+bx)}-\frac{d\int \frac{\sinh(c+dx)}{a+bx}\,dx}{a}+\frac{\cosh(c)\int \frac{\cosh(dx)}{x}\,dx}{a^2}-\frac{\left(b\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+d\right.}{a+bx}}{a^2}\\ &= \frac{\cosh(c+dx)}{a(a+bx)}+\frac{\cosh(c)\mathrm{Chi}(dx)}{a^2}-\frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^2}+\frac{\sinh(c)\mathrm{Shi}(dx)}{a^2}-\frac{\sin}{}\\ &= \frac{\cosh(c+dx)}{a(a+bx)}+\frac{\cosh(c)\mathrm{Chi}(dx)}{a^2}-\frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^2}-\frac{d\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh(}{ab}\end{aligned}$$

Mathematica [A]
time = 0.81, size = 139, normalized size = 0.93

$$\frac{\frac{a\cosh(c)\cosh(dx)}{a+bx}+\cosh(c)\mathrm{Chi}(dx)-\frac{\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(b\cosh\left(c-\frac{ad}{b}\right)+ad\sinh\left(c-\frac{ad}{b}\right)\right)}{b}+\frac{a\sinh(c)\sinh(dx)}{a+bx}+\sinh(c)\mathrm{Shi}(dx)-\frac{ad\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{b}-\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x)^2),x]
```

```
[Out] ((a*Cosh[c]*Cosh[d*x])/(a + b*x) + Cosh[c]*CoshIntegral[d*x] - (CoshIntegra
l[d*(a/b + x)]*(b*Cosh[c - (a*d)/b] + a*d*Sinh[c - (a*d)/b]))/b + (a*Sinh[c
]*Sinh[d*x])/(a + b*x) + Sinh[c]*SinhIntegral[d*x] - (a*d*Cosh[c - (a*d)/b]
*SinhIntegral[d*(a/b + x)])/b - Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)]
)/a^2
```

Maple [A]
time = 0.71, size = 254, normalized size = 1.69

method	result
risch	$\frac{e^{-dx-c}d}{2a(b(dx+c)+ad-bc)} - \frac{e^{-c}\,\text{expIntegral}(1,dx)}{2a^2} - \frac{e^{\frac{ad-bc}{b}}\,\text{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)d}{2ab} + \frac{e^{\frac{ad-bc}{b}}\,\text{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right.}{2a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(-d*x-c)*d/a/(b*(d*x+c)+a*d-b*c)-1/2/a^2*exp(-c)*Ei(1,d*x)-1/2/a/b*e
xp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*d+1/2/a^2*exp((a*d-b*c)/b)*Ei(1,d*x
+c+(a*d-b*c)/b)+1/2/a*d/b*exp(d*x+c)/(a*d/b+d*x)+1/2/a*d/b*exp(-(a*d-b*c)/b
)*Ei(1,-d*x-c-(a*d-b*c)/b)-1/2/a^2*exp(c)*Ei(1,-d*x)+1/2/a^2*exp(-(a*d-b*c)
/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Maxima [A]
time = 0.38, size = 227, normalized size = 1.51

$$-\frac{1}{2}d\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{ab} - \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{ab} - \frac{b\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{a^2d} - \frac{2\cosh(dx+c)\log(bx+a)}{a^2d} + \frac{2\cosh(dx+c)\log(x)}{a^2d} - \frac{\text{Ei}(-dx)e^{(-c)} + \text{Ei}(dx)e^{c}}{a^2d}\right) + \left(\frac{1}{abx+a^2} - \frac{\log(bx+a)}{a^2} + \frac{\log(x)}{a^2}\right)\cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*d*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/(a*b) - e^(c - a*d/
b)*exp_integral_e(1, -(b*x + a)*d/b)/(a*b) - b*(e^(-c + a*d/b)*exp_integral
_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)
/(a^2*d) - 2*cosh(d*x + c)*log(b*x + a)/(a^2*d) + 2*cosh(d*x + c)*log(x)/(a
^2*d) - (Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)/(a^2*d)) + (1/(a*b*x + a^2) - log(b
*x + a)/a^2 + log(x)/a^2)*cosh(d*x + c)
```

Fricas [A]
time = 0.37, size = 270, normalized size = 1.80

$$\frac{2\,ab\cosh(dx+c) + \left((b^2x+ab)\text{Ei}(dx) + (b^2x+ab)\text{Ei}(-dx)\right)\cosh(c) - \left((a^2d+ab+(abd+b^2)x)\text{Ei}\left(\frac{bdx+ad}{b}\right) - (a^2d-ab+(abd-b^2)x)\text{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right) + \left((b^2x+ab)\text{Ei}(dx) - (b^2x+ab)\text{Ei}(-dx)\right)\sinh(c) + \left((a^2d+ab+(abd+b^2)x)\text{Ei}\left(\frac{bdx+ad}{b}\right) + (a^2d-ab+(abd-b^2)x)\text{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2\left(a^2b^2x+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*cosh(d*x + c) + ((b^2*x + a*b)*Ei(d*x) + (b^2*x + a*b)*Ei(-d*x))
*cosh(c) - ((a^2*d + a*b + (a*b*d + b^2)*x)*Ei((b*d*x + a*d)/b) - (a^2*d -
a*b + (a*b*d - b^2)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + ((b^2*x
 + a*b)*Ei(d*x) - (b^2*x + a*b)*Ei(-d*x))*sinh(c) + ((a^2*d + a*b + (a*b*d
+ b^2)*x)*Ei((b*d*x + a*d)/b) + (a^2*d - a*b + (a*b*d - b^2)*x)*Ei(-(b*d*x
+ a*d)/b))*sinh(-(b*c - a*d)/b))/(a^2*b^2*x + a^3*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

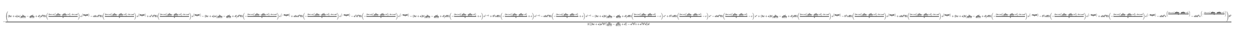
$$\int \frac{\cosh(c+dx)}{x(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)**2,x)
```

```
[Out] Integral(cosh(c + d*x)/(x*(a + b*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1329 vs. 2(152) = 304.
time = 0.46, size = 1329, normalized size = 8.86



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*((b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(b*
c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a*b*c*
d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b
*c - a*d)/b) + a^2*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) -
b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a
) + d)*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/
b)*e^(-(b*c - a*d)/b) + a*b*c*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x
+ a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^2*d^3*Ei(-((b*x + a)*(b*c/
(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - (b*x +
a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(-(b*x + a)*(b*c/(b*x + a) - a
*d/(b*x + a) + d)/b + c)*e^(-c) + b^2*c*d*Ei(-(b*x + a)*(b*c/(b*x + a) - a*
d/(b*x + a) + d)/b + c)*e^(-c) - a*b*d^2*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d
/(b*x + a) + d)/b + c)*e^(-c) - (b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a)
+ d)*d*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c + b^2*c*
d*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c - a*b*d^2*Ei(
(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c + (b*x + a)*b*(b*c
/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
 a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - b^2*c*d*Ei(((b*x + a)*(b*c/(b*
x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + a*b*d^2*Ei(
((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*
d)/b) + (b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(-((b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - b^2*
c*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-
(b*c - a*d)/b) + a*b*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)
 - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a*b*d^2*e^((b*x + a)*(b*c/(b*x + a) -
```

```
 a*d/(b*x + a) + d)/b) - a*b*d^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
a) + d)/b))*b^3/(((b*x + a)*a^2*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d) - a
^2*b^5*c + a^3*b^4*d)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)}{x\,(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x*(a + b*x)^2),x)
```

```
[Out] int(cosh(c + d*x)/(x*(a + b*x)^2), x)
```

3.32 $\int \frac{\cosh(c+dx)}{x^2(a+bx)^2}\,dx$

Optimal. Leaf size=186

$$-\frac{\cosh(c+dx)}{a^2x}-\frac{b\cosh(c+dx)}{a^2(a+bx)}-\frac{2b\cosh(c)\text{Chi}(dx)}{a^3}+\frac{2b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^3}+\frac{d\text{Chi}(dx)\sinh(c)}{a^2}+\frac{d\text{Cl}}{}$$

```
[Out] -2*b*Chi(d*x)*cosh(c)/a^3+2*b*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/a^3-cosh(d*x+c)
/a^2/x-b*cosh(d*x+c)/a^2/(b*x+a)+d*cosh(c)*Shi(d*x)/a^2+d*cosh(-c+a*d/b)*Sh
i(a*d/b+d*x)/a^2+d*Chi(d*x)*sinh(c)/a^2-2*b*Shi(d*x)*sinh(c)/a^3-d*Chi(a*d/
b+d*x)*sinh(-c+a*d/b)/a^2-2*b*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/a^3
```

Rubi [A]
time = 0.37, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3379, 3382}

$$-\frac{2b\cosh(c)\text{Chi}(dx)}{a^3}+\frac{2b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{2b\sinh(c)\text{Shi}(dx)}{a^3}+\frac{2b\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^3}+\frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^2}-\frac{b\cosh(c+dx)}{a^2(a+bx)}+\frac{d\sinh(c)\text{Chi}(dx)}{a^2}+\frac{d\cosh(c)\text{Shi}(dx)}{a^2}-\frac{\cosh(c+dx)}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x)^2),x]
```

```
[Out] -(Cosh[c + d*x]/(a^2*x)) - (b*Cosh[c + d*x])/(a^2*(a + b*x)) - (2*b*Cosh[c]
*CoshIntegral[d*x])/a^3 + (2*b*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x
])/a^3 + (d*CoshIntegral[d*x]*Sinh[c])/a^2 + (d*CoshIntegral[(a*d)/b + d*x]
*Sinh[c - (a*d)/b])/a^2 + (d*Cosh[c]*SinhIntegral[d*x])/a^2 - (2*b*Sinh[c]*
SinhIntegral[d*x])/a^3 + (d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/
a^2 + (2*b*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2}\,dx = \int \left(\frac{\cosh(c+dx)}{a^2x^2} - \frac{2b\cosh(c+dx)}{a^3x} + \frac{b^2\cosh(c+dx)}{a^2(a+bx)^2} + \frac{2b^2\cosh(c+dx)}{a^3(a+bx)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a^2} - \frac{(2b)\int \frac{\cosh(c+dx)}{x}\,dx}{a^3} + \frac{(2b^2)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^3} + \frac{b^2\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{a^2}$$

$$= -\frac{\cosh(c+dx)}{a^2x} - \frac{b\cosh(c+dx)}{a^2(a+bx)} + \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{a^2} + \frac{(bd)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{a^2} - \frac{(2b\cosh(\cdots}{\cdots}$$

$$= -\frac{\cosh(c+dx)}{a^2x} - \frac{b\cosh(c+dx)}{a^2(a+bx)} - \frac{2b\cosh(c)\text{Chi}(dx)}{a^3} + \frac{2b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+d\cdots\right.}{a^3}$$

$$= -\frac{\cosh(c+dx)}{a^2x} - \frac{b\cosh(c+dx)}{a^2(a+bx)} - \frac{2b\cosh(c)\text{Chi}(dx)}{a^3} + \frac{2b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+d\cdots\right.}{a^3}$$

Mathematica [A]
time = 0.90, size = 183, normalized size = 0.98

$$\frac{-\frac{a(a+2bx)\cosh(c)\cosh(dx)}{x(a+bx)} - 2b\cosh(c)\text{Chi}(dx) + 2b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right) + ad\text{Chi}(dx)\sinh(c) + ad\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\sinh\left(c-\frac{ad}{b}\right) - \frac{a(a+2bx)\sinh(c)\sinh(dx)}{x(a+bx)} + ad\cosh(c)\text{Shi}(dx) - 2b\sinh(c)\text{Shi}(dx) + ad\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + 2b\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x)^2),x]
```

```
[Out] (-((a*(a + 2*b*x)*Cosh[c]*Cosh[d*x])/(x*(a + b*x))) - 2*b*Cosh[c]*CoshInteg
ral[d*x] + 2*b*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] + a*d*CoshIntegr
al[d*x]*Sinh[c] + a*d*CoshIntegral[d*(a/b + x)]*Sinh[c - (a*d)/b] - (a*(a +
 2*b*x)*Sinh[c]*Sinh[d*x])/(x*(a + b*x)) + a*d*Cosh[c]*SinhIntegral[d*x] -
```

```
2*b*Sinh[c]*SinhIntegral[d*x] + a*d*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b +
 x)] + 2*b*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/a^3
```

Maple [A]
time = 0.74, size = 312, normalized size = 1.68

method	result
risch	$-\frac{d\,\mathrm{e}^{-dx-c}b}{a^{2}(bdx+ad)}-\frac{d\,\mathrm{e}^{-dx-c}}{2ax(bdx+ad)}+\frac{d\,\mathrm{e}^{-c}\operatorname{expIntegral}(1,dx)}{2a^{2}}+\frac{\mathrm{e}^{-c}\operatorname{expIntegral}(1,dx)b}{a^{3}}+\frac{d\,\mathrm{e}^{\frac{ad-bc}{b}}\operatorname{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)}{2a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d*exp(-d*x-c)/a^2/(b*d*x+a*d)*b-1/2*d*exp(-d*x-c)/a/x/(b*d*x+a*d)+1/2*d/a^
2*exp(-c)*Ei(1,d*x)+1/a^3*exp(-c)*Ei(1,d*x)*b+1/2*d/a^2*exp((a*d-b*c)/b)*Ei
(1,d*x+c+(a*d-b*c)/b)-1/a^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*b-1/2*
d/a^2*exp(d*x+c)/(a*d/b+d*x)-1/2*d/a^2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b
*c)/b)+1/a^3*b*exp(c)*Ei(1,-d*x)-b/a^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b
*c)/b)-1/2/a^2/x*exp(d*x+c)-1/2*d/a^2*exp(c)*Ei(1,-d*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x + a)^2*x^2), x)
```

Fricas [A]
time = 0.46, size = 377, normalized size = 2.03

$$-\frac{2\,(2\,abx+a^{2})\cosh\left(dx+c\right)-\left(\left(\left(abd-2\,b^{2}\right)x^{2}+\left(a^{2}d-2\,ab\right)x\right)\mathrm{Ei}(dx)-\left(\left(abd+2\,b^{2}\right)x^{2}+\left(a^{2}d+2\,ab\right)x\right)\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-\left(\left(\left(abd+2\,b^{2}\right)x^{2}+\left(a^{2}d+2\,ab\right)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-\left(\left(abd-2\,b^{2}\right)x^{2}+\left(a^{2}d-2\,ab\right)x\right)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-\left(\left(\left(abd-2\,b^{2}\right)x^{2}+\left(a^{2}d-2\,ab\right)x\right)\mathrm{Ei}(dx)+\left(\left(abd+2\,b^{2}\right)x^{2}+\left(a^{2}d+2\,ab\right)x\right)\mathrm{Ei}(-dx)\right)\sinh\left(c\right)+\left(\left(\left(abd+2\,b^{2}\right)x^{2}+\left(a^{2}d+2\,ab\right)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+\left(\left(abd-2\,b^{2}\right)x^{2}+\left(a^{2}d-2\,ab\right)x\right)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2\,(a^{3}bx^{2}+a^{4}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(2*a*b*x + a^2)*cosh(d*x + c) - (((a*b*d - 2*b^2)*x^2 + (a^2*d - 2*
a*b)*x)*Ei(d*x) - ((a*b*d + 2*b^2)*x^2 + (a^2*d + 2*a*b)*x)*Ei(-d*x))*cosh(
c) - (((a*b*d + 2*b^2)*x^2 + (a^2*d + 2*a*b)*x)*Ei((b*d*x + a*d)/b) - ((a*b
*d - 2*b^2)*x^2 + (a^2*d - 2*a*b)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d
)/b) - (((a*b*d - 2*b^2)*x^2 + (a^2*d - 2*a*b)*x)*Ei(d*x) + ((a*b*d + 2*b^2
)*x^2 + (a^2*d + 2*a*b)*x)*Ei(-d*x))*sinh(c) + (((a*b*d + 2*b^2)*x^2 + (a^2
*d + 2*a*b)*x)*Ei((b*d*x + a*d)/b) + ((a*b*d - 2*b^2)*x^2 + (a^2*d - 2*a*b)
*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^3*b*x^2 + a^4*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**2/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3353 vs. 2(191) = 382.
time = 0.47, size = 3353, normalized size = 18.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*((b*x + a)^2*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d^2*Ei(-(b*x + a)
*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c)/b - 2*(b*x + a)*a*(b*c/(
b*x + a) - a*d/(b*x + a) + d)*c*d^2*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x
 + a) + d)/b + c)*e^(-c) + a*b*c^2*d^2*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d/(
b*x + a) + d)/b + c)*e^(-c) + (b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a)
+ d)*d^3*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c)/b
- a^2*c*d^3*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c)
 - (b*x + a)^2*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d^2*Ei((b*x + a)*(b*
c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c/b + 2*(b*x + a)*a*(b*c/(b*x + a
) - a*d/(b*x + a) + d)*c*d^2*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
d)/b - c)*e^c - a*b*c^2*d^2*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d
)/b - c)*e^c - (b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*Ei((b*
x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c/b + a^2*c*d^3*Ei((b*x
 + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c - (b*x + a)^2*a*(b*c/(
b*x + a) - a*d/(b*x + a) + d)^2*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x
 + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b)/b + 2*(b*x + a)*a*(b*c/(b*x +
a) - a*d/(b*x + a) + d)*c*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
+ d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a*b*c^2*d^2*Ei(((b*x + a)*(b*c/(b*
x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a
^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a
*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b)/b + a^2*c*d^3*Ei(((b*x
+ a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b)
+ (b*x + a)^2*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d^2*Ei(-((b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b)/b - 2*
(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d^2*Ei(-((b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + a*b*c^2*
```

```
d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-
(b*c - a*d)/b) + (b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*Ei(-
((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a
*d)/b)/b - a^2*c*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b
*c + a*d)/b)*e^(-(b*c - a*d)/b) + 2*(b*x + a)^2*(b*c/(b*x + a) - a*d/(b*x +
 a) + d)^2*d*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c
) - 4*(b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d*Ei(-(b*x + a)*(b*
c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c) + 2*b^2*c^2*d*Ei(-(b*x + a)*
(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c) + 2*(b*x + a)*a*(b*c/(b*x
 + a) - a*d/(b*x + a) + d)*d^2*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
 + d)/b + c)*e^(-c) - 2*a*b*c*d^2*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
 a) + d)/b + c)*e^(-c) + 2*(b*x + a)^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^
2*d*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c - 4*(b*x +
a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d*Ei((b*x + a)*(b*c/(b*x + a) -
a*d/(b*x + a) + d)/b - c)*e^c + 2*b^2*c^2*d*Ei((b*x + a)*(b*c/(b*x + a) - a
*d/(b*x + a) + d)/b - c)*e^c + 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a)
 + d)*d^2*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c - 2*a
*b*c*d^2*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c - 2*(b
*x + a)^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d*Ei(((b*x + a)*(b*c/(b*x +
 a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + 4*(b*x + a)*b*
(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/
(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - 2*b^2*c^2*d*Ei(((b*x + a
)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - 2
*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(b*c/(b*
x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + 2*a*b*c*d^2
*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c
- a*d)/b) - 2*(b*x + a)^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d*Ei(-((b*x
 + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b
) + 4*(b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d*Ei(-((b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - 2*b^
2*c^2*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*
e^(-(b*c - a*d)/b) - 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*
Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c
 - a*d)/b) + 2*a*b*c*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)
 - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b
*x + a) + d)*d^2*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*a*
b*c*d^2*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + a^2*d^3*e^((b
*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2*(b*x + a)*a*(b*c/(b*x +
a) - a*d/(b*x + a) + d)*d^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
d)/b) - 2*a*b*c*d^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) +
a^2*d^3*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + d\,x)}{x^2\,(a + b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x^2*(a + b*x)^2),x)
[Out] int(cosh(c + d*x)/(x^2*(a + b*x)^2), x)
```

3.33 $\int \frac{x^3 \cosh(c+dx)}{(a+bx)^3} \, dx$

Optimal. Leaf size=264

$$\frac{a^3 \cosh(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2 \cosh(c+dx)}{b^4(a+bx)} - \frac{3a \cosh\left(c-\frac{ad}{b}\right) \operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} - \frac{a^3 d^2 \cosh\left(c-\frac{ad}{b}\right) \operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{2b^6} + \frac{3a^2 d}{}$$

```
[Out] -3*a*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^4-1/2*a^3*d^2*Chi(a*d/b+d*x)*cosh(-c+a
*d/b)/b^6+1/2*a^3*cosh(d*x+c)/b^4/(b*x+a)^2-3*a^2*cosh(d*x+c)/b^4/(b*x+a)+3
*a^2*d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/b^5-3*a^2*d*Chi(a*d/b+d*x)*sinh(-c+a*d
/b)/b^5+3*a*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^4+1/2*a^3*d^2*Shi(a*d/b+d*x)*si
nh(-c+a*d/b)/b^6+sinh(d*x+c)/b^3/d+1/2*a^3*d*sinh(d*x+c)/b^5/(b*x+a)
```

Rubi [A]
time = 0.47, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6874, 2717, 3378, 3384, 3379, 3382}

$$-\frac{a^3 d^2 \cosh\left(c-\frac{ad}{b}\right) \operatorname{Chi}\left(xd+\frac{ad}{b}\right)}{2b^6} - \frac{a^3 d^2 \sinh\left(c-\frac{ad}{b}\right) \operatorname{Shi}\left(xd+\frac{ad}{b}\right)}{2b^6} + \frac{a^3 d \sinh(c+dx)}{2b^5(a+bx)} + \frac{a^3 \cosh(c+dx)}{2b^4(a+bx)^2} + \frac{3a^2 d \sinh\left(c-\frac{ad}{b}\right) \operatorname{Chi}\left(xd+\frac{ad}{b}\right)}{b^5} + \frac{3a^2 d \cosh\left(c-\frac{ad}{b}\right) \operatorname{Shi}\left(xd+\frac{ad}{b}\right)}{b^5} - \frac{3a^2 \cosh(c+dx)}{b^4(a+bx)} - \frac{3a \cosh\left(c-\frac{ad}{b}\right) \operatorname{Chi}\left(xd+\frac{ad}{b}\right)}{b^4} - \frac{3a \sinh\left(c-\frac{ad}{b}\right) \operatorname{Shi}\left(xd+\frac{ad}{b}\right)}{b^4} + \frac{\sinh(c+dx)}{b^3 d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x)^3,x]
```

```
[Out] (a^3*Cosh[c + d*x])/(2*b^4*(a + b*x)^2) - (3*a^2*Cosh[c + d*x])/(b^4*(a + b
*x)) - (3*a*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^4 - (a^3*d^2*C
osh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*b^6) + (3*a^2*d*CoshIntegr
al[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^5 + Sinh[c + d*x]/(b^3*d) + (a^3*d*S
inh[c + d*x])/(2*b^5*(a + b*x)) + (3*a^2*d*Cosh[c - (a*d)/b]*SinhIntegral[(
a*d)/b + d*x])/b^5 - (3*a*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^
4 - (a^3*d^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*b^6)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^3\cosh(c+dx)}{(a+bx)^3}\,dx = \int \left(\frac{\cosh(c+dx)}{b^3} - \frac{a^3\cosh(c+dx)}{b^3(a+bx)^3} + \frac{3a^2\cosh(c+dx)}{b^3(a+bx)^2} - \frac{3a\cosh(c+dx)}{b^3(a+bx)}\right) dx$$

$$= \frac{\int \cosh(c+dx)\,dx}{b^3} - \frac{(3a)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b^3} + \frac{(3a^2)\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b^3} - \frac{a^3\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx}{b^3} \cdots$$

$$= \frac{a^3\cosh(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2\cosh(c+dx)}{b^4(a+bx)} + \frac{\sinh(c+dx)}{b^3d} + \frac{(3a^2d)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b^4} - \cdots$$

$$= \frac{a^3\cosh(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2\cosh(c+dx)}{b^4(a+bx)} - \frac{3a\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} + \cdots$$

$$= \frac{a^3\cosh(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2\cosh(c+dx)}{b^4(a+bx)} - \frac{3a\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} + \cdots$$

$$= \frac{a^3\cosh(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2\cosh(c+dx)}{b^4(a+bx)} - \frac{3a\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} - \cdots$$

Mathematica [A]
time = 0.67, size = 236, normalized size = 0.89

$$-\frac{b\cosh(dx)\left(a^2bd(5a+6bx)\cosh(c)-(a+bx)\left(2ab^2+a^3d^2+2b^3x\right)\sinh(c)\right)-b\left((a+bx)\left(2ab^2+a^3d^2+2b^3x\right)\cosh(c)-a^2bd(5a+6bx)\sinh(c)\right)\sinh(dx)+ad(a+bx)^2\left(\operatorname{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(\left(6b^2+a^2d^2\right)\cosh\left(c-\frac{ad}{b}\right)-6abd\sinh\left(c-\frac{ad}{b}\right)\right)+\left(-6abd\cosh\left(c-\frac{ad}{b}\right)+\left(6b^2+a^2d^2\right)\sinh\left(c-\frac{ad}{b}\right)\right)\operatorname{Shi}\left(d\left(\frac{a}{b}+x\right)\right)\right)}{2b^6d(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -1/2*(b*Cosh[d*x]*(a^2*b*d*(5*a + 6*b*x)*Cosh[c] - (a + b*x)*(2*a*b^2 + a^3
*d^2 + 2*b^3*x)*Sinh[c]) - b*((a + b*x)*(2*a*b^2 + a^3*d^2 + 2*b^3*x)*Cosh[
c] - a^2*b*d*(5*a + 6*b*x)*Sinh[c])*Sinh[d*x] + a*d*(a + b*x)^2*(CoshIntegr
al[d*(a/b + x)]*((6*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] - 6*a*b*d*Sinh[c - (a*
d)/b]) + (-6*a*b*d*Cosh[c - (a*d)/b] + (6*b^2 + a^2*d^2)*Sinh[c - (a*d)/b])
*SinhIntegral[d*(a/b + x)]))/(b^6*d*(a + b*x)^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(262) = 524$.
time = 0.91, size = 571, normalized size = 2.16

method	result
risch	$\frac{3d\,\mathrm{e}^{\frac{ad-bc}{b}}\,\mathrm{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)a^{2}}{2b^{5}}+\frac{3\,\mathrm{e}^{\frac{ad-bc}{b}}\,\mathrm{expIntegral}\left(1,dx+c+\frac{ad-bc}{b}\right)a}{2b^{4}}-\frac{5d^{2}\mathrm{e}^{-dx-c}a^{3}}{4b^{4}\left(b^{2}d^{2}x^{2}+2ab\,d^{2}x+a^{2}d^{2}\right)}-\frac{3d}{2b^{3}\left(b^{2}d^{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*d/b^5*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2+3/2/b^4*exp((a*d-b*c
)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a-5/4*d^2*exp(-d*x-c)/b^4/(b^2*d^2*x^2+2*a*b*d
^2*x+a^2*d^2)*a^3-3/2*d^2*exp(-d*x-c)/b^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)
*a^2*x-1/4*d^3*exp(-d*x-c)/b^4/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^3*x+1/4*
d^2/b^6*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3-1/2/d*exp(-d*x-c)/b^3-
1/4*d^3*exp(-d*x-c)/b^5/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^4+1/4*d^2/b^6*e
xp(d*x+c)/(a*d/b+d*x)^2*a^3+1/4*d^2/b^6*exp(d*x+c)/(a*d/b+d*x)*a^3+1/4*d^2/
b^6*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3+1/2/d/b^3*exp(d*x+c)+3/2
/b^4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a-3/2*d/b^5*exp(d*x+c)/(a*d
/b+d*x)*a^2-3/2*d/b^5*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/2*a^2*d*integrate(x*e^(d*x + c)/(b^5*d^2*x^4 + 4*a*b^4*d^2*x^3 + 6*a^2*b^
3*d^2*x^2 + 4*a^3*b^2*d^2*x + a^4*b*d^2), x) - 3/2*a^2*d*integrate(x/(b^5*d
^2*x^4*e^(d*x + c) + 4*a*b^4*d^2*x^3*e^(d*x + c) + 6*a^2*b^3*d^2*x^2*e^(d*x
 + c) + 4*a^3*b^2*d^2*x*e^(d*x + c) + a^4*b*d^2*e^(d*x + c)), x) - 3*a*b*in
```

```
tegrate(x*e^(d*x + c)/(b^5*d^2*x^4 + 4*a*b^4*d^2*x^3 + 6*a^2*b^3*d^2*x^2 +
4*a^3*b^2*d^2*x + a^4*b*d^2), x) - 3*a*b*integrate(x/(b^5*d^2*x^4*e^(d*x +
c) + 4*a*b^4*d^2*x^3*e^(d*x + c) + 6*a^2*b^3*d^2*x^2*e^(d*x + c) + 4*a^3*b^
2*d^2*x*e^(d*x + c) + a^4*b*d^2*e^(d*x + c)), x) + 1/2*((b*d*x^3*e^(2*c) -
3*a*x*e^(2*c))*e^(d*x) - (b*d*x^3 + 3*a*x)*e^(-d*x))/(b^4*d^2*x^3*e^c + 3*a
*b^3*d^2*x^2*e^c + 3*a^2*b^2*d^2*x*e^c + a^3*b*d^2*e^c) - 3/2*a^2*e^(-c + a
*d/b)*exp_integral_e(4, (b*x + a)*d/b)/((b*x + a)^3*b^2*d^2) - 3/2*a^2*e^(c
 - a*d/b)*exp_integral_e(4, -(b*x + a)*d/b)/((b*x + a)^3*b^2*d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(262) = 524$.
time = 0.37, size = 566, normalized size = 2.14

$$-\frac{2\left(6a^2b^3dx+5a^3b^2d\right)\cosh\left(dx+c\right)+\left(\left(a^5d^3-6a^4bd^2+6a^3b^2d+\left(a^3b^2d^3-6a^2b^3d^2+6ab^4d\right)x^2+2\left(a^4bd^3-6a^3b^2d^2+6a^2b^3d\right)x\right){\rm Ei}\left(\frac{bdx+ad}{b}\right)+\left(a^5d^3+6a^4bd^2+6a^3b^2d+\left(a^3b^2d^3+6a^2b^3d^2+6ab^4d\right)x^2+2\left(a^4bd^3+6a^3b^2d^2+6a^2b^3d\right)x\right){\rm Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-2\left(a^4bd^2+2b^5x^2+2a^2b^3+\left(a^3b^2d^2+4ab^4\right)x\right)\sinh\left(dx+c\right)-\left(\left(a^5d^3-6a^4bd^2+6a^3b^2d+\left(a^3b^2d^3-6a^2b^3d^2+6ab^4d\right)x^2+2\left(a^4bd^3-6a^3b^2d^2+6a^2b^3d\right)x\right){\rm Ei}\left(\frac{bdx+ad}{b}\right)-\left(a^5d^3+6a^4bd^2+6a^3b^2d+\left(a^3b^2d^3+6a^2b^3d^2+6ab^4d\right)x^2+2\left(a^4bd^3+6a^3b^2d^2+6a^2b^3d\right)x\right){\rm Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{4\left(b^8dx^2+2ab^7dx+a^2b^6d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(6*a^2*b^3*d*x + 5*a^3*b^2*d)*cosh(d*x + c) + ((a^5*d^3 - 6*a^4*b*d
^2 + 6*a^3*b^2*d + (a^3*b^2*d^3 - 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b
*d^3 - 6*a^3*b^2*d^2 + 6*a^2*b^3*d)*x)*Ei((b*d*x + a*d)/b) + (a^5*d^3 + 6*a
^4*b*d^2 + 6*a^3*b^2*d + (a^3*b^2*d^3 + 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*
(a^4*b*d^3 + 6*a^3*b^2*d^2 + 6*a^2*b^3*d)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b
*c - a*d)/b) - 2*(a^4*b*d^2 + 2*b^5*x^2 + 2*a^2*b^3 + (a^3*b^2*d^2 + 4*a*b^
4)*x)*sinh(d*x + c) - ((a^5*d^3 - 6*a^4*b*d^2 + 6*a^3*b^2*d + (a^3*b^2*d^3
- 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 - 6*a^3*b^2*d^2 + 6*a^2*b^3
*d)*x)*Ei((b*d*x + a*d)/b) - (a^5*d^3 + 6*a^4*b*d^2 + 6*a^3*b^2*d + (a^3*b^
2*d^3 + 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 + 6*a^3*b^2*d^2 + 6*a
^2*b^3*d)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^8*d*x^2 + 2*a*b
^7*d*x + a^2*b^6*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\cosh\left(c+dx\right)}{\left(a+bx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(262) = 524$.
time = 0.40, size = 879, normalized size = 3.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a^3*b^2*d^3*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^3*b^2*d^3*x^2*E
i(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*a^4*b*d^3*x*Ei((b*d*x + a*d)/b)*e^(c
 - a*d/b) - 6*a^2*b^3*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^4*b*d
^3*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 6*a^2*b^3*d^2*x^2*Ei(-(b*d*x + a
*d)/b)*e^(-c + a*d/b) + a^5*d^3*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 12*a^3*
b^2*d^2*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 6*a*b^4*d*x^2*Ei((b*d*x + a*d
)/b)*e^(c - a*d/b) + a^5*d^3*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 12*a^3*b
^2*d^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 6*a*b^4*d*x^2*Ei(-(b*d*x + a
*d)/b)*e^(-c + a*d/b) - a^3*b^2*d^2*x*e^(d*x + c) + a^3*b^2*d^2*x*e^(-d*x -
 c) - 6*a^4*b*d^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 12*a^2*b^3*d*x*Ei((b*
d*x + a*d)/b)*e^(c - a*d/b) + 6*a^4*b*d^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/
b) + 12*a^2*b^3*d*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^4*b*d^2*e^(d*x
+ c) + 6*a^2*b^3*d*x*e^(d*x + c) - 2*b^5*x^2*e^(d*x + c) + a^4*b*d^2*e^(-d*
x - c) + 6*a^2*b^3*d*x*e^(-d*x - c) + 2*b^5*x^2*e^(-d*x - c) + 6*a^3*b^2*d*
Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 6*a^3*b^2*d*Ei(-(b*d*x + a*d)/b)*e^(-c
+ a*d/b) + 5*a^3*b^2*d*e^(d*x + c) - 4*a*b^4*x*e^(d*x + c) + 5*a^3*b^2*d*e^
(-d*x - c) + 4*a*b^4*x*e^(-d*x - c) - 2*a^2*b^3*e^(d*x + c) + 2*a^2*b^3*e^(
-d*x - c))/(b^8*d*x^2 + 2*a*b^7*d*x + a^2*b^6*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \cosh(c + d\,x)}{(a + b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*cosh(c + d*x))/(a + b*x)^3,x)
```

```
[Out] int((x^3*cosh(c + d*x))/(a + b*x)^3, x)
```

3.34 $\int \frac{x^2 \cosh(c+dx)}{(a+bx)^3} \, dx$

Optimal. Leaf size=241

$$-\frac{a^2 \cosh(c+dx)}{2b^3(a+bx)^2}+\frac{2a \cosh(c+dx)}{b^3(a+bx)}+\frac{\cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{a^2d^2 \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b}+dx\right)}{2b^5}-\frac{2adC}{}$$

```
[Out] Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^3+1/2*a^2*d^2*Chi(a*d/b+d*x)*cosh(-c+a*d/b)
/b^5-1/2*a^2*cosh(d*x+c)/b^3/(b*x+a)^2+2*a*cosh(d*x+c)/b^3/(b*x+a)-2*a*d*co
sh(-c+a*d/b)*Shi(a*d/b+d*x)/b^4+2*a*d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/b^4-Shi
(a*d/b+d*x)*sinh(-c+a*d/b)/b^3-1/2*a^2*d^2*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^
5-1/2*a^2*d*sinh(d*x+c)/b^4/(b*x+a)
```

Rubi [A]
time = 0.41, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\frac{a^2d^2 \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{ad}{b}\right)}{2b^5}+\frac{a^2d^2 \sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(xd+\frac{ad}{b}\right)}{2b^5}-\frac{a^2d \sinh(c+dx)}{2b^4(a+bx)}-\frac{a^2 \cosh(c+dx)}{2b^3(a+bx)^2}-\frac{2ad \sinh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{2ad \cosh\left(c-\frac{ad}{b}\right) \text{Shi}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{\cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{\sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{2a \cosh(c+dx)}{b^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -1/2*(a^2*Cosh[c + d*x])/(b^3*(a + b*x)^2) + (2*a*Cosh[c + d*x])/(b^3*(a +
b*x)) + (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^3 + (a^2*d^2*Cosh
[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*b^5) - (2*a*d*CoshIntegral[(a
*d)/b + d*x]*Sinh[c - (a*d)/b])/b^4 - (a^2*d*Sinh[c + d*x])/(2*b^4*(a + b*x
)) - (2*a*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^4 + (Sinh[c -
(a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3 + (a^2*d^2*Sinh[c - (a*d)/b]*Sinh
Integral[(a*d)/b + d*x])/(2*b^5)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2\cosh(c+dx)}{(a+bx)^3}\,dx = \int \left(\frac{a^2\cosh(c+dx)}{b^2(a+bx)^3} - \frac{2a\cosh(c+dx)}{b^2(a+bx)^2} + \frac{\cosh(c+dx)}{b^2(a+bx)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b^2} - \frac{(2a)\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b^2} + \frac{a^2\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx}{b^2}$$

$$= -\frac{a^2\cosh(c+dx)}{2b^3(a+bx)^2} + \frac{2a\cosh(c+dx)}{b^3(a+bx)} - \frac{(2ad)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b^3} + \frac{(a^2d)\int \frac{\sinh(c+dx)}{(a+bx)^2}\,dx}{2b^3}$$

$$= -\frac{a^2\cosh(c+dx)}{2b^3(a+bx)^2} + \frac{2a\cosh(c+dx)}{b^3(a+bx)} + \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^3} - \frac{a^2d\sinh(c+}{2b^4(a+bx}$$

$$= -\frac{a^2\cosh(c+dx)}{2b^3(a+bx)^2} + \frac{2a\cosh(c+dx)}{b^3(a+bx)} + \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^3} - \frac{2ad\text{Chi}\left(\frac{ad}{b}+\right.}{}$$

$$= -\frac{a^2\cosh(c+dx)}{2b^3(a+bx)^2} + \frac{2a\cosh(c+dx)}{b^3(a+bx)} + \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{a^2d^2\cosh(c-}{}$$

Mathematica [A]
time = 0.63, size = 153, normalized size = 0.63

$$\frac{\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left((2b^2+a^2d^2)\cosh\left(c-\frac{ad}{b}\right)-4abd\sinh\left(c-\frac{ad}{b}\right)\right)-\frac{ab(-b(3a+4bx)\cosh(c+dx)+ad(a+bx)\sinh(c+dx))}{(a+bx)^2}+\left(-4abd\cosh\left(c-\frac{ad}{b}\right)+(2b^2+a^2d^2)\sinh\left(c-\frac{ad}{b}\right)\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x)^3,x]
```

[Out] (CoshIntegral[d*(a/b + x)]*((2*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] - 4*a*b*d*S
inh[c - (a*d)/b]) - (a*b*(-(b*(3*a + 4*b*x)*Cosh[c + d*x]) + a*d*(a + b*x)*
Sinh[c + d*x]))/(a + b*x)^2 + (-4*a*b*d*Cosh[c - (a*d)/b] + (2*b^2 + a^2*d^
2)*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)])/(2*b^5)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(240) = 480$.
time = 0.75, size = 527, normalized size = 2.19

method	result
risch	$\frac{d^{3} \mathrm{e}^{-d x -c} a^{2} x}{4 b^{3} \left(b^{2} d^{2} x^{2}+2 a b \,d^{2} x +a^{2} d^{2}\right)}+\frac{d^{3} \mathrm{e}^{-d x -c} a^{3}}{4 b^{4} \left(b^{2} d^{2} x^{2}+2 a b \,d^{2} x +a^{2} d^{2}\right)}+\frac{d^{2} \mathrm{e}^{-d x -c} a x}{b^{2} \left(b^{2} d^{2} x^{2}+2 a b \,d^{2} x +a^{2} d^{2}\right)}+\frac{3 d^{2} \mathrm{e}^{-d x -c} a^{2}}{4 b^{3} \left(b^{2} d^{2} x^{2}+2 a b \,d^{2} x +a^{2} d^{2}\right)}-$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cosh(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/4*d^3*exp(-d*x-c)/b^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^2*x+1/4*d^3*exp
(-d*x-c)/b^4/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^3+d^2*exp(-d*x-c)/b^2/(b^2
*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a*x+3/4*d^2*exp(-d*x-c)/b^3/(b^2*d^2*x^2+2*a*
b*d^2*x+a^2*d^2)*a^2-1/4*d^2/b^5*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a
^2-d/b^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a-1/2/b^3*exp((a*d-b*c)/b
)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2/b^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b
)-1/4*d^2/b^5*exp(d*x+c)/(a*d/b+d*x)^2*a^2-1/4*d^2/b^5*exp(d*x+c)/(a*d/b+d*
x)*a^2-1/4*d^2/b^5*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2+d/b^4*exp
(d*x+c)/(a*d/b+d*x)*a+d/b^4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")

[Out] -3/2*a*d*integrate(x*e^(d*x + c)/(b^4*d^2*x^4 + 4*a*b^3*d^2*x^3 + 6*a^2*b^2
*d^2*x^2 + 4*a^3*b*d^2*x + a^4*d^2), x) + 3/2*a*d*integrate(x/(b^4*d^2*x^4*
e^(d*x + c) + 4*a*b^3*d^2*x^3*e^(d*x + c) + 6*a^2*b^2*d^2*x^2*e^(d*x + c) +
4*a^3*b*d^2*x*e^(d*x + c) + a^4*d^2*e^(d*x + c)), x) + b*integrate(x*e^(d*
x + c)/(b^4*d^2*x^4 + 4*a*b^3*d^2*x^3 + 6*a^2*b^2*d^2*x^2 + 4*a^3*b*d^2*x +
a^4*d^2), x) + b*integrate(x/(b^4*d^2*x^4*e^(d*x + c) + 4*a*b^3*d^2*x^3*e^
(d*x + c) + 6*a^2*b^2*d^2*x^2*e^(d*x + c) + 4*a^3*b*d^2*x*e^(d*x + c) + a^4
*d^2*e^(d*x + c)), x) + 1/2*((d*x^2*e^(2*c) + x*e^(2*c))*e^(d*x) - (d*x^2 -
x)*e^(-d*x))/(b^3*d^2*x^3*e^c + 3*a*b^2*d^2*x^2*e^c + 3*a^2*b*d^2*x*e^c +
a^3*d^2*e^c) + 1/2*a*e^(-c + a*d/b)*exp_integral_e(4, (b*x + a)*d/b)/((b*x

```
+ a)^3*b*d^2) + 1/2*a*e^(c - a*d/b)*exp_integral_e(4, -(b*x + a)*d/b)/((b*x
 + a)^3*b*d^2)
```

Fricas [A]
time = 0.37, size = 475, normalized size = 1.97

$$\frac{2\left(4ab^3x+3a^2b^2\right)\cosh\left(dx+c\right)+\left(\left(a^4d^2-4a^3bd+2a^2b^2+\left(a^2b^2d^2-4ab^3d+2b^4\right)x^2+2\left(a^3bd^2-4a^2b^2d+2ab^3\right)x\right){\rm Ei}\left(\frac{bdx+ad}{b}\right)+\left(a^4d^2+4a^3bd+2a^2b^2+\left(a^2b^2d^2+4ab^3d+2b^4\right)x^2+2\left(a^3bd^2+4a^2b^2d+2ab^3\right)x\right){\rm Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-2\left(a^2b^2dx+a^3bd\right)\sinh\left(dx+c\right)-\left(\left(a^4d^2-4a^3bd+2a^2b^2+\left(a^2b^2d^2-4ab^3d+2b^4\right)x^2+2\left(a^3bd^2-4a^2b^2d+2ab^3\right)x\right){\rm Ei}\left(\frac{bdx+ad}{b}\right)-\left(a^4d^2+4a^3bd+2a^2b^2+\left(a^2b^2d^2+4ab^3d+2b^4\right)x^2+2\left(a^3bd^2+4a^2b^2d+2ab^3\right)x\right){\rm Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{4\left(b^7x^2+2ab^6x+a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(4*a*b^3*x + 3*a^2*b^2)*cosh(d*x + c) + ((a^4*d^2 - 4*a^3*b*d + 2*a^
2*b^2 + (a^2*b^2*d^2 - 4*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 - 4*a^2*b^2*d
+ 2*a*b^3)*x)*Ei((b*d*x + a*d)/b) + (a^4*d^2 + 4*a^3*b*d + 2*a^2*b^2 + (a^2
*b^2*d^2 + 4*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 + 4*a^2*b^2*d + 2*a*b^3)*x
)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(a^2*b^2*d*x + a^3*b*d)*si
nh(d*x + c) - ((a^4*d^2 - 4*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 - 4*a*b^3*d
+ 2*b^4)*x^2 + 2*(a^3*b*d^2 - 4*a^2*b^2*d + 2*a*b^3)*x)*Ei((b*d*x + a*d)/b)
 - (a^4*d^2 + 4*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 + 4*a*b^3*d + 2*b^4)*x^2
 + 2*(a^3*b*d^2 + 4*a^2*b^2*d + 2*a*b^3)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*
c - a*d)/b))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\cosh\left(c+dx\right)}{\left(a+bx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 741 vs. 2(240) = 480.
time = 0.42, size = 741, normalized size = 3.07

$$\frac{a^2b^2d^2x^2{\rm Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+a^2b^2d^2x^2{\rm Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+2a^3bd^2x{\rm Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}-4ab^3dx^2{\rm Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+2a^3bd^2x{\rm Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+4ab^3dx^2{\rm Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+a^4d^2{\rm Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}-8a^2b^2dx{\rm Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+2b^4x^2{\rm Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+a^4d^2{\rm Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+8a^2b^2dx{\rm Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+2b^4x^2{\rm Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}-a^2b^2dxe^{\left(dx+c\right)}+a^2b^2dxe^{\left(-dx-c\right)}-4a^3bd{\rm Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+4ab^3x{\rm Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+4a^3bd{\rm Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+4ab^3x{\rm Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}-a^3bde^{\left(dx+c\right)}+4ab^3xe^{\left(dx+c\right)}+a^3bde^{\left(-dx-c\right)}+4ab^3xe^{\left(-dx-c\right)}+2a^2b^2{\rm Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+2a^2b^2{\rm Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+3a^2b^2e^{\left(dx+c\right)}+3a^2b^2e^{\left(-dx-c\right)}}{4\left(b^7x^2+2ab^6x+a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*b^2*d^2*x^2*Ei
(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*a^3*b*d^2*x*Ei((b*d*x + a*d)/b)*e^(c
- a*d/b) - 4*a*b^3*d*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^3*b*d^2*x*
Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 4*a*b^3*d*x^2*Ei(-(b*d*x + a*d)/b)*e^
```

```
(-c + a*d/b) + a^4*d^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 8*a^2*b^2*d*x*Ei
((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*b^4*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/
b) + a^4*d^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 8*a^2*b^2*d*x*Ei(-(b*d*x
 + a*d)/b)*e^(-c + a*d/b) + 2*b^4*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) -
 a^2*b^2*d*x*e^(d*x + c) + a^2*b^2*d*x*e^(-d*x - c) - 4*a^3*b*d*Ei((b*d*x +
 a*d)/b)*e^(c - a*d/b) + 4*a*b^3*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 4*a^
3*b*d*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 4*a*b^3*x*Ei(-(b*d*x + a*d)/b)*
e^(-c + a*d/b) - a^3*b*d*e^(d*x + c) + 4*a*b^3*x*e^(d*x + c) + a^3*b*d*e^(-
d*x - c) + 4*a*b^3*x*e^(-d*x - c) + 2*a^2*b^2*Ei((b*d*x + a*d)/b)*e^(c - a*
d/b) + 2*a^2*b^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 3*a^2*b^2*e^(d*x + c
) + 3*a^2*b^2*e^(-d*x - c))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\cosh(c+d\,x)}{(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(c + d*x))/(a + b*x)^3,x)
```

```
[Out] int((x^2*cosh(c + d*x))/(a + b*x)^3, x)
```

3.35 $\int \frac{x \cosh(c+dx)}{(a+bx)^3} \, dx$

Optimal. Leaf size=178

$$\frac{a\cosh(c+dx)}{2b^2(a+bx)^2}-\frac{\cosh(c+dx)}{b^2(a+bx)}-\frac{ad^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{2b^4}+\frac{d\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^3}+\frac{ad\sinh(c+d}{2b^3(a+bx}$$

```
[Out] -1/2*a*d^2*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^4+1/2*a*cosh(d*x+c)/b^2/(b*x+a)^
2-cosh(d*x+c)/b^2/(b*x+a)+d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/b^3-d*Chi(a*d/b+d
*x)*sinh(-c+a*d/b)/b^3+1/2*a*d^2*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^4+1/2*a*d*
sinh(d*x+c)/b^3/(b*x+a)
```

Rubi [A]
time = 0.28, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6874, 3378, 3384, 3379, 3382}

$$-\frac{ad^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{2b^4}-\frac{ad^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{2b^4}+\frac{d\sinh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{d\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{ad\sinh(c+dx)}{2b^3(a+bx)}-\frac{\cosh(c+dx)}{b^2(a+bx)}+\frac{a\cosh(c+dx)}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x)^3,x]
```

```
[Out] (a*Cosh[c + d*x])/(2*b^2*(a + b*x)^2) - Cosh[c + d*x]/(b^2*(a + b*x)) - (a*
d^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*b^4) + (d*CoshIntegra
l[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^3 + (a*d*Sinh[c + d*x])/(2*b^3*(a + b
*x)) + (d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3 - (a*d^2*Sinh[
c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*b^4)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx &= \int \left(-\frac{a\cosh(c+dx)}{b(a+bx)^3}+\frac{\cosh(c+dx)}{b(a+bx)^2}\right)dx \\
&= \frac{\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b}-\frac{a\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx}{b} \\
&= \frac{a\cosh(c+dx)}{2b^2(a+bx)^2}-\frac{\cosh(c+dx)}{b^2(a+bx)}+\frac{d\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b^2}-\frac{(ad)\int \frac{\sinh(c+dx)}{(a+bx)^2}\,dx}{2b^2} \\
&= \frac{a\cosh(c+dx)}{2b^2(a+bx)^2}-\frac{\cosh(c+dx)}{b^2(a+bx)}+\frac{ad\sinh(c+dx)}{2b^3(a+bx)}-\frac{(ad^2)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{2b^3}+\frac{(d\cosh \\
&= \frac{a\cosh(c+dx)}{2b^2(a+bx)^2}-\frac{\cosh(c+dx)}{b^2(a+bx)}+\frac{d\operatorname{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^3}+\frac{ad\sinh(c+dx)}{2b^3(a+bx)}+ \\
&= \frac{a\cosh(c+dx)}{2b^2(a+bx)^2}-\frac{\cosh(c+dx)}{b^2(a+bx)}-\frac{ad^2\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{2b^4}+\frac{d\operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{b^3}
\end{aligned}
$$

Mathematica [A]
time = 0.40, size = 158, normalized size = 0.89

$$-\frac{b\cosh(dx)(b(a+2bx)\cosh(c)-ad(a+bx)\sinh(c))-b(ad(a+bx)\cosh(c)-b(a+2bx)\sinh(c))\sinh(dx)+d(a+bx)^2\left(\operatorname{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\cosh\left(c-\frac{ad}{b}\right)-2b\sinh\left(c-\frac{ad}{b}\right)\right)+\left(-2b\cosh\left(c-\frac{ad}{b}\right)+ad\sinh\left(c-\frac{ad}{b}\right)\right)\operatorname{Shi}\left(d\left(\frac{a}{b}+x\right)\right)\right)}{2b^4(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -1/2*(b*Cosh[d*x]*(b*(a + 2*b*x)*Cosh[c] - a*d*(a + b*x)*Sinh[c]) - b*(a*d*
(a + b*x)*Cosh[c] - b*(a + 2*b*x)*Sinh[c])*Sinh[d*x] + d*(a + b*x)^2*(CoshI
```

```
ntegral[d*(a/b + x)]*(a*d*Cosh[c - (a*d)/b] - 2*b*Sinh[c - (a*d)/b]) + (-2*
b*Cosh[c - (a*d)/b] + a*d*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)]))/(b
^4*(a + b*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(175) = 350.
time = 0.74, size = 435, normalized size = 2.44

method	result
risch	$-\frac{d^3 e^{-dx-c} a x}{4b^2(b^2d^2x^2+2ab\,d^2x+a^2d^2)}-\frac{d^3 e^{-dx-c} a^2}{4b^3(b^2d^2x^2+2ab\,d^2x+a^2d^2)}-\frac{d^2 e^{-dx-c} x}{2b(b^2d^2x^2+2ab\,d^2x+a^2d^2)}-\frac{d^2 e^{-dx-c} a}{4b^2(b^2d^2x^2+2ab\,d^2x+a^2d^2)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d^3*exp(-d*x-c)/b^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a*x-1/4*d^3*exp(
-d*x-c)/b^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^2-1/2*d^2*exp(-d*x-c)/b/(b^
2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*x-1/4*d^2*exp(-d*x-c)/b^2/(b^2*d^2*x^2+2*a*b
*d^2*x+a^2*d^2)*a+1/4*d^2/b^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a+1/
2*d/b^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+1/4*d^2/b^4*exp(d*x+c)/(a*
d/b+d*x)^2*a+1/4*d^2/b^4*exp(d*x+c)/(a*d/b+d*x)*a+1/4*d^2/b^4*exp(-(a*d-b*c
)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a-1/2*d/b^3*exp(d*x+c)/(a*d/b+d*x)-1/2*d/b^3*
exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] b*integrate(x*e^(d*x + c)/(b^4*d*x^4 + 4*a*b^3*d*x^3 + 6*a^2*b^2*d*x^2 + 4*
a^3*b*d*x + a^4*d), x) - b*integrate(x/(b^4*d*x^4*e^(d*x + c) + 4*a*b^3*d*x
^3*e^(d*x + c) + 6*a^2*b^2*d*x^2*e^(d*x + c) + 4*a^3*b*d*x*e^(d*x + c) + a^
4*d*e^(d*x + c)), x) + 1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b^3*d*x^3*e^c +
3*a*b^2*d*x^2*e^c + 3*a^2*b*d*x*e^c + a^3*d*e^c) - 1/2*a*e^(-c + a*d/b)*exp
_integral_e(4, (b*x + a)*d/b)/((b*x + a)^3*b*d) + 1/2*a*e^(c - a*d/b)*exp_i
ntegral_e(4, -(b*x + a)*d/b)/((b*x + a)^3*b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(175) = 350.
time = 0.44, size = 373, normalized size = 2.10

$$-\frac{2\,(2\,b^3x+ab^2)\cosh(dx+c)+\left(\left(a^3d^2-2\,a^2bd+(ab^2d^2-2\,b^3d)x^2+2\,(a^2bd^2-2\,ab^2d)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+\left(a^3d^2+2\,a^2bd+(ab^2d^2+2\,b^3d)x^2+2\,(a^2bd^2+2\,ab^2d)x\right)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-2\,(ab^2dx+a^2bd)\sinh(dx+c)-\left(\left(a^3d^2-2\,a^2bd+(ab^2d^2-2\,b^3d)x^2+2\,(a^2bd^2-2\,ab^2d)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-\left(a^3d^2+2\,a^2bd+(ab^2d^2+2\,b^3d)x^2+2\,(a^2bd^2+2\,ab^2d)x\right)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{4\,(b^6x^2+2\,ab^5x+a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*b^3*x + a*b^2)*cosh(d*x + c) + ((a^3*d^2 - 2*a^2*b*d + (a*b^2*d^
2 - 2*b^3*d)*x^2 + 2*(a^2*b*d^2 - 2*a*b^2*d)*x)*Ei((b*d*x + a*d)/b) + (a^3*
d^2 + 2*a^2*b*d + (a*b^2*d^2 + 2*b^3*d)*x^2 + 2*(a^2*b*d^2 + 2*a*b^2*d)*x)*
Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(a*b^2*d*x + a^2*b*d)*sinh(d
*x + c) - ((a^3*d^2 - 2*a^2*b*d + (a*b^2*d^2 - 2*b^3*d)*x^2 + 2*(a^2*b*d^2
- 2*a*b^2*d)*x)*Ei((b*d*x + a*d)/b) - (a^3*d^2 + 2*a^2*b*d + (a*b^2*d^2 + 2
*b^3*d)*x^2 + 2*(a^2*b*d^2 + 2*a*b^2*d)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c
 - a*d)/b))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(175) = 350.
time = 0.41, size = 529, normalized size = 2.97

$$-\frac{ab^2d^2x^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+ab^2d^2x^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+2\,a^2bd^2x\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}-2\,b^3dx^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+2\,a^2bd^2x\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+2\,b^3dx^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+a^3d^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}-4\,ab^2dx\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+a^3d^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+4\,ab^2dx\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}-ab^2dxe^{(dx+c)}+ab^2dxe^{(-dx-c)}-2\,a^2bd\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+2\,a^2bd\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}-a^2bde^{(dx+c)}+2\,b^3xe^{(dx+c)}+a^2bde^{(-dx-c)}+2\,b^3xe^{(-dx-c)}+ab^2e^{(dx+c)}+ab^2e^{(-dx-c)}}{4\,(b^6x^2+2\,ab^5x+a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a*b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a*b^2*d^2*x^2*Ei(-(
b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*a^2*b*d^2*x*Ei((b*d*x + a*d)/b)*e^(c - a
*d/b) - 2*b^3*d*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^2*b*d^2*x*Ei(-(
b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*b^3*d*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a
*d/b) + a^3*d^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 4*a*b^2*d*x*Ei((b*d*x +
 a*d)/b)*e^(c - a*d/b) + a^3*d^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 4*a*
b^2*d*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a*b^2*d*x*e^(d*x + c) + a*b^2
*d*x*e^(-d*x - c) - 2*a^2*b*d*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^2*b*d
*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^2*b*d*e^(d*x + c) + 2*b^3*x*e^(d*x
 + c) + a^2*b*d*e^(-d*x - c) + 2*b^3*x*e^(-d*x - c) + a*b^2*e^(d*x + c) + a
*b^2*e^(-d*x - c))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\cosh(c+d\,x)}{{(a+b\,x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(c + d*x))/(a + b*x)^3,x)
[Out] int((x*cosh(c + d*x))/(a + b*x)^3, x)
```

3.36 $\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx$

Optimal. Leaf size=104

$$-\frac{\cosh(c+dx)}{2b(a+bx)^2}+\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{2b^3}-\frac{d\sinh(c+dx)}{2b^2(a+bx)}+\frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{2b^3}$$

```
[Out] 1/2*d^2*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/b^3-1/2*cosh(d*x+c)/b/(b*x+a)^2-1/2*d
^2*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/b^3-1/2*d*sinh(d*x+c)/b^2/(b*x+a)
```

Rubi [A]
time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3378, 3384, 3379, 3382}

$$\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2b^3}+\frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2b^3}-\frac{d\sinh(c+dx)}{2b^2(a+bx)}-\frac{\cosh(c+dx)}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*x)^3,x]
```

```
[Out] -1/2*Cosh[c + d*x]/(b*(a + b*x)^2) + (d^2*Cosh[c - (a*d)/b]*CoshIntegral[(a
*d)/b + d*x])/(2*b^3) - (d*Sinh[c + d*x])/(2*b^2*(a + b*x)) + (d^2*Sinh[c -
 (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*b^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx &= -\frac{\cosh(c+dx)}{2b(a+bx)^2}+\frac{d\int \frac{\sinh(c+dx)}{(a+bx)^2}\,dx}{2b}\\ &= -\frac{\cosh(c+dx)}{2b(a+bx)^2}-\frac{d\sinh(c+dx)}{2b^2(a+bx)}+\frac{d^2\int \frac{\cosh(c+dx)}{a+bx}\,dx}{2b^2}\\ &= -\frac{\cosh(c+dx)}{2b(a+bx)^2}-\frac{d\sinh(c+dx)}{2b^2(a+bx)}+\frac{\left(d^2\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2b^2}+\frac{\left(d^2\sinh\left(c-\right.\right.}{}\\ &= -\frac{\cosh(c+dx)}{2b(a+bx)^2}+\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{2b^3}-\frac{d\sinh(c+dx)}{2b^2(a+bx)}+\frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{S}}{2b^3}\end{aligned}$$

Mathematica [A]
time = 0.40, size = 88, normalized size = 0.85

$$\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)-\frac{b(b\cosh(c+dx)+d(a+bx)\sinh(c+dx))}{(a+bx)^2}+d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x)^3,x]
```

```
[Out] (d^2*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] - (b*(b*Cosh[c + d*x] + d*
(a + b*x)*Sinh[c + d*x]))/(a + b*x)^2 + d^2*Sinh[c - (a*d)/b]*SinhIntegral[
d*(a/b + x)])/(2*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(98) = 196$.
time = 0.72, size = 276, normalized size = 2.65

method	result
risch	$\frac{d^3 e^{-dx-c} x}{4b(b^2d^2x^2+2ab\,d^2x+a^2d^2)}+\frac{d^3 e^{-dx-c} a}{4b^2(b^2d^2x^2+2ab\,d^2x+a^2d^2)}-\frac{d^2 e^{-dx-c}}{4b(b^2d^2x^2+2ab\,d^2x+a^2d^2)}-\frac{d^2 e^{\frac{ad-bc}{b}}\,\text{expIntegral}\left(1,dx+c+\frac{ad-b}{b}\right.}{4b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d^3*exp(-d*x-c)/b/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*x+1/4*d^3*exp(-d*x-
c)/b^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a-1/4*d^2*exp(-d*x-c)/b/(b^2*d^2*x
^2+2*a*b*d^2*x+a^2*d^2)-1/4*d^2/b^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b
)-1/4*d^2/b^3*exp(d*x+c)/(a*d/b+d*x)^2-1/4*d^2/b^3*exp(d*x+c)/(a*d/b+d*x)-1
/4*d^2/b^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Maxima [A]
time = 0.29, size = 95, normalized size = 0.91

$$\frac{d\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_2\left(\frac{(bx+a)d}{b}\right)}{(bx+a)b}-\frac{e^{\left(c-\frac{ad}{b}\right)}E_2\left(-\frac{(bx+a)d}{b}\right)}{(bx+a)b}\right)}{4\,b}-\frac{\cosh\left(dx+c\right)}{2\left(bx+a\right)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*d*(e^(-c + a*d/b)*exp_integral_e(2, (b*x + a)*d/b)/((b*x + a)*b) - e^(c
 - a*d/b)*exp_integral_e(2, -(b*x + a)*d/b)/((b*x + a)*b))/b - 1/2*cosh(d*x
 + c)/((b*x + a)^2*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(98) = 196$.
time = 0.34, size = 253, normalized size = 2.43

$$-\frac{2\,b^2\cosh\left(dx+c\right)-\left(\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)+2\left(b^2dx+abd\right)\sinh\left(dx+c\right)+\left(\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{4\left(b^5x^2+2\,ab^4x+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*cosh(d*x + c) - ((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*Ei((b*d*
x + a*d)/b) + (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*Ei(-(b*d*x + a*d)/b))*c
osh(-(b*c - a*d)/b) + 2*(b^2*d*x + a*b*d)*sinh(d*x + c) + ((b^2*d^2*x^2 + 2
*a*b*d^2*x + a^2*d^2)*Ei((b*d*x + a*d)/b) - (b^2*d^2*x^2 + 2*a*b*d^2*x + a^
2*d^2)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^5*x^2 + 2*a*b^4*x + a
^2*b^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(98) = 196.
time = 0.42, size = 298, normalized size = 2.87

$$\frac{b^{2} d^{2} x^{2} \mathrm{Ei}\left(\frac{b d x + a d}{b}\right) e^{\left(c - \frac{a d}{b}\right)} + b^{2} d^{2} x^{2} \mathrm{Ei}\left(-\frac{b d x + a d}{b}\right) e^{\left(-c + \frac{a d}{b}\right)} + 2 \, a b d^{2} x \mathrm{Ei}\left(\frac{b d x + a d}{b}\right) e^{\left(c - \frac{a d}{b}\right)} + 2 \, a b d^{2} x \mathrm{Ei}\left(-\frac{b d x + a d}{b}\right) e^{\left(-c + \frac{a d}{b}\right)} + a^{2} d^{2} \mathrm{Ei}\left(\frac{b d x + a d}{b}\right) e^{\left(c - \frac{a d}{b}\right)} + a^{2} d^{2} \mathrm{Ei}\left(-\frac{b d x + a d}{b}\right) e^{\left(-c + \frac{a d}{b}\right)} - b^{2} d x e^{\left(d x + c\right)} + b^{2} d x e^{\left(-d x - c\right)} - a b d e^{\left(d x + c\right)} + a b d e^{\left(-d x - c\right)} - b^{2} e^{\left(d x + c\right)} - b^{2} e^{\left(-d x - c\right)}}{4 \, {\left(b^{5} x^{2} + 2 \, a b^{4} x + a^{2} b^{3}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + b^2*d^2*x^2*Ei(-(b*d*x
 + a*d)/b)*e^(-c + a*d/b) + 2*a*b*d^2*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) +
 2*a*b*d^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + a^2*d^2*Ei((b*d*x + a*d)
/b)*e^(c - a*d/b) + a^2*d^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - b^2*d*x*e
^(d*x + c) + b^2*d*x*e^(-d*x - c) - a*b*d*e^(d*x + c) + a*b*d*e^(-d*x - c)
- b^2*e^(d*x + c) - b^2*e^(-d*x - c))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + d\,x)}{{(a + b\,x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*x)^3,x)
```

```
[Out] int(cosh(c + d*x)/(a + b*x)^3, x)
```

3.37 $\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx$

Optimal. Leaf size=262

$$\frac{\cosh(c+dx)}{2a(a+bx)^2}+\frac{\cosh(c+dx)}{a^2(a+bx)}+\frac{\cosh(c)\text{Chi}(dx)}{a^3}-\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^3}-\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{2ab^2}$$

```
[Out] Chi(d*x)*cosh(c)/a^3-Chi(a*d/b+d*x)*cosh(-c+a*d/b)/a^3-1/2*d^2*Chi(a*d/b+d*
x)*cosh(-c+a*d/b)/a/b^2+1/2*cosh(d*x+c)/a/(b*x+a)^2+cosh(d*x+c)/a^2/(b*x+a)
-d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/a^2/b+Shi(d*x)*sinh(c)/a^3+d*Chi(a*d/b+d*x
)*sinh(-c+a*d/b)/a^2/b+Shi(a*d/b+d*x)*sinh(-c+a*d/b)/a^3+1/2*d^2*Shi(a*d/b+
d*x)*sinh(-c+a*d/b)/a/b^2+1/2*d*sinh(d*x+c)/a/b/(b*x+a)
```

Rubi [A]
time = 0.43, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3384, 3379, 3382, 3378}

$$-\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^3}+\frac{\cosh(c)\text{Chi}(dx)}{a^3}+\frac{\sinh(c)\text{Shi}(dx)}{a^3}-\frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^2b}-\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^2b}+\frac{\cosh(c+dx)}{a^2(a+bx)}-\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2ab^2}-\frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2ab^2}+\frac{d\sinh(c+dx)}{2ab(a+bx)}+\frac{\cosh(c+dx)}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x)^3),x]
```

```
[Out] Cosh[c + d*x]/(2*a*(a + b*x)^2) + Cosh[c + d*x]/(a^2*(a + b*x)) + (Cosh[c]*
CoshIntegral[d*x])/a^3 - (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^
3 - (d^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*a*b^2) - (d*Cosh
Integral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/(a^2*b) + (d*Sinh[c + d*x])/(2*a
*b*(a + b*x)) + (Sinh[c]*SinhIntegral[d*x])/a^3 - (d*Cosh[c - (a*d)/b]*Sinh
Integral[(a*d)/b + d*x])/(a^2*b) - (Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b
+ d*x])/a^3 - (d^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*a*b^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx = \int \left(\frac{\cosh(c+dx)}{a^3x} - \frac{b\cosh(c+dx)}{a(a+bx)^3} - \frac{b\cosh(c+dx)}{a^2(a+bx)^2} - \frac{b\cosh(c+dx)}{a^3(a+bx)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a^3} - \frac{b\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^3} - \frac{b\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{a^2} - \frac{b\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx}{a}$$

$$= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} - \frac{d\int \frac{\sinh(c+dx)}{a+bx}\,dx}{a^2} - \frac{d\int \frac{\sinh(c+dx)}{(a+bx)^2}\,dx}{2a} + \frac{\cosh(c)\int \frac{\cosh(dx)}{x}}{a^3} \cdots$$

$$= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} + \frac{\cosh(c)\mathrm{Chi}(dx)}{a^3} - \frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} + \frac{d\sin \cdots}{2ab \cdots}$$

$$= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} + \frac{\cosh(c)\mathrm{Chi}(dx)}{a^3} - \frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{d\mathrm{Chi} \cdots}{\cdots}$$

$$= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} + \frac{\cosh(c)\mathrm{Chi}(dx)}{a^3} - \frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{d^2\cosh \cdots}{\cdots}$$

Mathematica [A]
time = 0.87, size = 451, normalized size = 1.72

$$-\frac{-3a^2b^2\cosh(c+dx) - 2ab^3x\cosh(c+dx) - 2b^2(a+bx)^2\cosh(c)\mathrm{Chi}(dx) + (a+bx)^2\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(\left(2b^2+a^2d^2\right)\cosh\left(c-\frac{ad}{b}\right) + 2abd\sinh\left(c-\frac{ad}{b}\right)\right) - a^3bd\sinh(c+dx) - a^2b^2dx\sinh(c+dx) - 2a^2b^2\sinh(c)\mathrm{Shi}(dx) - 4ab^3x\sinh(c)\mathrm{Shi}(dx) - 2b^4x^2\sinh(c)\mathrm{Shi}(dx) + 2a^3bd\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + 4a^2b^2dx\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + 2ab^3dx^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + 2a^2b^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + a^4d^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + 4ab^3x\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + 2a^3bd^2x\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + 2b^4x^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + a^2b^2d^2x^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{2a^3b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x)^3),x]
```

```
[Out] -1/2*(-3*a^2*b^2*Cosh[c + d*x] - 2*a*b^3*x*Cosh[c + d*x] - 2*b^2*(a + b*x)^
2*Cosh[c]*CoshIntegral[d*x] + (a + b*x)^2*CoshIntegral[d*(a/b + x)]*((2*b^2
 + a^2*d^2)*Cosh[c - (a*d)/b] + 2*a*b*d*Sinh[c - (a*d)/b]) - a^3*b*d*Sinh[c
 + d*x] - a^2*b^2*d*x*Sinh[c + d*x] - 2*a^2*b^2*Sinh[c]*SinhIntegral[d*x] -
 4*a*b^3*x*Sinh[c]*SinhIntegral[d*x] - 2*b^4*x^2*Sinh[c]*SinhIntegral[d*x]
+ 2*a^3*b*d*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 4*a^2*b^2*d*x*Cos
h[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*a*b^3*d*x^2*Cosh[c - (a*d)/b]*
SinhIntegral[d*(a/b + x)] + 2*a^2*b^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b
 + x)] + a^4*d^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 4*a*b^3*x*Si
nh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*a^3*b*d^2*x*Sinh[c - (a*d)/b]
*SinhIntegral[d*(a/b + x)] + 2*b^4*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/
b + x)] + a^2*b^2*d^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/(a^3
*b^2*(a + b*x)^2)
```

Maple [A]
time = 0.73, size = 488, normalized size = 1.86

method	result
risch	$-\frac{e^{-dx-c} d\left(\left(dx+c\right) abd+a^{2} d^{2}-abcd-2\left(dx+c\right) b^{2}-3bda+2b^{2} c\right)}{4a^{2} b\left(\left(dx+c\right)^{2} b^{2}+2\left(dx+c\right) abd-2\left(dx+c\right) b^{2} c+a^{2} d^{2}-2abcd+b^{2} c^{2}\right)}-\frac{e^{-c} \operatorname{expIntegral}\left(1,dx\right)}{2a^{3}}+\frac{e^{\frac{ad-bc}{b}} \operatorname{expIntegral}\left(1,dx+c+\frac{ad}{}\right.}{4a\, b^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*exp(-d*x-c)*d*((d*x+c)*a*b*d+a^2*d^2-a*b*c*d-2*(d*x+c)*b^2-3*b*d*a+2*b
^2*c)/a^2/b/((d*x+c)^2*b^2+2*(d*x+c)*a*b*d-2*(d*x+c)*b^2*c+a^2*d^2-2*a*b*c*
d+b^2*c^2)-1/2/a^3*exp(-c)*Ei(1,d*x)+1/4/a/b^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+
(a*d-b*c)/b)*d^2-1/2/a^2/b*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*d+1/2/a
^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+1/4/a*d^2/b^2*exp(d*x+c)/(a*d/b
+d*x)^2+1/4/a*d^2/b^2*exp(d*x+c)/(a*d/b+d*x)+1/4/a*d^2/b^2*exp(-(a*d-b*c)/b
)*Ei(1,-d*x-c-(a*d-b*c)/b)+1/2*d/b/a^2*exp(d*x+c)/(a*d/b+d*x)+1/2*d/b/a^2*e
xp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-1/2/a^3*exp(c)*Ei(1,-d*x)+1/2/a^3
*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x + a)^3*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(258) = 516$.
time = 0.41, size = 601, normalized size = 2.29

$$\frac{2\,(2\,ab^3x + 3\,a^2b^2)\cosh(dx+c) + 2\,((b^4x^2 + 2\,ab^3x + a^2b^2)\mathrm{Ei}(dx) + (b^4x^2 + 2\,ab^3x + a^2b^2)\mathrm{Ei}(-dx))\cosh(c) - ((a^4d^2 + 2\,a^3bd + 2\,a^2b^2 + (a^2b^2d^2 + 2\,ab^3d + 2\,b^4)x^2 + 2\,(a^3bd^2 + 2\,a^2b^2d + 2\,ab^3)x)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right) + (a^4d^2 - 2\,a^3bd + 2\,a^2b^2 + (a^2b^2d^2 - 2\,ab^3d + 2\,b^4)x^2 + 2\,(a^3bd^2 - 2\,a^2b^2d + 2\,ab^3)x)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right))\cosh\left(-\frac{bc-ad}{b}\right) + 2\,(a^2b^2dx + a^3bd)\sinh(dx+c) + 2\,((b^4x^2 + 2\,ab^3x + a^2b^2)\mathrm{Ei}(dx) - (b^4x^2 + 2\,ab^3x + a^2b^2)\mathrm{Ei}(-dx))\sinh(c) + ((a^4d^2 + 2\,a^3bd + 2\,a^2b^2 + (a^2b^2d^2 + 2\,ab^3d + 2\,b^4)x^2 + 2\,(a^3bd^2 + 2\,a^2b^2d + 2\,ab^3)x)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right) - (a^4d^2 - 2\,a^3bd + 2\,a^2b^2 + (a^2b^2d^2 - 2\,ab^3d + 2\,b^4)x^2 + 2\,(a^3bd^2 - 2\,a^2b^2d + 2\,ab^3)x)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right))\sinh\left(-\frac{bc-ad}{b}\right)}{4\,(a^3b^4x^2 + 2\,a^4b^3x + a^5b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(2*a*b^3*x + 3*a^2*b^2)*cosh(d*x + c) + 2*((b^4*x^2 + 2*a*b^3*x + a^
2*b^2)*Ei(d*x) + (b^4*x^2 + 2*a*b^3*x + a^2*b^2)*Ei(-d*x))*cosh(c) - ((a^4*
d^2 + 2*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 + 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^
3*b*d^2 + 2*a^2*b^2*d + 2*a*b^3)*x)*Ei((b*d*x + a*d)/b) + (a^4*d^2 - 2*a^3*
b*d + 2*a^2*b^2 + (a^2*b^2*d^2 - 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*
a^2*b^2*d + 2*a*b^3)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + 2*(a^2
*b^2*d*x + a^3*b*d)*sinh(d*x + c) + 2*((b^4*x^2 + 2*a*b^3*x + a^2*b^2)*Ei(d
*x) - (b^4*x^2 + 2*a*b^3*x + a^2*b^2)*Ei(-d*x))*sinh(c) + ((a^4*d^2 + 2*a^3
*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 + 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 + 2
*a^2*b^2*d + 2*a*b^3)*x)*Ei((b*d*x + a*d)/b) - (a^4*d^2 - 2*a^3*b*d + 2*a^2
*b^2 + (a^2*b^2*d^2 - 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*a^2*b^2*d +
 2*a*b^3)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^3*b^4*x^2 + 2*a
^4*b^3*x + a^5*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c+dx)}{x\,(a+bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)**3,x)
```

```
[Out] Integral(cosh(c + d*x)/(x*(a + b*x)**3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(258) = 516$.
time = 0.42, size = 837, normalized size = 3.19

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a^2*b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*b^2*d^2*x^2*E
i(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*a^3*b*d^2*x*Ei((b*d*x + a*d)/b)*e^(c
```

```
 - a*d/b) + 2*a*b^3*d*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^3*b*d^2*x
*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 2*a*b^3*d*x^2*Ei(-(b*d*x + a*d)/b)*e
^(-c + a*d/b) - 2*b^4*x^2*Ei(-d*x)*e^(-c) + a^4*d^2*Ei((b*d*x + a*d)/b)*e^(
c - a*d/b) + 4*a^2*b^2*d*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*b^4*x^2*Ei
((b*d*x + a*d)/b)*e^(c - a*d/b) - 2*b^4*x^2*Ei(d*x)*e^c + a^4*d^2*Ei(-(b*d*
x + a*d)/b)*e^(-c + a*d/b) - 4*a^2*b^2*d*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d
/b) + 2*b^4*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^2*b^2*d*x*e^(d*x +
c) + a^2*b^2*d*x*e^(-d*x - c) - 4*a*b^3*x*Ei(-d*x)*e^(-c) + 2*a^3*b*d*Ei((b
*d*x + a*d)/b)*e^(c - a*d/b) + 4*a*b^3*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b)
- 4*a*b^3*x*Ei(d*x)*e^c - 2*a^3*b*d*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 4
*a*b^3*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^3*b*d*e^(d*x + c) - 2*a*b^
3*x*e^(d*x + c) + a^3*b*d*e^(-d*x - c) - 2*a*b^3*x*e^(-d*x - c) - 2*a^2*b^2
*Ei(-d*x)*e^(-c) + 2*a^2*b^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 2*a^2*b^2*
Ei(d*x)*e^c + 2*a^2*b^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 3*a^2*b^2*e^(
d*x + c) - 3*a^2*b^2*e^(-d*x - c))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)}{x\,{\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x*(a + b*x)^3),x)
```

```
[Out] int(cosh(c + d*x)/(x*(a + b*x)^3), x)
```

3.38 $\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx$

Optimal. Leaf size=298

$$-\frac{\cosh(c+dx)}{a^3x}-\frac{b\cosh(c+dx)}{2a^2(a+bx)^2}-\frac{2b\cosh(c+dx)}{a^3(a+bx)}-\frac{3b\cosh(c)\mathrm{Chi}(dx)}{a^4}+\frac{3b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^4}+\frac{d^2\mathrm{co}}{}$$

```
[Out] -3*b*Chi(d*x)*cosh(c)/a^4+3*b*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/a^4+1/2*d^2*Chi
(a*d/b+d*x)*cosh(-c+a*d/b)/a^2/b-cosh(d*x+c)/a^3/x-1/2*b*cosh(d*x+c)/a^2/(b
*x+a)^2-2*b*cosh(d*x+c)/a^3/(b*x+a)+d*cosh(c)*Shi(d*x)/a^3+2*d*cosh(-c+a*d/
b)*Shi(a*d/b+d*x)/a^3+d*Chi(d*x)*sinh(c)/a^3-3*b*Shi(d*x)*sinh(c)/a^4-2*d*C
hi(a*d/b+d*x)*sinh(-c+a*d/b)/a^3-3*b*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/a^4-1/2*
d^2*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/a^2/b-1/2*d*sinh(d*x+c)/a^2/(b*x+a)
```

Rubi [A]
time = 0.52, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3379, 3382}

$$-\frac{3b\cosh(c)\mathrm{Chi}(dx)}{a^4}+\frac{3b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{a^4}-\frac{3b\sinh(c)\mathrm{Shi}(dx)}{a^4}+\frac{3b\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{a^4}+\frac{2d\sinh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{a^3}+\frac{2d\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{2b\cosh(c+dx)}{a^3(a+bx)}+\frac{d\sinh(c)\mathrm{Chi}(dx)}{a^3}+\frac{d\cosh(c)\mathrm{Shi}(dx)}{a^3}-\frac{\cosh(c+dx)}{a^3x}+\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{2a^2b}+\frac{d^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{2a^2b}-\frac{d\sinh(c+dx)}{2a^2(a+bx)}-\frac{b\cosh(c+dx)}{2a^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x)^3),x]
```

```
[Out] -(Cosh[c + d*x]/(a^3*x)) - (b*Cosh[c + d*x])/(2*a^2*(a + b*x)^2) - (2*b*Cos
h[c + d*x])/(a^3*(a + b*x)) - (3*b*Cosh[c]*CoshIntegral[d*x])/a^4 + (3*b*Co
sh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^4 + (d^2*Cosh[c - (a*d)/b]*C
oshIntegral[(a*d)/b + d*x])/(2*a^2*b) + (d*CoshIntegral[d*x]*Sinh[c])/a^3 +
 (2*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/a^3 - (d*Sinh[c + d*x]
)/(2*a^2*(a + b*x)) + (d*Cosh[c]*SinhIntegral[d*x])/a^3 - (3*b*Sinh[c]*Sinh
Integral[d*x])/a^4 + (2*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^
3 + (3*b*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^4 + (d^2*Sinh[c -
 (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*a^2*b)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx = \int \left(\frac{\cosh(c+dx)}{a^3x^2} - \frac{3b\cosh(c+dx)}{a^4x} + \frac{b^2\cosh(c+dx)}{a^2(a+bx)^3} + \frac{2b^2\cosh(c+dx)}{a^3(a+bx)^2} + \frac{3b^2\cosh}{a^4(a}\right.$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a^3} - \frac{(3b)\int \frac{\cosh(c+dx)}{x}\,dx}{a^4} + \frac{(3b^2)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^4} + \frac{(2b^2)\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{a^3} + \cdots$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{b\cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b\cosh(c+dx)}{a^3(a+bx)} + \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{a^3} + \frac{(2bd)\int \frac{\sinh(}{a+}}{a^3}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{b\cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b\cosh(c+dx)}{a^3(a+bx)} - \frac{3b\cosh(c)\text{Chi}(dx)}{a^4} + \frac{3b\cosh\left(\right.}{}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{b\cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b\cosh(c+dx)}{a^3(a+bx)} - \frac{3b\cosh(c)\text{Chi}(dx)}{a^4} + \frac{3b\cosh\left(\right.}{}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{b\cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b\cosh(c+dx)}{a^3(a+bx)} - \frac{3b\cosh(c)\text{Chi}(dx)}{a^4} + \frac{3b\cosh\left(\right.}{}$$

Mathematica [A]
time = 0.98, size = 541, normalized size = 1.82

$$\frac{-2a^3b\cosh(c+dx) - 9a^2b^2x\cosh(c+dx) - 6ab^3x^2\cosh(c+dx) + 2bx(a+bx)^2\text{Chi}(dx)(-3b\cosh(c) + ad\sinh(c)) + x(a+bx)^2\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left((6b^2+a^2d^2)\cosh\left(c-\frac{ad}{b}\right) + 4abd\sinh\left(c-\frac{ad}{b}\right)\right) - a^3bdx\sinh(c+dx) - a^2b^2dx^2\sinh(c+dx) + 2a^3bdx\cosh(c)\text{Shi}(dx) + 4a^2b^2dx^2\cosh(c)\text{Shi}(dx) + 2ab^3dx^3\cosh(c)\text{Shi}(dx) - 6a^2b^2x\sinh(c)\text{Shi}(dx) - 12ab^3x^2\sinh(c)\text{Shi}(dx) - 6b^4x^3\sinh(c)\text{Shi}(dx) + 4a^3bdx\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + 8a^2b^2dx^2\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + 4ab^3dx^3\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + 6a^2b^2x\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + a^4d^2x\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + 12ab^3x^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + 2a^3bd^2x^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + 6b^4x^3\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + a^2b^2d^2x^3\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{2a^4bx(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x)^3),x]

[Out] (-2*a^3*b*Cosh[c + d*x] - 9*a^2*b^2*x*Cosh[c + d*x] - 6*a*b^3*x^2*Cosh[c + d*x] + 2*b*x*(a + b*x)^2*CoshIntegral[d*x]*(-3*b*Cosh[c] + a*d*Sinh[c]) + x *(a + b*x)^2*CoshIntegral[d*(a/b + x)]*((6*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] + 4*a*b*d*Sinh[c - (a*d)/b]) - a^3*b*d*x*Sinh[c + d*x] - a^2*b^2*d*x^2*Sin h[c + d*x] + 2*a^3*b*d*x*Cosh[c]*SinhIntegral[d*x] + 4*a^2*b^2*d*x^2*Cosh[c]*SinhIntegral[d*x] + 2*a*b^3*d*x^3*Cosh[c]*SinhIntegral[d*x] - 6*a^2*b^2*x *Sinh[c]*SinhIntegral[d*x] - 12*a*b^3*x^2*Sinh[c]*SinhIntegral[d*x] - 6*b^4 *x^3*Sinh[c]*SinhIntegral[d*x] + 4*a^3*b*d*x*Cosh[c - (a*d)/b]*SinhIntegral [d*(a/b + x)] + 8*a^2*b^2*d*x^2*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 4*a*b^3*d*x^3*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 6*a^2*b^2*x* Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + a^4*d^2*x*Sinh[c - (a*d)/b]*S inhIntegral[d*(a/b + x)] + 12*a*b^3*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a /b + x)] + 2*a^3*b*d^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 6* b^4*x^3*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + a^2*b^2*d^2*x^3*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/(2*a^4*b*x*(a + b*x)^2)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(296) = 592$.
time = 0.76, size = 643, normalized size = 2.16

method	result
risch	$\frac{e^{-dx-c}x\,d^3 b}{4a^2(b^2d^2x^2+2ab\,d^2x+a^2d^2)}+\frac{e^{-dx-c}d^3}{4a(b^2d^2x^2+2ab\,d^2x+a^2d^2)}-\frac{3\,e^{-dx-c}x\,d^2b^2}{2a^3(b^2d^2x^2+2ab\,d^2x+a^2d^2)}-\frac{9\,e^{-dx-c}d^2b}{4a^2(b^2d^2x^2+2ab\,d^2x+a^2d^2)}-$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x^2/(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/4*exp(-d*x-c)/a^2*x*d^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b+1/4*exp(-d*x-c)/a*d^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)-3/2*exp(-d*x-c)/a^3*x*d^2/(b^2*d ^2*x^2+2*a*b*d^2*x+a^2*d^2)*b^2-9/4*exp(-d*x-c)/a^2*d^2/(b^2*d^2*x^2+2*a*b* d^2*x+a^2*d^2)*b-1/2*exp(-d*x-c)/a/x*d^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)+ 1/2*d/a^3*exp(-c)*Ei(1,d*x)+3/2/a^4*exp(-c)*Ei(1,d*x)*b-1/4/b/a^2*d^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+d/a^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d- b*c)/b)-3/2*b/a^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/4/a^2*d^2/b*ex p(d*x+c)/(a*d/b+d*x)^2-1/4/a^2*d^2/b*exp(d*x+c)/(a*d/b+d*x)-1/4/a^2*d^2/b*e xp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-d/a^3*exp(d*x+c)/(a*d/b+d*x)-d/a^ 3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)+3/2/a^4*b*exp(c)*Ei(1,-d*x)-3/ 2*b/a^4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-1/2/a^3/x*exp(d*x+c)-1/2 *d/a^3*exp(c)*Ei(1,-d*x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x + a)^3*x^2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(296) = 592$.
time = 0.43, size = 762, normalized size = 2.56

$$-\frac{2\,(6\,ab^3x^2+9\,a^2b^2x+2\,a^3b)\cosh(dx+c)-2\,\big(((ab^3d-3\,b^4)x^3+2\,(a^2b^2d-3\,ab^3)x^2+(a^3bd-3\,a^2b^2)x)\mathrm{Ei}(dx)-((ab^3d+3\,b^4)x^3+2\,(a^2b^2d+3\,ab^3)x^2+(a^3bd+3\,a^2b^2)x)\mathrm{Ei}(-dx)\big)\cosh(c)-\big(((a^2b^2d^2+4\,ab^3d+6\,b^4)x^3+2\,(a^3bd^2+4\,a^2b^2d+6\,ab^3)x^2+(a^4d^2+4\,a^3bd+6\,a^2b^2)x)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+((a^2b^2d^2-4\,ab^3d+6\,b^4)x^3+2\,(a^3bd^2-4\,a^2b^2d+6\,ab^3)x^2+(a^4d^2-4\,a^3bd+6\,a^2b^2)x)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\big)\cosh\left(-\frac{bc-ad}{b}\right)+2\,(a^2b^2dx^2+a^3bdx)\sinh(dx+c)-2\,\big(((ab^3d-3\,b^4)x^3+2\,(a^2b^2d-3\,ab^3)x^2+(a^3bd-3\,a^2b^2)x)\mathrm{Ei}(dx)+((ab^3d+3\,b^4)x^3+2\,(a^2b^2d+3\,ab^3)x^2+(a^3bd+3\,a^2b^2)x)\mathrm{Ei}(-dx)\big)\sinh(c)+\big(((a^2b^2d^2+4\,ab^3d+6\,b^4)x^3+2\,(a^3bd^2+4\,a^2b^2d+6\,ab^3)x^2+(a^4d^2+4\,a^3bd+6\,a^2b^2)x)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-((a^2b^2d^2-4\,ab^3d+6\,b^4)x^3+2\,(a^3bd^2-4\,a^2b^2d+6\,ab^3)x^2+(a^4d^2-4\,a^3bd+6\,a^2b^2)x)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\big)\sinh\left(-\frac{bc-ad}{b}\right)}{4\,(a^4b^3x^3+2\,a^5b^2x^2+a^6bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(6*a*b^3*x^2 + 9*a^2*b^2*x + 2*a^3*b)*cosh(d*x + c) - 2*(((a*b^3*d
- 3*b^4)*x^3 + 2*(a^2*b^2*d - 3*a*b^3)*x^2 + (a^3*b*d - 3*a^2*b^2)*x)*Ei(d*
x) - ((a*b^3*d + 3*b^4)*x^3 + 2*(a^2*b^2*d + 3*a*b^3)*x^2 + (a^3*b*d + 3*a^
2*b^2)*x)*Ei(-d*x))*cosh(c) - (((a^2*b^2*d^2 + 4*a*b^3*d + 6*b^4)*x^3 + 2*(
a^3*b*d^2 + 4*a^2*b^2*d + 6*a*b^3)*x^2 + (a^4*d^2 + 4*a^3*b*d + 6*a^2*b^2)*
x)*Ei((b*d*x + a*d)/b) + ((a^2*b^2*d^2 - 4*a*b^3*d + 6*b^4)*x^3 + 2*(a^3*b*
d^2 - 4*a^2*b^2*d + 6*a*b^3)*x^2 + (a^4*d^2 - 4*a^3*b*d + 6*a^2*b^2)*x)*Ei(
-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + 2*(a^2*b^2*d*x^2 + a^3*b*d*x)*sin
h(d*x + c) - 2*(((a*b^3*d - 3*b^4)*x^3 + 2*(a^2*b^2*d - 3*a*b^3)*x^2 + (a^3
*b*d - 3*a^2*b^2)*x)*Ei(d*x) + ((a*b^3*d + 3*b^4)*x^3 + 2*(a^2*b^2*d + 3*a*
b^3)*x^2 + (a^3*b*d + 3*a^2*b^2)*x)*Ei(-d*x))*sinh(c) + (((a^2*b^2*d^2 + 4*
a*b^3*d + 6*b^4)*x^3 + 2*(a^3*b*d^2 + 4*a^2*b^2*d + 6*a*b^3)*x^2 + (a^4*d^2
 + 4*a^3*b*d + 6*a^2*b^2)*x)*Ei((b*d*x + a*d)/b) - ((a^2*b^2*d^2 - 4*a*b^3*
d + 6*b^4)*x^3 + 2*(a^3*b*d^2 - 4*a^2*b^2*d + 6*a*b^3)*x^2 + (a^4*d^2 - 4*a
^3*b*d + 6*a^2*b^2)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^4*b^3
*x^3 + 2*a^5*b^2*x^2 + a^6*b*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**2/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1006 vs. $2(296) = 592$.

time = 0.40, size = 1006, normalized size = 3.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*b^2*d^2*x^3*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*b^2*d^2*x^3*Ei
(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 2*a*b^3*d*x^3*Ei(-d*x)*e^(-c) + 2*a^3*b
*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 4*a*b^3*d*x^3*Ei((b*d*x + a*d)
/b)*e^(c - a*d/b) + 2*a*b^3*d*x^3*Ei(d*x)*e^c + 2*a^3*b*d^2*x^2*Ei(-(b*d*x
+ a*d)/b)*e^(-c + a*d/b) - 4*a*b^3*d*x^3*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b
) - 4*a^2*b^2*d*x^2*Ei(-d*x)*e^(-c) - 6*b^4*x^3*Ei(-d*x)*e^(-c) + a^4*d^2*x
*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 8*a^2*b^2*d*x^2*Ei((b*d*x + a*d)/b)*e^
(c - a*d/b) + 6*b^4*x^3*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 4*a^2*b^2*d*x^2
*Ei(d*x)*e^c - 6*b^4*x^3*Ei(d*x)*e^c + a^4*d^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c
 + a*d/b) - 8*a^2*b^2*d*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 6*b^4*x^3
*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^2*b^2*d*x^2*e^(d*x + c) + a^2*b^2*
d*x^2*e^(-d*x - c) - 2*a^3*b*d*x*Ei(-d*x)*e^(-c) - 12*a*b^3*x^2*Ei(-d*x)*e^
(-c) + 4*a^3*b*d*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 12*a*b^3*x^2*Ei((b*d
*x + a*d)/b)*e^(c - a*d/b) + 2*a^3*b*d*x*Ei(d*x)*e^c - 12*a*b^3*x^2*Ei(d*x)
*e^c - 4*a^3*b*d*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 12*a*b^3*x^2*Ei(-(
b*d*x + a*d)/b)*e^(-c + a*d/b) - a^3*b*d*x*e^(d*x + c) - 6*a*b^3*x^2*e^(d*x
 + c) + a^3*b*d*x*e^(-d*x - c) - 6*a*b^3*x^2*e^(-d*x - c) - 6*a^2*b^2*x*Ei(
-d*x)*e^(-c) + 6*a^2*b^2*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 6*a^2*b^2*x*
Ei(d*x)*e^c + 6*a^2*b^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 9*a^2*b^2*x
*e^(d*x + c) - 9*a^2*b^2*x*e^(-d*x - c) - 2*a^3*b*e^(d*x + c) - 2*a^3*b*e^(
-d*x - c))/(a^4*b^3*x^3 + 2*a^5*b^2*x^2 + a^6*b*x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)}{x^2\,{\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x^2*(a + b*x)^3),x)
```

```
[Out] int(cosh(c + d*x)/(x^2*(a + b*x)^3), x)
```

3.39 $\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx$

Optimal. Leaf size=377

$$-\frac{\cosh(c+dx)}{2a^3x^2}+\frac{3b\cosh(c+dx)}{a^4x}+\frac{b^2\cosh(c+dx)}{2a^3(a+bx)^2}+\frac{3b^2\cosh(c+dx)}{a^4(a+bx)}+\frac{6b^2\cosh(c)\text{Chi}(dx)}{a^5}+\frac{d^2\cosh(c)\text{Chi}(}{2a^3}$$

```
[Out] 6*b^2*Chi(d*x)*cosh(c)/a^5+1/2*d^2*Chi(d*x)*cosh(c)/a^3-6*b^2*Chi(a*d/b+d*x
)*cosh(-c+a*d/b)/a^5-1/2*d^2*Chi(a*d/b+d*x)*cosh(-c+a*d/b)/a^3-1/2*cosh(d*x
+c)/a^3/x^2+3*b*cosh(d*x+c)/a^4/x+1/2*b^2*cosh(d*x+c)/a^3/(b*x+a)^2+3*b^2*c
osh(d*x+c)/a^4/(b*x+a)-3*b*d*cosh(c)*Shi(d*x)/a^4-3*b*d*cosh(-c+a*d/b)*Shi(
a*d/b+d*x)/a^4-3*b*d*Chi(d*x)*sinh(c)/a^4+6*b^2*Shi(d*x)*sinh(c)/a^5+1/2*d^
2*Shi(d*x)*sinh(c)/a^3+3*b*d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/a^4+6*b^2*Shi(a*
d/b+d*x)*sinh(-c+a*d/b)/a^5+1/2*d^2*Shi(a*d/b+d*x)*sinh(-c+a*d/b)/a^3-1/2*d
*sinh(d*x+c)/a^3/x+1/2*b*d*sinh(d*x+c)/a^3/(b*x+a)
```

Rubi [A]
time = 0.63, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3379, 3382}

$$\frac{6b^2\cosh(c)\text{Chi}(dx)}{a^5}-\frac{6b^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^5}+\frac{6b^2\sinh(c)\text{Shi}(dx)}{a^5}-\frac{6b^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^5}+\frac{3b^2\cosh(c+dx)}{a^4(a+bx)}-\frac{3bd\sinh(c)\text{Chi}(dx)}{a^4}-\frac{3bd\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^4}-\frac{3bd\cosh(c)\text{Shi}(dx)}{a^4}-\frac{3bd\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^4}+\frac{3b\cosh(c+dx)}{a^4x}+\frac{b^2\cosh(c+dx)}{2a^3(a+bx)^2}-\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2a^3}-\frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2a^3}+\frac{bd\sinh(c+dx)}{2a^3(a+bx)}+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a^3}+\frac{d^2\sinh(c)\text{Shi}(dx)}{2a^3}-\frac{\cosh(c+dx)}{2a^3x^2}-\frac{d\sinh(c+dx)}{2a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^3*(a + b*x)^3),x]
```

```
[Out] -1/2*Cosh[c + d*x]/(a^3*x^2) + (3*b*Cosh[c + d*x])/(a^4*x) + (b^2*Cosh[c +
d*x])/(2*a^3*(a + b*x)^2) + (3*b^2*Cosh[c + d*x])/(a^4*(a + b*x)) + (6*b^2*
Cosh[c]*CoshIntegral[d*x])/a^5 + (d^2*Cosh[c]*CoshIntegral[d*x])/(2*a^3) -
(6*b^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^5 - (d^2*Cosh[c - (
a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*a^3) - (3*b*d*CoshIntegral[d*x]*Sin
h[c])/a^4 - (3*b*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/a^4 - (d*
Sinh[c + d*x])/(2*a^3*x) + (b*d*Sinh[c + d*x])/(2*a^3*(a + b*x)) - (3*b*d*C
osh[c]*SinhIntegral[d*x])/a^4 + (6*b^2*Sinh[c]*SinhIntegral[d*x])/a^5 + (d^
2*Sinh[c]*SinhIntegral[d*x])/(2*a^3) - (3*b*d*Cosh[c - (a*d)/b]*SinhIntegra
l[(a*d)/b + d*x])/a^4 - (6*b^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x
])/a^5 - (d^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*a^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx = \int \left(\frac{\cosh(c+dx)}{a^3x^3} - \frac{3b\cosh(c+dx)}{a^4x^2} + \frac{6b^2\cosh(c+dx)}{a^5x} - \frac{b^3\cosh(c+dx)}{a^3(a+bx)^3} - \frac{3b^3\cosh(}{a^4(a+}\right.$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^3}\,dx}{a^3} - \frac{(3b)\int \frac{\cosh(c+dx)}{x^2}\,dx}{a^4} + \frac{(6b^2)\int \frac{\cosh(c+dx)}{x}\,dx}{a^5} - \frac{(6b^3)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^5} - \frac{(3}{}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b\cosh(c+dx)}{a^4x} + \frac{b^2\cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2\cosh(c+dx)}{a^4(a+bx)} + \frac{d\int \frac{\sinh(c+dx}{x^2}}{2a^3}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b\cosh(c+dx)}{a^4x} + \frac{b^2\cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2\cosh(c+dx)}{a^4(a+bx)} + \frac{6b^2\cosh(c)\mathrm{C}}{a^5}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b\cosh(c+dx)}{a^4x} + \frac{b^2\cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2\cosh(c+dx)}{a^4(a+bx)} + \frac{6b^2\cosh(c)\mathrm{C}}{a^5}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b\cosh(c+dx)}{a^4x} + \frac{b^2\cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2\cosh(c+dx)}{a^4(a+bx)} + \frac{6b^2\cosh(c)\mathrm{C}}{a^5}$$

Mathematica [**A**]
time = 1.21, size = 627, normalized size = 1.66

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x)^3),x]
```

```
[Out] -1/2*(a^4*Cosh[c + d*x] - 4*a^3*b*x*Cosh[c + d*x] - 18*a^2*b^2*x^2*Cosh[c +
 d*x] - 12*a*b^3*x^3*Cosh[c + d*x] - x^2*(a + b*x)^2*CoshIntegral[d*x]*((12
*b^2 + a^2*d^2)*Cosh[c] - 6*a*b*d*Sinh[c]) + x^2*(a + b*x)^2*CoshIntegral[d
*(a/b + x)]*((12*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] + 6*a*b*d*Sinh[c - (a*d)/
b]) + a^4*d*x*Sinh[c + d*x] + a^3*b*d*x^2*Sinh[c + d*x] + 6*a^3*b*d*x^2*Cos
h[c]*SinhIntegral[d*x] + 12*a^2*b^2*d*x^3*Cosh[c]*SinhIntegral[d*x] + 6*a*b
^3*d*x^4*Cosh[c]*SinhIntegral[d*x] - 12*a^2*b^2*x^2*Sinh[c]*SinhIntegral[d*
x] - a^4*d^2*x^2*Sinh[c]*SinhIntegral[d*x] - 24*a*b^3*x^3*Sinh[c]*SinhInteg
ral[d*x] - 2*a^3*b*d^2*x^3*Sinh[c]*SinhIntegral[d*x] - 12*b^4*x^4*Sinh[c]*S
inhIntegral[d*x] - a^2*b^2*d^2*x^4*Sinh[c]*SinhIntegral[d*x] + 6*a^3*b*d*x^
2*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 12*a^2*b^2*d*x^3*Cosh[c - (
a*d)/b]*SinhIntegral[d*(a/b + x)] + 6*a*b^3*d*x^4*Cosh[c - (a*d)/b]*SinhInt
egral[d*(a/b + x)] + 12*a^2*b^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b +
 x)] + a^4*d^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 24*a*b^3*x
^3*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*a^3*b*d^2*x^3*Sinh[c - (
a*d)/b]*SinhIntegral[d*(a/b + x)] + 12*b^4*x^4*Sinh[c - (a*d)/b]*SinhIntegr
al[d*(a/b + x)] + a^2*b^2*d^2*x^4*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x
)])/(a^5*x^2*(a + b*x)^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 759 vs. $2(367) = 734$.
time = 0.77, size = 760, normalized size = 2.02

method	result
risch	$\frac{d^3 e^{-dx-c} b}{4a^2(b^2d^2x^2+2ab\,d^2x+a^2d^2)} + \frac{3d^2 e^{-dx-c} x\, b^3}{a^4(b^2d^2x^2+2ab\,d^2x+a^2d^2)} + \frac{d^3 e^{-dx-c}}{4ax(b^2d^2x^2+2ab\,d^2x+a^2d^2)} + \frac{9d^2 e^{-dx-c} b^2}{2a^3(b^2d^2x^2+2ab\,d^2x+a^2d^2)} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d^3*exp(-d*x-c)/a^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b+3*d^2*exp(-d*x-
c)/a^4*x/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b^3+1/4*d^3*exp(-d*x-c)/a/x/(b^2
*d^2*x^2+2*a*b*d^2*x+a^2*d^2)+9/2*d^2*exp(-d*x-c)/a^3/(b^2*d^2*x^2+2*a*b*d^
2*x+a^2*d^2)*b^2+d^2*exp(-d*x-c)/a^2/x/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b-
1/4*d^2*exp(-d*x-c)/a/x^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)-1/4*d^2/a^3*exp
(-c)*Ei(1,d*x)-3/2*d/a^4*exp(-c)*Ei(1,d*x)*b-3/a^5*exp(-c)*Ei(1,d*x)*b^2+1/
```

4*d^2/a^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-3/2*d/a^4*exp((a*d-b*c)/
b)*Ei(1,d*x+c+(a*d-b*c)/b)*b+3/a^5*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)
*b^2+1/4*d^2/a^3*exp(d*x+c)/(a*d/b+d*x)^2+1/4*d^2/a^3*exp(d*x+c)/(a*d/b+d*x
)+1/4*d^2/a^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)+3/2*d/a^4*b*exp(d*
x+c)/(a*d/b+d*x)+3/2*d/a^4*b*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-3/a
^5*b^2*exp(c)*Ei(1,-d*x)+3*b^2/a^5*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/
b)+3/2/a^4*b/x*exp(d*x+c)+3/2*d/a^4*b*exp(c)*Ei(1,-d*x)-1/4/a^3/x^2*exp(d*x
+c)-1/4*d/a^3/x*exp(d*x+c)-1/4*d^2/a^3*exp(c)*Ei(1,-d*x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^3/(b*x+a)^3,x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/((b*x + a)^3*x^3), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 892 vs. 2(367) = 734.
time = 0.39, size = 892, normalized size = 2.37

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^3/(b*x+a)^3,x, algorithm="fricas")

[Out] 1/4*(2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4)*cosh(d*x + c) + ((
(a^2*b^2*d^2 - 6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*
b^3)*x^3 + (a^4*d^2 - 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(d*x) + ((a^2*b^2*d^2
+ 6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a
^4*d^2 + 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(-d*x))*cosh(c) - (((a^2*b^2*d^2 +
6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4
*d^2 + 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei((b*d*x + a*d)/b) + ((a^2*b^2*d^2 - 6
*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*
d^2 - 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/
b) - 2*(a^3*b*d*x^2 + a^4*d*x)*sinh(d*x + c) + (((a^2*b^2*d^2 - 6*a*b^3*d +
 12*b^4)*x^4 + 2*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 - 6*a^
3*b*d + 12*a^2*b^2)*x^2)*Ei(d*x) - ((a^2*b^2*d^2 + 6*a*b^3*d + 12*b^4)*x^4
+ 2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 + 6*a^3*b*d + 12*a^
2*b^2)*x^2)*Ei(-d*x))*sinh(c) + (((a^2*b^2*d^2 + 6*a*b^3*d + 12*b^4)*x^4 +
2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 + 6*a^3*b*d + 12*a^2*
b^2)*x^2)*Ei((b*d*x + a*d)/b) - ((a^2*b^2*d^2 - 6*a*b^3*d + 12*b^4)*x^4 + 2
*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 - 6*a^3*b*d + 12*a^2*b

```
^2)*x^2)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^5*b^2*x^4 + 2*a^6*b
*x^3 + a^7*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**3/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1169 vs. $2(367) = 734$.
time = 0.43, size = 1169, normalized size = 3.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*b^2*d^2*x^4*Ei(-d*x)*e^(-c) - a^2*b^2*d^2*x^4*Ei((b*d*x + a*d)/b)*
e^(c - a*d/b) + a^2*b^2*d^2*x^4*Ei(d*x)*e^c - a^2*b^2*d^2*x^4*Ei(-(b*d*x +
a*d)/b)*e^(-c + a*d/b) + 2*a^3*b*d^2*x^3*Ei(-d*x)*e^(-c) + 6*a*b^3*d*x^4*Ei
(-d*x)*e^(-c) - 2*a^3*b*d^2*x^3*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 6*a*b^3
*d*x^4*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^3*b*d^2*x^3*Ei(d*x)*e^c - 6*
a*b^3*d*x^4*Ei(d*x)*e^c - 2*a^3*b*d^2*x^3*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/
b) + 6*a*b^3*d*x^4*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + a^4*d^2*x^2*Ei(-d*
x)*e^(-c) + 12*a^2*b^2*d*x^3*Ei(-d*x)*e^(-c) + 12*b^4*x^4*Ei(-d*x)*e^(-c) -
 a^4*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 12*a^2*b^2*d*x^3*Ei((b*d*x
 + a*d)/b)*e^(c - a*d/b) - 12*b^4*x^4*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a
^4*d^2*x^2*Ei(d*x)*e^c - 12*a^2*b^2*d*x^3*Ei(d*x)*e^c + 12*b^4*x^4*Ei(d*x)*
e^c - a^4*d^2*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 12*a^2*b^2*d*x^3*Ei
(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 12*b^4*x^4*Ei(-(b*d*x + a*d)/b)*e^(-c +
 a*d/b) + 6*a^3*b*d*x^2*Ei(-d*x)*e^(-c) + 24*a*b^3*x^3*Ei(-d*x)*e^(-c) - 6*
a^3*b*d*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 24*a*b^3*x^3*Ei((b*d*x + a*
d)/b)*e^(c - a*d/b) - 6*a^3*b*d*x^2*Ei(d*x)*e^c + 24*a*b^3*x^3*Ei(d*x)*e^c
+ 6*a^3*b*d*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 24*a*b^3*x^3*Ei(-(b*d
*x + a*d)/b)*e^(-c + a*d/b) - a^3*b*d*x^2*e^(d*x + c) + 12*a*b^3*x^3*e^(d*x
 + c) + a^3*b*d*x^2*e^(-d*x - c) + 12*a*b^3*x^3*e^(-d*x - c) + 12*a^2*b^2*x
^2*Ei(-d*x)*e^(-c) - 12*a^2*b^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 12*
a^2*b^2*x^2*Ei(d*x)*e^c - 12*a^2*b^2*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b
) - a^4*d*x*e^(d*x + c) + 18*a^2*b^2*x^2*e^(d*x + c) + a^4*d*x*e^(-d*x - c)
 + 18*a^2*b^2*x^2*e^(-d*x - c) + 4*a^3*b*x*e^(d*x + c) + 4*a^3*b*x*e^(-d*x
```

```
- c) - a^4*e^(d*x + c) - a^4*e^(-d*x - c))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7
*x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + d\,x)}{x^3\,{(a + b\,x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x^3*(a + b*x)^3),x)
[Out] int(cosh(c + d*x)/(x^3*(a + b*x)^3), x)
```

3.40 $\int x^3(a+bx^2)\cosh(c+dx)\,dx$

Optimal. Leaf size=139

$$-\frac{120b\cosh(c+dx)}{d^6}-\frac{6a\cosh(c+dx)}{d^4}-\frac{60bx^2\cosh(c+dx)}{d^4}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{5bx^4\cosh(c+dx)}{d^2}+\frac{120bx}{}$$

```
[Out] -120*b*cosh(d*x+c)/d^6-6*a*cosh(d*x+c)/d^4-60*b*x^2*cosh(d*x+c)/d^4-3*a*x^2
*cosh(d*x+c)/d^2-5*b*x^4*cosh(d*x+c)/d^2+120*b*x*sinh(d*x+c)/d^5+6*a*x*sinh
(d*x+c)/d^3+20*b*x^3*sinh(d*x+c)/d^3+a*x^3*sinh(d*x+c)/d+b*x^5*sinh(d*x+c)/
d
```

Rubi [A]

time = 0.18, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.176$, Rules used = {5395, 3377, 2718}

$$-\frac{6a\cosh(c+dx)}{d^4}+\frac{6ax\sinh(c+dx)}{d^3}-\frac{3ax^2\cosh(c+dx)}{d^2}+\frac{ax^3\sinh(c+dx)}{d}-\frac{120b\cosh(c+dx)}{d^6}+\frac{120bx\sinh(c+dx)}{d^5}-\frac{60bx^2\cosh(c+dx)}{d^4}+\frac{20bx^3\sinh(c+dx)}{d^3}-\frac{5bx^4\cosh(c+dx)}{d^2}+\frac{bx^5\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-120*b*Cosh[c + d*x])/d^6 - (6*a*Cosh[c + d*x])/d^4 - (60*b*x^2*Cosh[c + d
*x])/d^4 - (3*a*x^2*Cosh[c + d*x])/d^2 - (5*b*x^4*Cosh[c + d*x])/d^2 + (120
*b*x*Sinh[c + d*x])/d^5 + (6*a*x*Sinh[c + d*x])/d^3 + (20*b*x^3*Sinh[c + d*
x])/d^3 + (a*x^3*Sinh[c + d*x])/d + (b*x^5*Sinh[c + d*x])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx^2)\cosh(c+dx)\,dx &= \int \left(ax^3\cosh(c+dx)+bx^5\cosh(c+dx)\right)\,dx\\ &= a\int x^3\cosh(c+dx)\,dx + b\int x^5\cosh(c+dx)\,dx\\ &= \frac{ax^3\sinh(c+dx)}{d}+\frac{bx^5\sinh(c+dx)}{d}-\frac{(3a)\int x^2\sinh(c+dx)\,dx}{d}-\frac{(5b)\int \\ &= -\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{5bx^4\cosh(c+dx)}{d^2}+\frac{ax^3\sinh(c+dx)}{d}+\frac{bx^5\sinh(c}{d}\\ &= -\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{5bx^4\cosh(c+dx)}{d^2}+\frac{6ax\sinh(c+dx)}{d^3}+\frac{20bx^3\sinh}{d^3}\\ &= -\frac{6a\cosh(c+dx)}{d^4}-\frac{60bx^2\cosh(c+dx)}{d^4}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{5bx^4\cosh(}{d^2}\\ &= -\frac{6a\cosh(c+dx)}{d^4}-\frac{60bx^2\cosh(c+dx)}{d^4}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{5bx^4\cosh(}{d^2}\\ &= -\frac{120b\cosh(c+dx)}{d^6}-\frac{6a\cosh(c+dx)}{d^4}-\frac{60bx^2\cosh(c+dx)}{d^4}-\frac{3ax^2\cosh(}{d^2}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 92, normalized size = 0.66

$$\frac{-((3ad^2(2+d^2x^2)+5b(24+12d^2x^2+d^4x^4))\cosh(c+dx))+dx(ad^2(6+d^2x^2)+b(120+20d^2x^2+d^4x^4))\sinh(c+dx)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-((3*a*d^2*(2 + d^2*x^2) + 5*b*(24 + 12*d^2*x^2 + d^4*x^4))*Cosh[c + d*x])
 + d*x*(a*d^2*(6 + d^2*x^2) + b*(120 + 20*d^2*x^2 + d^4*x^4))*Sinh[c + d*x]
)/d^6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 446 vs. 2(139) = 278.
time = 0.69, size = 447, normalized size = 3.22 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(-1/d^2*b*c^5*sinh(d*x+c)+5/d^2*b*c^4*((d*x+c)*sinh(d*x+c)-cosh(d*x+c
))-10/d^2*b*c^3*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))
+10/d^2*b*c^2*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh
(d*x+c)-6*cosh(d*x+c))-5/d^2*b*c*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*
x+c)+12*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))+1/d^2*
b*((d*x+c)^5*sinh(d*x+c)-5*(d*x+c)^4*cosh(d*x+c)+20*(d*x+c)^3*sinh(d*x+c)-6
```

```
0*(d*x+c)^2*cosh(d*x+c)+120*(d*x+c)*sinh(d*x+c)-120*cosh(d*x+c))-a*c^3*sinh
(d*x+c)+3*a*c^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-3*a*c*((d*x+c)^2*sinh(d*x
+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+a*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)
^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c)))
```

Maxima [A]
time = 0.26, size = 250, normalized size = 1.80

$$-\frac{1}{24}\,d\left(\frac{3\,(d^4x^4e^c - 4\,d^3x^3e^c + 12\,d^2x^2e^c - 24\,dxe^c + 24\,e^c)ae^{(dx)}}{d^5} + \frac{3\,(d^4x^4 + 4\,d^3x^3 + 12\,d^2x^2 + 24\,dx + 24)ae^{(-dx-c)}}{d^5} + \frac{2\,(d^6x^6e^c - 6\,d^5x^5e^c + 30\,d^4x^4e^c - 120\,d^3x^3e^c + 360\,d^2x^2e^c - 720\,dxe^c + 720\,e^c)be^{(dx)}}{d^7} + \frac{2\,(d^6x^6 + 6\,d^5x^5 + 30\,d^4x^4 + 120\,d^3x^3 + 360\,d^2x^2 + 720\,dx + 720)be^{(-dx-c)}}{d^7}\right) + \frac{1}{12}\,(2\,bx^6 + 3\,ax^4)\cosh\left(dx + c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/24*d*(3*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*
e^c)*a*e^(d*x)/d^5 + 3*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a*e
^(-d*x - c)/d^5 + 2*(d^6*x^6*e^c - 6*d^5*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3
*x^3*e^c + 360*d^2*x^2*e^c - 720*d*x*e^c + 720*e^c)*b*e^(d*x)/d^7 + 2*(d^6*
x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 120*d^3*x^3 + 360*d^2*x^2 + 720*d*x + 720)*b
*e^(-d*x - c)/d^7) + 1/12*(2*b*x^6 + 3*a*x^4)*cosh(d*x + c)
```

Fricas [A]
time = 0.42, size = 95, normalized size = 0.68

$$-\frac{(5\,bd^4x^4 + 6\,ad^2 + 3\,(ad^4 + 20\,bd^2)x^2 + 120\,b)\cosh\left(dx + c\right) - (bd^5x^5 + (ad^5 + 20\,bd^3)x^3 + 6\,(ad^3 + 20\,bd)x)\sinh\left(dx + c\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((5*b*d^4*x^4 + 6*a*d^2 + 3*(a*d^4 + 20*b*d^2)*x^2 + 120*b)*cosh(d*x + c)
- (b*d^5*x^5 + (a*d^5 + 20*b*d^3)*x^3 + 6*(a*d^3 + 20*b*d)*x)*sinh(d*x + c)
)/d^6
```

Sympy [A]
time = 0.49, size = 168, normalized size = 1.21

$$\begin{cases} \frac{ax^3\sinh\left(c+dx\right)}{d} - \frac{3ax^2\cosh\left(c+dx\right)}{d^2} + \frac{6ax\sinh\left(c+dx\right)}{d^3} - \frac{6a\cosh\left(c+dx\right)}{d^4} + \frac{bx^5\sinh\left(c+dx\right)}{d} - \frac{5bx^4\cosh\left(c+dx\right)}{d^2} + \frac{20bx^3\sinh\left(c+dx\right)}{d^3} - \frac{60bx^2\cosh\left(c+dx\right)}{d^4} + \frac{120bx\sinh\left(c+dx\right)}{d^5} - \frac{120b\cosh\left(c+dx\right)}{d^6} & \text{for}\: d \neq 0 \\ \left(\frac{ax^4}{4} + \frac{bx^6}{6}\right)\cosh\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x**3*sinh(c + d*x)/d - 3*a*x**2*cosh(c + d*x)/d**2 + 6*a*x*sin
h(c + d*x)/d**3 - 6*a*cosh(c + d*x)/d**4 + b*x**5*sinh(c + d*x)/d - 5*b*x**
4*cosh(c + d*x)/d**2 + 20*b*x**3*sinh(c + d*x)/d**3 - 60*b*x**2*cosh(c + d*
x)/d**4 + 120*b*x*sinh(c + d*x)/d**5 - 120*b*cosh(c + d*x)/d**6, Ne(d, 0)),
 ((a*x**4/4 + b*x**6/6)*cosh(c), True))
```

Giac [A]
time = 0.42, size = 174, normalized size = 1.25

$$\frac{\left(bd^{5}x^{5}+ad^{5}x^{3}-5\,bd^{4}x^{4}-3\,ad^{4}x^{2}+20\,bd^{3}x^{3}+6\,ad^{3}x-60\,bd^{2}x^{2}-6\,ad^{2}+120\,bdx-120\,b\right)e^{(dx+c)}}{2\,d^{6}}-\frac{\left(bd^{5}x^{5}+ad^{5}x^{3}+5\,bd^{4}x^{4}+3\,ad^{4}x^{2}+20\,bd^{3}x^{3}+6\,ad^{3}x+60\,bd^{2}x^{2}+6\,ad^{2}+120\,bdx+120\,b\right)e^{(-dx-c)}}{2\,d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^5*x^5 + a*d^5*x^3 - 5*b*d^4*x^4 - 3*a*d^4*x^2 + 20*b*d^3*x^3 + 6*a
*d^3*x - 60*b*d^2*x^2 - 6*a*d^2 + 120*b*d*x - 120*b)*e^(d*x + c)/d^6 - 1/2*
(b*d^5*x^5 + a*d^5*x^3 + 5*b*d^4*x^4 + 3*a*d^4*x^2 + 20*b*d^3*x^3 + 6*a*d^3
*x + 60*b*d^2*x^2 + 6*a*d^2 + 120*b*d*x + 120*b)*e^(-d*x - c)/d^6
```

Mupad [B]
time = 0.99, size = 116, normalized size = 0.83

$$\frac{x^{3}\sinh(c+d\,x)\,\left(a\,d^{2}+20\,b\right)}{d^{3}}-\frac{3\,x^{2}\cosh(c+d\,x)\,\left(a\,d^{2}+20\,b\right)}{d^{4}}-\frac{6\cosh(c+d\,x)\,\left(a\,d^{2}+20\,b\right)}{d^{6}}+\frac{6\,x\sinh(c+d\,x)\,\left(a\,d^{2}+20\,b\right)}{d^{5}}-\frac{5\,b\,x^{4}\cosh(c+d\,x)}{d^{2}}+\frac{b\,x^{5}\sinh(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(c + d*x)*(a + b*x^2),x)
```

```
[Out] (x^3*sinh(c + d*x)*(20*b + a*d^2))/d^3 - (3*x^2*cosh(c + d*x)*(20*b + a*d^2
))/d^4 - (6*cosh(c + d*x)*(20*b + a*d^2))/d^6 + (6*x*sinh(c + d*x)*(20*b +
a*d^2))/d^5 - (5*b*x^4*cosh(c + d*x))/d^2 + (b*x^5*sinh(c + d*x))/d
```

3.41 $\int x^2(a+bx^2)\cosh(c+dx)\,dx$

Optimal. Leaf size=109

$$-\frac{24bx\cosh(c+dx)}{d^4}-\frac{2ax\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{24b\sinh(c+dx)}{d^5}+\frac{2a\sinh(c+dx)}{d^3}+\frac{12bx^2\sinh}{d}$$

```
[Out] -24*b*x*cosh(d*x+c)/d^4-2*a*x*cosh(d*x+c)/d^2-4*b*x^3*cosh(d*x+c)/d^2+24*b*
sinh(d*x+c)/d^5+2*a*sinh(d*x+c)/d^3+12*b*x^2*sinh(d*x+c)/d^3+a*x^2*sinh(d*x
+c)/d+b*x^4*sinh(d*x+c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5395, 3377, 2717}

$$\frac{2a\sinh(c+dx)}{d^3}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{ax^2\sinh(c+dx)}{d}+\frac{24b\sinh(c+dx)}{d^5}-\frac{24bx\cosh(c+dx)}{d^4}+\frac{12bx^2\sinh(c+dx)}{d^3}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{bx^4\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-24*b*x*Cosh[c + d*x])/d^4 - (2*a*x*Cosh[c + d*x])/d^2 - (4*b*x^3*Cosh[c +
 d*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (2*a*Sinh[c + d*x])/d^3 + (12*b*x^2
*Sinh[c + d*x])/d^3 + (a*x^2*Sinh[c + d*x])/d + (b*x^4*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2(a+bx^2)\cosh(c+dx)\,dx = \int \left(ax^2\cosh(c+dx)+bx^4\cosh(c+dx)\right)\,dx$$
$$= a\int x^2\cosh(c+dx)\,dx + b\int x^4\cosh(c+dx)\,dx$$
$$= \frac{ax^2\sinh(c+dx)}{d} + \frac{bx^4\sinh(c+dx)}{d} - \frac{(2a)\int x\sinh(c+dx)\,dx}{d} - \frac{(4b)\int x$$
$$= -\frac{2ax\cosh(c+dx)}{d^2} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{ax^2\sinh(c+dx)}{d} + \frac{bx^4\sinh(c+$$
$$= -\frac{2ax\cosh(c+dx)}{d^2} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{2a\sinh(c+dx)}{d^3} + \frac{12bx^2\sinh(c$$
$$= -\frac{24bx\cosh(c+dx)}{d^4} - \frac{2ax\cosh(c+dx)}{d^2} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{2a\sinh(c$$
$$= -\frac{24bx\cosh(c+dx)}{d^4} - \frac{2ax\cosh(c+dx)}{d^2} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{24b\sinh(c$$

Mathematica [A]
time = 0.10, size = 74, normalized size = 0.68

$$\frac{-2dx(ad^2+2b(6+d^2x^2))\cosh(c+dx)+(ad^2(2+d^2x^2)+b(24+12d^2x^2+d^4x^4))\sinh(c+dx)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-2*d*x*(a*d^2 + 2*b*(6 + d^2*x^2))*Cosh[c + d*x] + (a*d^2*(2 + d^2*x^2) +
b*(24 + 12*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(109) = 218$.
time = 0.66, size = 298, normalized size = 2.73

method	result
risch	$\frac{(b x^4 d^4+a d^4 x^2-4b d^3 x^3-2a d^3 x+12b d^2 x^2+2a d^2-24b d x+24b)e^{dx+c}}{2d^5} - \frac{(b x^4 d^4+a d^4 x^2+4b d^3 x^3+2a d^3 x+12b d^2 x^2+}{2d^5}$
meijerg	$-\frac{16ib\cosh(c)\sqrt{\pi}\left(-\frac{ixd\left(\frac{5d^2x^2}{2}+15\right)\cosh(dx)}{10\sqrt{\pi}}+\frac{i\left(\frac{5}{8}d^4x^4+\frac{15}{2}d^2x^2+15\right)\sinh(dx)}{10\sqrt{\pi}}\right)}{d^5} - \frac{16b\sinh(c)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d\right.}{}\right.}{}$
derivativedivides	$\frac{b c^4\sinh(dx+c)}{d^2} - \frac{4b c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^2} + \frac{6b c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2} - \frac{4bc\left((dx+\right.}{}$
default	$\frac{b c^4\sinh(dx+c)}{d^2} - \frac{4b c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^2} + \frac{6b c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2} - \frac{4bc\left((dx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(1/d^2*b*c^4*sinh(d*x+c)-4/d^2*b*c^3*((d*x+c)*sinh(d*x+c)-cosh(d*x+c)
)+6/d^2*b*c^2*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-4
/d^2*b*c*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+
c)-6*cosh(d*x+c))+1/d^2*b*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12
*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))+a*c^2*sinh(d*
x+c)-2*a*c*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+a*((d*x+c)^2*sinh(d*x+c)-2*(d*
x+c)*cosh(d*x+c)+2*sinh(d*x+c)))
```

Maxima [A]
time = 0.27, size = 214, normalized size = 1.96

$$-\frac{1}{30}\,d\left(\frac{5\,(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c)ae^{(dx)}}{d^4}+\frac{5\,(d^3x^3+3\,d^2x^2+6\,dx+6)ae^{(-dx-c)}}{d^4}+\frac{3\,(d^5x^5e^c-5\,d^4x^4e^c+20\,d^3x^3e^c-60\,d^2x^2e^c+120\,dxe^c-120\,e^c)be^{(dx)}}{d^6}+\frac{3\,(d^5x^5+5\,d^4x^4+20\,d^3x^3+60\,d^2x^2+120\,dx+120)be^{(-dx-c)}}{d^6}\right)+\frac{1}{15}\,(3\,bx^5+5\,ax^3)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/30*d*(5*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*e^(d*x)/d^4
+ 5*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*e^(-d*x - c)/d^4 + 3*(d^5*x^5*e^c -
 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*b
*e^(d*x)/d^6 + 3*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x +
 120)*b*e^(-d*x - c)/d^6) + 1/15*(3*b*x^5 + 5*a*x^3)*cosh(d*x + c)
```

Fricas [A]
time = 0.39, size = 78, normalized size = 0.72

$$-\frac{2\,(2\,bd^3x^3+(ad^3+12\,bd)x)\cosh\left(dx+c\right)-(bd^4x^4+2\,ad^2+(ad^4+12\,bd^2)x^2+24\,b)\sinh\left(dx+c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(2*(2*b*d^3*x^3 + (a*d^3 + 12*b*d)*x)*cosh(d*x + c) - (b*d^4*x^4 + 2*a*d^2
 + (a*d^4 + 12*b*d^2)*x^2 + 24*b)*sinh(d*x + c))/d^5
```

Sympy [A]
time = 0.31, size = 134, normalized size = 1.23

$$\begin{cases}\frac{ax^2\sinh\left(c+dx\right)}{d}-\frac{2ax\cosh\left(c+dx\right)}{d^2}+\frac{2a\sinh\left(c+dx\right)}{d^3}+\frac{bx^4\sinh\left(c+dx\right)}{d}-\frac{4bx^3\cosh\left(c+dx\right)}{d^2}+\frac{12bx^2\sinh\left(c+dx\right)}{d^3}-\frac{24bx\cosh\left(c+dx\right)}{d^4}+\frac{24b\sinh\left(c+dx\right)}{d^5} & \text{for}\ d\neq 0\\ \left(\frac{ax^3}{3}+\frac{bx^5}{5}\right)\cosh\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x**2*sinh(c + d*x)/d - 2*a*x*cosh(c + d*x)/d**2 + 2*a*sinh(c +
 d*x)/d**3 + b*x**4*sinh(c + d*x)/d - 4*b*x**3*cosh(c + d*x)/d**2 + 12*b*x*
*2*sinh(c + d*x)/d**3 - 24*b*x*cosh(c + d*x)/d**4 + 24*b*sinh(c + d*x)/d**5
, Ne(d, 0)), ((a*x**3/3 + b*x**5/5)*cosh(c), True))
```

Giac [A]
time = 0.42, size = 138, normalized size = 1.27

$$\frac{(bd^4x^4 + ad^4x^2 - 4\,bd^3x^3 - 2\,ad^3x + 12\,bd^2x^2 + 2\,ad^2 - 24\,bdx + 24\,b)e^{(dx+c)}}{2\,d^5} - \frac{(bd^4x^4 + ad^4x^2 + 4\,bd^3x^3 + 2\,ad^3x + 12\,bd^2x^2 + 2\,ad^2 + 24\,bdx + 24\,b)e^{(-dx-c)}}{2\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^4*x^4 + a*d^4*x^2 - 4*b*d^3*x^3 - 2*a*d^3*x + 12*b*d^2*x^2 + 2*a*d
^2 - 24*b*d*x + 24*b)*e^(d*x + c)/d^5 - 1/2*(b*d^4*x^4 + a*d^4*x^2 + 4*b*d^
3*x^3 + 2*a*d^3*x + 12*b*d^2*x^2 + 2*a*d^2 + 24*b*d*x + 24*b)*e^(-d*x - c)/
d^5
```

Mupad [B]
time = 0.93, size = 93, normalized size = 0.85

$$\frac{2\sinh(c+d\,x)\,\left(a\,d^2+12\,b\right)}{d^5}+\frac{x^2\sinh(c+d\,x)\,\left(a\,d^2+12\,b\right)}{d^3}-\frac{2\,x\cosh(c+d\,x)\,\left(a\,d^2+12\,b\right)}{d^4}-\frac{4\,b\,x^3\cosh(c+d\,x)}{d^2}+\frac{b\,x^4\sinh(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(c + d*x)*(a + b*x^2),x)
```

```
[Out] (2*sinh(c + d*x)*(12*b + a*d^2))/d^5 + (x^2*sinh(c + d*x)*(12*b + a*d^2))/d
^3 - (2*x*cosh(c + d*x)*(12*b + a*d^2))/d^4 - (4*b*x^3*cosh(c + d*x))/d^2 +
 (b*x^4*sinh(c + d*x))/d
```

3.42 $\int x(a+bx^2)\cosh(c+dx)\,dx$

Optimal. Leaf size=79

$$-\frac{6b\cosh(c+dx)}{d^4}-\frac{a\cosh(c+dx)}{d^2}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{6bx\sinh(c+dx)}{d^3}+\frac{ax\sinh(c+dx)}{d}+\frac{bx^3\sinh(c+d$$

```
[Out] -6*b*cosh(d*x+c)/d^4-a*cosh(d*x+c)/d^2-3*b*x^2*cosh(d*x+c)/d^2+6*b*x*sinh(d
*x+c)/d^3+a*x*sinh(d*x+c)/d+b*x^3*sinh(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5395, 3377, 2718}

$$-\frac{a\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}-\frac{6b\cosh(c+dx)}{d^4}+\frac{6bx\sinh(c+dx)}{d^3}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{bx^3\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-6*b*Cosh[c + d*x])/d^4 - (a*Cosh[c + d*x])/d^2 - (3*b*x^2*Cosh[c + d*x])/
d^2 + (6*b*x*Sinh[c + d*x])/d^3 + (a*x*Sinh[c + d*x])/d + (b*x^3*Sinh[c + d
*x])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x(a+bx^2)\cosh(c+dx)\,dx = \int \left(ax\cosh(c+dx)+bx^3\cosh(c+dx)\right)\,dx$$

$$= a\int x\cosh(c+dx)\,dx + b\int x^3\cosh(c+dx)\,dx$$

$$= \frac{ax\sinh(c+dx)}{d} + \frac{bx^3\sinh(c+dx)}{d} - \frac{a\int \sinh(c+dx)\,dx}{d} - \frac{(3b)\int x^2\sinh(}{d}$$

$$= -\frac{a\cosh(c+dx)}{d^2} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{ax\sinh(c+dx)}{d} + \frac{bx^3\sinh(c+dx)}{d}$$

$$= -\frac{a\cosh(c+dx)}{d^2} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{6bx\sinh(c+dx)}{d^3} + \frac{ax\sinh(c+dx)}{d}$$

$$= -\frac{6b\cosh(c+dx)}{d^4} - \frac{a\cosh(c+dx)}{d^2} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{6bx\sinh(c+dx)}{d^3}$$

Mathematica [A]
time = 0.09, size = 57, normalized size = 0.72

$$\frac{-((ad^2+3b(2+d^2x^2))\cosh(c+dx))+dx(ad^2+b(6+d^2x^2))\sinh(c+dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-((a*d^2 + 3*b*(2 + d^2*x^2))*Cosh[c + d*x]) + d*x*(a*d^2 + b*(6 + d^2*x^2
))*Sinh[c + d*x])/d^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(79) = 158.
time = 0.65, size = 183, normalized size = 2.32

method	result
risch	$\frac{(b\,d^3x^3+a\,d^3x-3b\,d^2x^2-a\,d^2+6bdx-6b)\,\mathrm{e}^{dx+c}}{2d^4} - \frac{(b\,d^3x^3+a\,d^3x+3b\,d^2x^2+a\,d^2+6bdx+6b)\,\mathrm{e}^{-dx-c}}{2d^4}$
meijerg	$\frac{8b\cosh(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(\frac{3d^2x^2}{2}+3\right)\cosh(dx)}{4\sqrt{\pi}}+\frac{dx\left(\frac{d^2x^2}{2}+3\right)\sinh(dx)}{4\sqrt{\pi}}\right)}{d^4} - \frac{8ib\sinh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{5d^2x^2}{2}+15\right)\cosh(dx)}{20\sqrt{\pi}}\right.}{d^4}$
derivativedivides	$\frac{3b\,c^2((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^2} - \frac{3bc\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2} + \frac{b\left((dx+c)^3\sinh(dx+c)-3(dx\right.}{d^2}$
default	$\frac{3b\,c^2((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^2} - \frac{3bc\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2} + \frac{b\left((dx+c)^3\sinh(dx+c)-3(dx\right.}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(3/d^2*b*c^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-3/d^2*b*c*((d*x+c)^2*s
inh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+1/d^2*b*((d*x+c)^3*sinh(d*x
+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))+a*((d*x+c)
*sinh(d*x+c)-cosh(d*x+c))-1/d^2*b*c^3*sinh(d*x+c)-c*a*sinh(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(79) = 158$.
time = 0.27, size = 213, normalized size = 2.70

$$\frac{(bx^2+a)^2\cosh(dx+c)}{4\,b} - \frac{\left(\frac{a^2e^{(dx+c)}}{d} + \frac{a^2e^{(-dx-c)}}{d} + \frac{2\,(d^2x^2e^c - 2\,dxe^c + 2\,e^c)abe^{(dx)}}{d^3} + \frac{2\,(d^2x^2 + 2\,dx + 2)abe^{(-dx-c)}}{d^3} + \frac{(d^4x^4e^c - 4\,d^3x^3e^c + 12\,d^2x^2e^c - 24\,dxe^c + 24\,e^c)b^2e^{(dx)}}{d^5} + \frac{(d^4x^4 + 4\,d^3x^3 + 12\,d^2x^2 + 24\,dx + 24)b^2e^{(-dx-c)}}{d^5}\right)d}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^2*cosh(d*x + c)/b - 1/8*(a^2*e^(d*x + c)/d + a^2*e^(-d*x -
c)/d + 2*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*b*e^(d*x)/d^3 + 2*(d^2*x^2 + 2
*d*x + 2)*a*b*e^(-d*x - c)/d^3 + (d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*
e^c - 24*d*x*e^c + 24*e^c)*b^2*e^(d*x)/d^5 + (d^4*x^4 + 4*d^3*x^3 + 12*d^2*
x^2 + 24*d*x + 24)*b^2*e^(-d*x - c)/d^5)*d/b
```

Fricas [A]
time = 0.37, size = 60, normalized size = 0.76

$$-\frac{(3\,bd^2x^2 + ad^2 + 6\,b)\cosh(dx+c) - (bd^3x^3 + (ad^3 + 6\,bd)x)\sinh(dx+c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((3*b*d^2*x^2 + a*d^2 + 6*b)*cosh(d*x + c) - (b*d^3*x^3 + (a*d^3 + 6*b*d)*
x)*sinh(d*x + c))/d^4
```

Sympy [A]
time = 0.22, size = 99, normalized size = 1.25

$$\begin{cases} \frac{ax\sinh(c+dx)}{d} - \frac{a\cosh(c+dx)}{d^2} + \frac{bx^3\sinh(c+dx)}{d} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{6bx\sinh(c+dx)}{d^3} - \frac{6b\cosh(c+dx)}{d^4} & \text{for } d \neq 0 \\ \left(\frac{ax^2}{2} + \frac{bx^4}{4}\right)\cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x*sinh(c + d*x)/d - a*cosh(c + d*x)/d**2 + b*x**3*sinh(c + d*x
)/d - 3*b*x**2*cosh(c + d*x)/d**2 + 6*b*x*sinh(c + d*x)/d**3 - 6*b*cosh(c +
 d*x)/d**4, Ne(d, 0)), ((a*x**2/2 + b*x**4/4)*cosh(c), True))
```

Giac [A]
time = 0.44, size = 101, normalized size = 1.28

$$\frac{(bd^3x^3 + ad^3x - 3\,bd^2x^2 - ad^2 + 6\,bdx - 6\,b)e^{(dx+c)}}{2\,d^4} - \frac{(bd^3x^3 + ad^3x + 3\,bd^2x^2 + ad^2 + 6\,bdx + 6\,b)e^{(-dx-c)}}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^3*x^3 + a*d^3*x - 3*b*d^2*x^2 - a*d^2 + 6*b*d*x - 6*b)*e^(d*x + c)
/d^4 - 1/2*(b*d^3*x^3 + a*d^3*x + 3*b*d^2*x^2 + a*d^2 + 6*b*d*x + 6*b)*e^(-
d*x - c)/d^4
```

Mupad [B]
time = 0.98, size = 70, normalized size = 0.89

$$\frac{x\sinh(c+d\,x)\,\left(a\,d^2+6\,b\right)}{d^3} - \frac{\cosh(c+d\,x)\,\left(a\,d^2+6\,b\right)}{d^4} - \frac{3\,b\,x^2\cosh(c+d\,x)}{d^2} + \frac{b\,x^3\sinh(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(c + d*x)*(a + b*x^2),x)
```

```
[Out] (x*sinh(c + d*x)*(6*b + a*d^2))/d^3 - (cosh(c + d*x)*(6*b + a*d^2))/d^4 - (
3*b*x^2*cosh(c + d*x))/d^2 + (b*x^3*sinh(c + d*x))/d
```

3.43 $\int (a+bx^2)\cosh(c+dx)\,dx$

Optimal. Leaf size=51

$$-\frac{2bx\cosh(c+dx)}{d^2}+\frac{2b\sinh(c+dx)}{d^3}+\frac{a\sinh(c+dx)}{d}+\frac{bx^2\sinh(c+dx)}{d}$$

```
[Out] -2*b*x*cosh(d*x+c)/d^2+2*b*sinh(d*x+c)/d^3+a*sinh(d*x+c)/d+b*x^2*sinh(d*x+c
)/d
```

Rubi [A]
time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5385, 2717, 3377}

$$\frac{a\sinh(c+dx)}{d}+\frac{2b\sinh(c+dx)}{d^3}-\frac{2bx\cosh(c+dx)}{d^2}+\frac{bx^2\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-2*b*x*Cosh[c + d*x])/d^2 + (2*b*Sinh[c + d*x])/d^3 + (a*Sinh[c + d*x])/d
+ (b*x^2*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5385

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c,
d, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+bx^2\right)\cosh(c+dx)\,dx &= \int \left(a\cosh(c+dx)+bx^2\cosh(c+dx)\right)\,dx \\
&= a\int \cosh(c+dx)\,dx + b\int x^2\cosh(c+dx)\,dx \\
&= \frac{a\sinh(c+dx)}{d}+\frac{bx^2\sinh(c+dx)}{d}-\frac{(2b)\int x\sinh(c+dx)\,dx}{d} \\
&= -\frac{2bx\cosh(c+dx)}{d^2}+\frac{a\sinh(c+dx)}{d}+\frac{bx^2\sinh(c+dx)}{d}+\frac{(2b)\int \cosh(c+dx}{d^2} \\
&= -\frac{2bx\cosh(c+dx)}{d^2}+\frac{2b\sinh(c+dx)}{d^3}+\frac{a\sinh(c+dx)}{d}+\frac{bx^2\sinh(c+dx)}{d}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 40, normalized size = 0.78

$$\frac{-2bdx\cosh(c+dx)+\left(ad^2+b\left(2+d^2x^2\right)\right)\sinh(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-2*b*d*x*Cosh[c + d*x] + (a*d^2 + b*(2 + d^2*x^2))*Sinh[c + d*x])/d^3
```

Maple [A]
time = 0.65, size = 97, normalized size = 1.90

method	result
risch	$\frac{\left(b\,d^2x^2+a\,d^2-2bdx+2b\right)\mathrm{e}^{dx+c}}{2d^3}-\frac{\left(b\,d^2x^2+a\,d^2+2bdx+2b\right)\mathrm{e}^{-dx-c}}{2d^3}$
derivativedivides	$\frac{\frac{b\,c^2\sinh(dx+c)}{d^2}-\frac{2bc((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^2}+\frac{b\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}+a\sinh(dx+c)}{d}$
default	$\frac{\frac{b\,c^2\sinh(dx+c)}{d^2}-\frac{2bc((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^2}+\frac{b\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}+a\sinh(dx+c)}{d}$
meijerg	$\frac{4ib\cosh(c)\sqrt{\pi}\left(\frac{ixd\cosh(dx)}{2\sqrt{\pi}}-\frac{i\left(\frac{3d^2x^2}{2}+3\right)\sinh(dx)}{6\sqrt{\pi}}\right)}{d^3}+\frac{4b\sinh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(\frac{d^2x^2}{2}+1\right)\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right.}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/d^2*b*c^2*sinh(d*x+c)-2/d^2*b*c*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+1/
d^2*b*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+a*sinh(d*
x+c))
```

Maxima [A]
time = 0.27, size = 86, normalized size = 1.69

$$\frac{ae^{(dx+c)}}{2\,d} - \frac{ae^{(-dx-c)}}{2\,d} + \frac{(d^2x^2e^c - 2\,dxe^c + 2\,e^c)be^{(dx)}}{2\,d^3} - \frac{(d^2x^2 + 2\,dx + 2)be^{(-dx-c)}}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a*e^(d*x + c)/d - 1/2*a*e^(-d*x - c)/d + 1/2*(d^2*x^2*e^c - 2*d*x*e^c +
 2*e^c)*b*e^(d*x)/d^3 - 1/2*(d^2*x^2 + 2*d*x + 2)*b*e^(-d*x - c)/d^3
```

Fricas [A]
time = 0.52, size = 42, normalized size = 0.82

$$-\frac{2\,bdx\cosh\left(dx+c\right) - \left(bd^2x^2 + ad^2 + 2\,b\right)\sinh\left(dx+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(2*b*d*x*cosh(d*x + c) - (b*d^2*x^2 + a*d^2 + 2*b)*sinh(d*x + c))/d^3
```

Sympy [A]
time = 0.13, size = 65, normalized size = 1.27

$$\begin{cases} \frac{a\sinh(c+dx)}{d} + \frac{bx^2\sinh(c+dx)}{d} - \frac{2bx\cosh(c+dx)}{d^2} + \frac{2b\sinh(c+dx)}{d^3} & \text{for}\ d \neq 0 \\ \left(ax + \frac{bx^3}{3}\right)\cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)/d + b*x**2*sinh(c + d*x)/d - 2*b*x*cosh(c + d*x)
/d**2 + 2*b*sinh(c + d*x)/d**3, Ne(d, 0)), ((a*x + b*x**3/3)*cosh(c), True)
)
```

Giac [A]
time = 0.41, size = 70, normalized size = 1.37

$$\frac{(bd^2x^2 + ad^2 - 2\,bdx + 2\,b)e^{(dx+c)}}{2\,d^3} - \frac{(bd^2x^2 + ad^2 + 2\,bdx + 2\,b)e^{(-dx-c)}}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c),x, algorithm="giac")
```

[Out] 1/2*(b*d^2*x^2 + a*d^2 - 2*b*d*x + 2*b)*e^(d*x + c)/d^3 - 1/2*(b*d^2*x^2 + a*d^2 + 2*b*d*x + 2*b)*e^(-d*x - c)/d^3

Mupad [B]
time = 0.93, size = 47, normalized size = 0.92

$$\frac{\sinh(c+d\,x)\,\left(a\,d^2+2\,b\right)}{d^3}-\frac{2\,b\,x\cosh(c+d\,x)}{d^2}+\frac{b\,x^2\sinh(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)*(a + b*x^2),x)

[Out] (sinh(c + d*x)*(2*b + a*d^2))/d^3 - (2*b*x*cosh(c + d*x))/d^2 + (b*x^2*sinh(c + d*x))/d

3.44 $\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx$

Optimal. Leaf size=41

$$-\frac{b\cosh(c+dx)}{d^2}+a\cosh(c)\mathrm{Chi}(dx)+\frac{bx\sinh(c+dx)}{d}+a\sinh(c)\mathrm{Shi}(dx)$$

```
[Out] a*Chi(d*x)*cosh(c)-b*cosh(d*x+c)/d^2+a*Shi(d*x)*sinh(c)+b*x*sinh(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.353$, Rules used = {5395, 3384, 3379, 3382, 3377, 2718}

$$a\cosh(c)\mathrm{Chi}(dx)+a\sinh(c)\mathrm{Shi}(dx)-\frac{b\cosh(c+dx)}{d^2}+\frac{bx\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Cosh[c + d*x])/x,x]
```

```
[Out] -((b*Cosh[c + d*x])/d^2) + a*Cosh[c]*CoshIntegral[d*x] + (b*x*Sinh[c + d*x]
)/d + a*Sinh[c]*SinhIntegral[d*x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx &= \int \left(\frac{a\cosh(c+dx)}{x}+bx\cosh(c+dx)\right)dx\\ &= a\int \frac{\cosh(c+dx)}{x}\,dx+b\int x\cosh(c+dx)\,dx\\ &= \frac{bx\sinh(c+dx)}{d}-\frac{b\int \sinh(c+dx)\,dx}{d}+(a\cosh(c))\int \frac{\cosh(dx)}{x}\,dx+(a\sinh\\ &= -\frac{b\cosh(c+dx)}{d^2}+a\cosh(c)\mathrm{Chi}(dx)+\frac{bx\sinh(c+dx)}{d}+a\sinh(c)\mathrm{Shi}(dx)\end{aligned}$$

Mathematica [A]
time = 0.09, size = 55, normalized size = 1.34

$$a\cosh(c)\mathrm{Chi}(dx)+\frac{b\cosh(dx)(-\cosh(c)+dx\sinh(c))}{d^2}+\frac{b(dx\cosh(c)-\sinh(c))\sinh(dx)}{d^2}+a\sinh(c)\mathrm{Shi}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x,x]
[Out] a*Cosh[c]*CoshIntegral[d*x] + (b*Cosh[d*x]*(-Cosh[c] + d*x*Sinh[c]))/d^2 +
(b*(d*x*Cosh[c] - Sinh[c])*Sinh[d*x])/d^2 + a*Sinh[c]*SinhIntegral[d*x]
```

Maple [A]
time = 0.91, size = 81, normalized size = 1.98

method	result
risch	$-\frac{a\,e^{-c}\,\mathrm{expIntegral}(1,dx)}{2}-\frac{b\,e^{-dx-c}x}{2d}-\frac{b\,e^{-dx-c}}{2d^2}-\frac{a\,e^{c}\,\mathrm{expIntegral}(1,-dx)}{2}+\frac{b\,e^{dx+c}x}{2d}-\frac{b\,e^{dx+c}}{2d^2}$
meijerg	$-\frac{2b\cosh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{b\sinh(c)(\cosh(dx)dx-\sinh(dx))}{d^2}+\frac{a\cosh(c)\sqrt{\pi}\left(\frac{2\,\mathrm{hyperbolicCosineInte}}{\sqrt{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*cosh(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*exp(-c)*Ei(1,d*x)-1/2/d*b*exp(-d*x-c)*x-1/2/d^2*b*exp(-d*x-c)-1/2*a*
exp(c)*Ei(1,-d*x)+1/2/d*b*exp(d*x+c)*x-1/2/d^2*b*exp(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(41) = 82$.
time = 0.31, size = 122, normalized size = 2.98

$$-\frac{1}{4}\left(b\left(\frac{(d^2x^2e^c-2\,dxe^c+2\,e^c)e^{(dx)}}{d^3}+\frac{(d^2x^2+2\,dx+2)e^{(-dx-c)}}{d^3}\right)+\frac{2\,a\cosh\left(dx+c\right)\log\left(x^2\right)}{d}-\frac{2\left(\mathrm{Ei}(-dx)\,e^{(-c)}+\mathrm{Ei}(dx)\,e^c\right)a}{d}\right)d+\frac{1}{2}\left(bx^2+a\log\left(x^2\right)\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(b*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x +
 2)*e^(-d*x - c)/d^3) + 2*a*cosh(d*x + c)*log(x^2)/d - 2*(Ei(-d*x)*e^(-c) +
 Ei(d*x)*e^c)*a/d)*d + 1/2*(b*x^2 + a*log(x^2))*cosh(d*x + c)
```

Fricas [A]
time = 0.40, size = 73, normalized size = 1.78

$$\frac{2\,bdx\sinh\left(dx+c\right)-2\,b\cosh\left(dx+c\right)+\left(ad^2\mathrm{Ei}(dx)+ad^2\mathrm{Ei}(-dx)\right)\cosh\left(c\right)+\left(ad^2\mathrm{Ei}(dx)-ad^2\mathrm{Ei}(-dx)\right)\sinh\left(c\right)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x*sinh(d*x + c) - 2*b*cosh(d*x + c) + (a*d^2*Ei(d*x) + a*d^2*Ei(
-d*x))*cosh(c) + (a*d^2*Ei(d*x) - a*d^2*Ei(-d*x))*sinh(c))/d^2
```

Sympy [A]
time = 1.94, size = 49, normalized size = 1.20

$$a\sinh\left(c\right)\operatorname{Shi}\left(dx\right)+a\cosh\left(c\right)\operatorname{Chi}\left(dx\right)+b\left(\begin{cases}\frac{x\sinh\left(c+dx\right)}{d}-\frac{\cosh\left(c+dx\right)}{d^2} & \text{for}\: d\neq 0\\ \frac{x^2\cosh\left(c\right)}{2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*cosh(d*x+c)/x,x)
```

```
[Out] a*sinh(c)*Shi(d*x) + a*cosh(c)*Chi(d*x) + b*Piecewise((x*sinh(c + d*x)/d -
cosh(c + d*x)/d**2, Ne(d, 0)), (x**2*cosh(c)/2, True))
```

Giac [A]
time = 0.40, size = 76, normalized size = 1.85

$$\frac{ad^2\mathrm{Ei}(-dx)\,e^{(-c)} + ad^2\mathrm{Ei}(dx)\,e^c + bdxe^{(dx+c)} - bdxe^{(-dx-c)} - be^{(dx+c)} - be^{(-dx-c)}}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(a*d^2*Ei(-d*x)*e^(-c) + a*d^2*Ei(d*x)*e^c + b*d*x*e^(d*x + c) - b*d*x*
e^(-d*x - c) - b*e^(d*x + c) - b*e^(-d*x - c))/d^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$a\,\mathrm{coshint}(d\,x)\,\cosh(c) + a\,\mathrm{sinhint}(d\,x)\,\sinh(c) - \frac{b\,(\cosh(c+d\,x) - d\,x\,\sinh(c+d\,x))}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^2))/x,x)
```

```
[Out] a*coshint(d*x)*cosh(c) + a*sinhint(d*x)*sinh(c) - (b*(cosh(c + d*x) - d*x*s
inh(c + d*x)))/d^2
```

3.45 $\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx$

Optimal. Leaf size=42

$$-\frac{a\cosh(c+dx)}{x}+ad\text{Chi}(dx)\sinh(c)+\frac{b\sinh(c+dx)}{d}+ad\cosh(c)\text{Shi}(dx)$$

```
[Out] -a*cosh(d*x+c)/x+a*d*cosh(c)*Shi(d*x)+a*d*Chi(d*x)*sinh(c)+b*sinh(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5395, 2717, 3378, 3384, 3379, 3382}

$$ad\sinh(c)\text{Chi}(dx)+ad\cosh(c)\text{Shi}(dx)-\frac{a\cosh(c+dx)}{x}+\frac{b\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a*Cosh[c + d*x])/x) + a*d*CoshIntegral[d*x]*Sinh[c] + (b*Sinh[c + d*x])/
d + a*d*Cosh[c]*SinhIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx &= \int \left(b\cosh(c+dx)+\frac{a\cosh(c+dx)}{x^2}\right)dx\\ &= a\int \frac{\cosh(c+dx)}{x^2}\,dx + b\int \cosh(c+dx)\,dx\\ &= -\frac{a\cosh(c+dx)}{x}+\frac{b\sinh(c+dx)}{d}+(ad)\int \frac{\sinh(c+dx)}{x}\,dx\\ &= -\frac{a\cosh(c+dx)}{x}+\frac{b\sinh(c+dx)}{d}+(ad\cosh(c))\int \frac{\sinh(dx)}{x}\,dx+(ad\sinh(c\\ &= -\frac{a\cosh(c+dx)}{x}+ad\text{Chi}(dx)\sinh(c)+\frac{b\sinh(c+dx)}{d}+ad\cosh(c)\text{Shi}(dx)\end{aligned}$$

Mathematica [**A**]
time = 0.08, size = 42, normalized size = 1.00

$$-\frac{a\cosh(c+dx)}{x}+ad\text{Chi}(dx)\sinh(c)+\frac{b\sinh(c+dx)}{d}+ad\cosh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a*Cosh[c + d*x])/x) + a*d*CoshIntegral[d*x]*Sinh[c] + (b*Sinh[c + d*x])/
d + a*d*Cosh[c]*SinhIntegral[d*x]
```

Maple [**A**]
time = 0.86, size = 81, normalized size = 1.93

method	result

risch	$-\frac{a\,\mathrm{e}^{-dx-c}}{2x}+\frac{da\,\mathrm{e}^{-c}\,\mathrm{expIntegral}(1,dx)}{2}-\frac{b\,\mathrm{e}^{-dx-c}}{2d}-\frac{a\,\mathrm{e}^{dx+c}}{2x}-\frac{da\,\mathrm{e}^{c}\,\mathrm{expIntegral}(1,-dx)}{2}+\frac{b\,\mathrm{e}^{dx+c}}{2d}$
meijerg	$\frac{b\cosh(c)\sinh(dx)}{d}-\frac{b\sinh(c)\sqrt{\pi}\,\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(dx)}{\sqrt{\pi}}\right)}{d}+\frac{ia\cosh(c)\sqrt{\pi}\,d\left(\frac{4i\cosh(dx)}{dx\sqrt{\pi}}-\frac{4i\,\mathrm{hyperbolicSineIntegral}(dx)}{\sqrt{\pi}}\right)}{4}+\frac{a\sinh}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*cosh(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*exp(-d*x-c)/x+1/2*d*a*exp(-c)*Ei(1,d*x)-1/2/d*b*exp(-d*x-c)-1/2*a/x*
exp(d*x+c)-1/2*d*a*exp(c)*Ei(1,-d*x)+1/2*b/d*exp(d*x+c)
```

Maxima [A]
time = 0.32, size = 80, normalized size = 1.90

$$-\frac{1}{2}\left(a\mathrm{Ei}(-dx)\,e^{(-c)}-a\mathrm{Ei}(dx)\,e^{c}+\frac{(dxe^{c}-e^{c})be^{(dx)}}{d^2}+\frac{(dx+1)be^{(-dx-c)}}{d^2}\right)d+\left(bx-\frac{a}{x}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*Ei(-d*x)*e^(-c) - a*Ei(d*x)*e^c + (d*x*e^c - e^c)*b*e^(d*x)/d^2 + (
d*x + 1)*b*e^(-d*x - c)/d^2)*d + (b*x - a/x)*cosh(d*x + c)
```

Fricas [A]
time = 0.47, size = 82, normalized size = 1.95

$$-\frac{2\,ad\cosh\left(dx+c\right)-2\,bx\sinh\left(dx+c\right)-\left(ad^2x\mathrm{Ei}(dx)-ad^2x\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-\left(ad^2x\mathrm{Ei}(dx)+ad^2x\mathrm{Ei}(-dx)\right)\sinh\left(c\right)}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*cosh(d*x + c) - 2*b*x*sinh(d*x + c) - (a*d^2*x*Ei(d*x) - a*d^2*
x*Ei(-d*x))*cosh(c) - (a*d^2*x*Ei(d*x) + a*d^2*x*Ei(-d*x))*sinh(c))/(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^2\right)\cosh\left(c+dx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*cosh(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**2)*cosh(c + d*x)/x**2, x)
```

Giac [A]
time = 0.40, size = 80, normalized size = 1.90

$$-\frac{ad^2x\text{Ei}(-dx)\,e^{(-c)} - ad^2x\text{Ei}(dx)\,e^c + ade^{(dx+c)} - bxe^{(dx+c)} + ade^{(-dx-c)} + bxe^{(-dx-c)}}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*d^2*x*Ei(-d*x)*e^(-c) - a*d^2*x*Ei(d*x)*e^c + a*d*e^(d*x + c) - b*x
*e^(d*x + c) + a*d*e^(-d*x - c) + b*x*e^(-d*x - c))/(d*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^2))/x^2,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^2))/x^2, x)
```

3.46 $\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx$

Optimal. Leaf size=74

$$-\frac{a\cosh(c+dx)}{2x^2}+b\cosh(c)\text{Chi}(dx)+\frac{1}{2}ad^2\cosh(c)\text{Chi}(dx)-\frac{ad\sinh(c+dx)}{2x}+b\sinh(c)\text{Shi}(dx)+\frac{1}{2}ad^2\sinh(c$$

```
[Out] b*Chi(d*x)*cosh(c)+1/2*a*d^2*Chi(d*x)*cosh(c)-1/2*a*cosh(d*x+c)/x^2+b*Shi(d
*x)*sinh(c)+1/2*a*d^2*Shi(d*x)*sinh(c)-1/2*a*d*sinh(d*x+c)/x
```

Rubi [A]
time = 0.13, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5395, 3378, 3384, 3379, 3382}

$$\frac{1}{2}ad^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}ad^2\sinh(c)\text{Shi}(dx)-\frac{a\cosh(c+dx)}{2x^2}-\frac{ad\sinh(c+dx)}{2x}+b\cosh(c)\text{Chi}(dx)+b\sinh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Cosh[c + d*x])/x^3,x]
```

```
[Out] -1/2*(a*Cosh[c + d*x])/x^2 + b*Cosh[c]*CoshIntegral[d*x] + (a*d^2*Cosh[c]*C
oshIntegral[d*x])/2 - (a*d*Sinh[c + d*x])/(2*x) + b*Sinh[c]*SinhIntegral[d*
x] + (a*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx &= \int \left(\frac{a\cosh(c+dx)}{x^3}+\frac{b\cosh(c+dx)}{x}\right)dx\\ &= a\int \frac{\cosh(c+dx)}{x^3}\,dx+b\int \frac{\cosh(c+dx)}{x}\,dx\\ &= -\frac{a\cosh(c+dx)}{2x^2}+\frac{1}{2}(ad)\int \frac{\sinh(c+dx)}{x^2}\,dx+(b\cosh(c))\int \frac{\cosh(dx)}{x}\,dx+\\ &= -\frac{a\cosh(c+dx)}{2x^2}+b\cosh(c)\mathrm{Chi}(dx)-\frac{ad\sinh(c+dx)}{2x}+b\sinh(c)\mathrm{Shi}(dx)+\\ &= -\frac{a\cosh(c+dx)}{2x^2}+b\cosh(c)\mathrm{Chi}(dx)-\frac{ad\sinh(c+dx)}{2x}+b\sinh(c)\mathrm{Shi}(dx)+\\ &= -\frac{a\cosh(c+dx)}{2x^2}+b\cosh(c)\mathrm{Chi}(dx)+\frac{1}{2}ad^2\cosh(c)\mathrm{Chi}(dx)-\frac{ad\sinh(c+d}{2x}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 80, normalized size = 1.08

$$b\cosh(c)\mathrm{Chi}(dx)-\frac{a\cosh(dx)(\cosh(c)+dx\sinh(c))}{2x^2}-\frac{a(dx\cosh(c)+\sinh(c))\sinh(dx)}{2x^2}+b\sinh(c)\mathrm{Shi}(dx)+\frac{1}{2}ad^2(\cosh(c)\mathrm{Chi}(dx)+\sinh(c)\mathrm{Shi}(dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x^3,x]
[Out] b*Cosh[c]*CoshIntegral[d*x] - (a*Cosh[d*x]*(Cosh[c] + d*x*Sinh[c]))/(2*x^2)
 - (a*(d*x*Cosh[c] + Sinh[c])*Sinh[d*x])/(2*x^2) + b*Sinh[c]*SinhIntegral[d
*x] + (a*d^2*(Cosh[c]*CoshIntegral[d*x] + Sinh[c]*SinhIntegral[d*x]))/2
```

Maple [A]
time = 0.92, size = 110, normalized size = 1.49

method	result

risch	$\frac{da\,\mathrm{e}^{-dx-c}}{4x}-\frac{a\,\mathrm{e}^{-dx-c}}{4x^{2}}-\frac{d^{2}a\,\mathrm{e}^{-c}\,\mathrm{expIntegral}(1,dx)}{4}-\frac{b\,\mathrm{e}^{-c}\,\mathrm{expIntegral}(1,dx)}{2}-\frac{a\,\mathrm{e}^{dx+c}}{4x^{2}}-\frac{da\,\mathrm{e}^{dx+c}}{4x}-\frac{d^{2}a\,\mathrm{e}^{c}\,\mathrm{expIntegral}(}{4}$
meijerg	$\frac{b\cosh(c)\sqrt{\pi}\left(\frac{2\,\mathrm{hyperbolicCosineIntegral}(dx)-2\ln(dx)-2\gamma}{\sqrt{\pi}}+\frac{2\gamma+2\ln(x)+2\ln(id)}{\sqrt{\pi}}\right)}{2}+b\,\mathrm{hyperbolicSineIntegral}\,(dx)\sinh\,($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*cosh(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d*a*exp(-d*x-c)/x-1/4*a*exp(-d*x-c)/x^2-1/4*d^2*a*exp(-c)*Ei(1,d*x)-1/2
*b*exp(-c)*Ei(1,d*x)-1/4*a/x^2*exp(d*x+c)-1/4*d*a/x*exp(d*x+c)-1/4*d^2*a*ex
p(c)*Ei(1,-d*x)-1/2*b*exp(c)*Ei(1,-d*x)
```

Maxima [A]
time = 0.35, size = 90, normalized size = 1.22

$$\frac{1}{4}\left(\left(de^{(-c)}\Gamma(-1,dx)+de^{c}\Gamma(-1,-dx)\right)a-\frac{2\,b\cosh\left(dx+c\right)\log\left(x^{2}\right)}{d}+\frac{2\left(\mathrm{Ei}(-dx)\,e^{(-c)}+\mathrm{Ei}(dx)\,e^{c}\right)b}{d}\right)d+\frac{1}{2}\left(b\log\left(x^{2}\right)-\frac{a}{x^{2}}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*((d*e^(-c)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))*a - 2*b*cosh(d*x + c
)*log(x^2)/d + 2*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b/d)*d + 1/2*(b*log(x^2) -
 a/x^2)*cosh(d*x + c)
```

Fricas [A]
time = 0.41, size = 107, normalized size = 1.45

$$-\frac{2\,adx\sinh\left(dx+c\right)+2\,a\cosh\left(dx+c\right)-\left((ad^{2}+2\,b)x^{2}\mathrm{Ei}(dx)+(ad^{2}+2\,b)x^{2}\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-\left((ad^{2}+2\,b)x^{2}\mathrm{Ei}(dx)-(ad^{2}+2\,b)x^{2}\mathrm{Ei}(-dx)\right)\sinh\left(c\right)}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d*x*sinh(d*x + c) + 2*a*cosh(d*x + c) - ((a*d^2 + 2*b)*x^2*Ei(d*x
) + (a*d^2 + 2*b)*x^2*Ei(-d*x))*cosh(c) - ((a*d^2 + 2*b)*x^2*Ei(d*x) - (a*d
^2 + 2*b)*x^2*Ei(-d*x))*sinh(c))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+bx^{2})\cosh\left(c+dx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*cosh(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**2)*cosh(c + d*x)/x**3, x)
```

Giac [A]
time = 0.42, size = 109, normalized size = 1.47

$$\frac{ad^2x^2\mathrm{Ei}(-dx)\,e^{(-c)} + ad^2x^2\mathrm{Ei}(dx)\,e^c + 2\,bx^2\mathrm{Ei}(-dx)\,e^{(-c)} + 2\,bx^2\mathrm{Ei}(dx)\,e^c - adxe^{(dx+c)} + adxe^{(-dx-c)} - ae^{(dx+c)} - ae^{(-dx-c)}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^2*x^2*Ei(-d*x)*e^(-c) + a*d^2*x^2*Ei(d*x)*e^c + 2*b*x^2*Ei(-d*x)*e
^(-c) + 2*b*x^2*Ei(d*x)*e^c - a*d*x*e^(d*x + c) + a*d*x*e^(-d*x - c) - a*e^
(d*x + c) - a*e^(-d*x - c))/x^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^2))/x^3,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^2))/x^3, x)
```

3.47 $\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx$

Optimal. Leaf size=105

$$-\frac{a\cosh(c+dx)}{3x^3}-\frac{b\cosh(c+dx)}{x}-\frac{ad^2\cosh(c+dx)}{6x}+bd\text{Chi}(dx)\sinh(c)+\frac{1}{6}ad^3\text{Chi}(dx)\sinh(c)-\frac{ad\sinh(c+$$

[Out] -1/3*a*cosh(d*x+c)/x^3-b*cosh(d*x+c)/x-1/6*a*d^2*cosh(d*x+c)/x+b*d*cosh(c)*Shi(d*x)+1/6*a*d^3*cosh(c)*Shi(d*x)+b*d*Chi(d*x)*sinh(c)+1/6*a*d^3*Chi(d*x)*sinh(c)-1/6*a*d*sinh(d*x+c)/x^2

Rubi [A]
time = 0.17, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5395, 3378, 3384, 3379, 3382}

$$\frac{1}{6}ad^3\sinh(c)\text{Chi}(dx)+\frac{1}{6}ad^3\cosh(c)\text{Shi}(dx)-\frac{ad^2\cosh(c+dx)}{6x}-\frac{a\cosh(c+dx)}{3x^3}-\frac{ad\sinh(c+dx)}{6x^2}+bd\sinh(c)\text{Chi}(dx)+bd\cosh(c)\text{Shi}(dx)-\frac{b\cosh(c+dx)}{x}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^2)*Cosh[c + d*x])/x^4,x]

[Out] -1/3*(a*Cosh[c + d*x])/x^3 - (b*Cosh[c + d*x])/x - (a*d^2*Cosh[c + d*x])/(6*x) + b*d*CoshIntegral[d*x]*Sinh[c] + (a*d^3*CoshIntegral[d*x]*Sinh[c])/6 - (a*d*Sinh[c + d*x])/(6*x^2) + b*d*Cosh[c]*SinhIntegral[d*x] + (a*d^3*Cosh[c]*SinhIntegral[d*x])/6

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx &= \int \left(\frac{a\cosh(c+dx)}{x^4}+\frac{b\cosh(c+dx)}{x^2}\right)dx \\
&= a\int \frac{\cosh(c+dx)}{x^4}\,dx + b\int \frac{\cosh(c+dx)}{x^2}\,dx \\
&= -\frac{a\cosh(c+dx)}{3x^3}-\frac{b\cosh(c+dx)}{x}+\frac{1}{3}(ad)\int \frac{\sinh(c+dx)}{x^3}\,dx+(bd)\int \frac{\text{sinh}}{} \\
&= -\frac{a\cosh(c+dx)}{3x^3}-\frac{b\cosh(c+dx)}{x}-\frac{ad\sinh(c+dx)}{6x^2}+\frac{1}{6}(ad^2)\int \frac{\cosh(c+d}{x^2} \\
&= -\frac{a\cosh(c+dx)}{3x^3}-\frac{b\cosh(c+dx)}{x}-\frac{ad^2\cosh(c+dx)}{6x}+bd\text{Chi}(dx)\sinh(c)- \\
&= -\frac{a\cosh(c+dx)}{3x^3}-\frac{b\cosh(c+dx)}{x}-\frac{ad^2\cosh(c+dx)}{6x}+bd\text{Chi}(dx)\sinh(c)- \\
&= -\frac{a\cosh(c+dx)}{3x^3}-\frac{b\cosh(c+dx)}{x}-\frac{ad^2\cosh(c+dx)}{6x}+bd\text{Chi}(dx)\sinh(c)+
\end{aligned}$$

Mathematica [A]
time = 0.18, size = 95, normalized size = 0.90

$$-\frac{2a\cosh(c+dx)+6bx^2\cosh(c+dx)+ad^2x^2\cosh(c+dx)-d(6b+ad^2)\,x^3\text{Chi}(dx)\sinh(c)+adx\sinh(c+dx)-d(6b+ad^2)\,x^3\cosh(c)\text{Shi}(dx)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x^4,x]
```

```
[Out] -1/6*(2*a*Cosh[c + d*x] + 6*b*x^2*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x] -
 d*(6*b + a*d^2)*x^3*CoshIntegral[d*x]*Sinh[c] + a*d*x*Sinh[c + d*x] - d*(6
*b + a*d^2)*x^3*Cosh[c]*SinhIntegral[d*x])/x^3
```

Maple [A]
time = 0.93, size = 172, normalized size = 1.64

method	result
risch	$-\frac{d^{2} a\,\mathrm{e}^{-d x-c}}{12 x}+\frac{d a\,\mathrm{e}^{-d x-c}}{12 x^{2}}-\frac{a\,\mathrm{e}^{-d x-c}}{6 x^{3}}+\frac{d^{3} a\,\mathrm{e}^{-c} \operatorname{expIntegral}(1,d x)}{12}-\frac{b\,\mathrm{e}^{-d x-c}}{2 x}+\frac{d b\,\mathrm{e}^{-c} \operatorname{expIntegral}(1,d x)}{2}-\frac{a\,\mathrm{e}^{d x+c}}{6 x^{3}}-$
meijerg	$\frac{i d b \cosh(c) \sqrt{\pi}\,\left(\frac{4 i \cosh(d x)}{d x \sqrt{\pi}}-\frac{4 i \,\mathrm{hyperbolicSineIntegral}(d x)}{\sqrt{\pi}}\right)}{4}+\frac{d b \sinh(c) \sqrt{\pi}\,\left(\frac{4}{\sqrt{\pi}}-\frac{4 \sinh(d x)}{\sqrt{\pi}\, d x}+\frac{4\,\mathrm{hyperbolicCosineIntegral}(d x)-4}{\sqrt{\pi}}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*cosh(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*d^2*a*exp(-d*x-c)/x+1/12*d*a*exp(-d*x-c)/x^2-1/6*a*exp(-d*x-c)/x^3+1/
12*d^3*a*exp(-c)*Ei(1,d*x)-1/2*b*exp(-d*x-c)/x+1/2*d*b*exp(-c)*Ei(1,d*x)-1/
6*a/x^3*exp(d*x+c)-1/12*d*a/x^2*exp(d*x+c)-1/12*d^2*a/x*exp(d*x+c)-1/12*d^3
*a*exp(c)*Ei(1,-d*x)-1/2*b/x*exp(d*x+c)-1/2*d*b*exp(c)*Ei(1,-d*x)
```

Maxima [A]
time = 0.35, size = 73, normalized size = 0.70

$$\frac{1}{6}\left(ad^2e^{(-c)}\Gamma(-2,dx)-ad^2e^c\Gamma(-2,-dx)-3\,b\mathrm{Ei}(-dx)\,e^{(-c)}+3\,b\mathrm{Ei}(dx)\,e^c\right)d-\frac{(3\,bx^2+a)\cosh(dx+c)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(a*d^2*e^(-c)*gamma(-2, d*x) - a*d^2*e^c*gamma(-2, -d*x) - 3*b*Ei(-d*x)
*e^(-c) + 3*b*Ei(d*x)*e^c)*d - 1/3*(3*b*x^2 + a)*cosh(d*x + c)/x^3
```

Fricas [A]
time = 0.47, size = 127, normalized size = 1.21

$$-\frac{2\,adx\sinh(dx+c)+2\left((ad^2+6\,b)x^2+2\,a\right)\cosh(dx+c)-\left((ad^3+6\,bd)x^3\mathrm{Ei}(dx)-(ad^3+6\,bd)x^3\mathrm{Ei}(-dx)\right)\cosh(c)-\left((ad^3+6\,bd)x^3\mathrm{Ei}(dx)+(ad^3+6\,bd)x^3\mathrm{Ei}(-dx)\right)\sinh(c)}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*a*d*x*sinh(d*x + c) + 2*((a*d^2 + 6*b)*x^2 + 2*a)*cosh(d*x + c) -
((a*d^3 + 6*b*d)*x^3*Ei(d*x) - (a*d^3 + 6*b*d)*x^3*Ei(-d*x))*cosh(c) - ((a*
d^3 + 6*b*d)*x^3*Ei(d*x) + (a*d^3 + 6*b*d)*x^3*Ei(-d*x))*sinh(c))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*cosh(d*x+c)/x**4,x)
[Out] Integral((a + b*x**2)*cosh(c + d*x)/x**4, x)
```

Giac [A]
time = 0.42, size = 170, normalized size = 1.62

$$-\frac{ad^3x^3\mathrm{Ei}(-dx)\,e^{(-c)} - ad^3x^3\mathrm{Ei}(dx)\,e^c + 6\,bdx^3\mathrm{Ei}(-dx)\,e^{(-c)} - 6\,bdx^3\mathrm{Ei}(dx)\,e^c + ad^2x^2e^{(dx+c)} + ad^2x^2e^{(-dx-c)} + adxe^{(dx+c)} + 6\,bx^2e^{(dx+c)} - adxe^{(-dx-c)} + 6\,bx^2e^{(-dx-c)} + 2\,ae^{(dx+c)} + 2\,ae^{(-dx-c)}}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^4,x, algorithm="giac")
[Out] -1/12*(a*d^3*x^3*Ei(-d*x)*e^(-c) - a*d^3*x^3*Ei(d*x)*e^c + 6*b*d*x^3*Ei(-d*
x)*e^(-c) - 6*b*d*x^3*Ei(d*x)*e^c + a*d^2*x^2*e^(d*x + c) + a*d^2*x^2*e^(-d
*x - c) + a*d*x*e^(d*x + c) + 6*b*x^2*e^(d*x + c) - a*d*x*e^(-d*x - c) + 6*
b*x^2*e^(-d*x - c) + 2*a*e^(d*x + c) + 2*a*e^(-d*x - c))/x^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^2))/x^4,x)
[Out] int((cosh(c + d*x)*(a + b*x^2))/x^4, x)
```

3.48 $\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx$

Optimal. Leaf size=149

$$-\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{2x^2}-\frac{ad^2\cosh(c+dx)}{24x^2}+\frac{1}{2}bd^2\cosh(c)\text{Chi}(dx)+\frac{1}{24}ad^4\cosh(c)\text{Chi}(dx)-\frac{ad\sinh}{1}$$

```
[Out] 1/2*b*d^2*Chi(d*x)*cosh(c)+1/24*a*d^4*Chi(d*x)*cosh(c)-1/4*a*cosh(d*x+c)/x^
4-1/2*b*cosh(d*x+c)/x^2-1/24*a*d^2*cosh(d*x+c)/x^2+1/2*b*d^2*Shi(d*x)*sinh(
c)+1/24*a*d^4*Shi(d*x)*sinh(c)-1/12*a*d*sinh(d*x+c)/x^3-1/2*b*d*sinh(d*x+c)
/x-1/24*a*d^3*sinh(d*x+c)/x
```

Rubi [A]
time = 0.22, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5395, 3378, 3384, 3379, 3382}

$$\frac{1}{24}ad^4\cosh(c)\text{Chi}(dx)+\frac{1}{24}ad^4\sinh(c)\text{Shi}(dx)-\frac{ad^3\sinh(c+dx)}{24x}-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{a\cosh(c+dx)}{4x^4}-\frac{ad\sinh(c+dx)}{12x^3}+\frac{1}{2}bd^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}bd^2\sinh(c)\text{Shi}(dx)-\frac{b\cosh(c+dx)}{2x^2}-\frac{bd\sinh(c+dx)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Cosh[c + d*x])/x^5,x]
```

```
[Out] -1/4*(a*Cosh[c + d*x])/x^4 - (b*Cosh[c + d*x])/(2*x^2) - (a*d^2*Cosh[c + d*
x])/(24*x^2) + (b*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (a*d^4*Cosh[c]*CoshInt
egral[d*x])/24 - (a*d*Sinh[c + d*x])/(12*x^3) - (b*d*Sinh[c + d*x])/(2*x) -
 (a*d^3*Sinh[c + d*x])/(24*x) + (b*d^2*Sinh[c]*SinhIntegral[d*x])/2 + (a*d^
4*Sinh[c]*SinhIntegral[d*x])/24
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx = \int \left(\frac{a\cosh(c+dx)}{x^5} + \frac{b\cosh(c+dx)}{x^3}\right)dx$$

$$= a\int \frac{\cosh(c+dx)}{x^5}\,dx + b\int \frac{\cosh(c+dx)}{x^3}\,dx$$

$$= -\frac{a\cosh(c+dx)}{4x^4} - \frac{b\cosh(c+dx)}{2x^2} + \frac{1}{4}(ad)\int \frac{\sinh(c+dx)}{x^4}\,dx + \frac{1}{2}(bd)\int$$

$$= -\frac{a\cosh(c+dx)}{4x^4} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad\sinh(c+dx)}{12x^3} - \frac{bd\sinh(c+dx)}{2x} +$$

$$= -\frac{a\cosh(c+dx)}{4x^4} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad^2\cosh(c+dx)}{24x^2} - \frac{ad\sinh(c+dx)}{12x^3} -$$

$$= -\frac{a\cosh(c+dx)}{4x^4} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad^2\cosh(c+dx)}{24x^2} + \frac{1}{2}bd^2\cosh(c)\mathrm{Chi}(dx$$

$$= -\frac{a\cosh(c+dx)}{4x^4} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad^2\cosh(c+dx)}{24x^2} + \frac{1}{2}bd^2\cosh(c)\mathrm{Chi}(dx$$

$$= -\frac{a\cosh(c+dx)}{4x^4} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad^2\cosh(c+dx)}{24x^2} + \frac{1}{2}bd^2\cosh(c)\mathrm{Chi}(dx$$

Mathematica [A]
time = 0.25, size = 127, normalized size = 0.85

$$-\frac{6a\cosh(c+dx)+12bx^2\cosh(c+dx)+ad^2x^2\cosh(c+dx)-d^2(12b+ad^2)\,x^4\cosh(c)\mathrm{Chi}(dx)+2adx\sinh(c+dx)+12bdx^3\sinh(c+dx)+ad^3x^3\sinh(c+dx)-d^2(12b+ad^2)\,x^4\sinh(c)\mathrm{Shi}(dx)}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x^5,x]
```

```
[Out] -1/24*(6*a*Cosh[c + d*x] + 12*b*x^2*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x]
 - d^2*(12*b + a*d^2)*x^4*Cosh[c]*CoshIntegral[d*x] + 2*a*d*x*Sinh[c + d*x]
```

```
 + 12*b*d*x^3*Sinh[c + d*x] + a*d^3*x^3*Sinh[c + d*x] - d^2*(12*b + a*d^2)*
x^4*Sinh[c]*SinhIntegral[d*x])/x^4
```

Maple [A]
time = 1.00, size = 238, normalized size = 1.60

method	result
risch	$-\frac{d^4 a\,\mathrm{e}^{-c}\,\mathrm{expIntegral}(1,dx)}{48}-\frac{d^2 a\,\mathrm{e}^{-dx-c}}{48x^2}+\frac{da\,\mathrm{e}^{-dx-c}}{24x^3}-\frac{a\,\mathrm{e}^{-dx-c}}{8x^4}+\frac{db\,\mathrm{e}^{-dx-c}}{4x}-\frac{b\,\mathrm{e}^{-dx-c}}{4x^2}-\frac{d^2 b\,\mathrm{e}^{-c}\,\mathrm{expIntegral}(1,dx}{4}$
meijerg	$-\frac{d^2 b\cosh(c)\sqrt{\pi}\,\left(-\frac{4\left(\frac{9d^2x^2}{2}+3\right)}{3\sqrt{\pi}\,d^2x^2}+\frac{4\cosh(dx)}{\sqrt{\pi}\,d^2x^2}+\frac{4\sinh(dx)}{\sqrt{\pi}\,dx}-\frac{4(\mathrm{hyperbolicCosineIntegral}(dx)-\ln(dx)-\gamma)}{\sqrt{\pi}}-\frac{2(2\gamma-3+2\ln(x)+2\ln(id))}{\sqrt{\pi}}+\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*cosh(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*d^4*a*exp(-c)*Ei(1,d*x)-1/48*d^2*a*exp(-d*x-c)/x^2+1/24*d*a*exp(-d*x-
c)/x^3-1/8*a*exp(-d*x-c)/x^4+1/4*d*b*exp(-d*x-c)/x-1/4*b*exp(-d*x-c)/x^2-1/
4*d^2*b*exp(-c)*Ei(1,d*x)+1/48*d^3*a*exp(-d*x-c)/x-1/48*d^4*a*exp(c)*Ei(1,-
d*x)-1/4*b/x^2*exp(d*x+c)-1/4*d*b/x*exp(d*x+c)-1/4*d^2*b*exp(c)*Ei(1,-d*x)-
1/8*a/x^4*exp(d*x+c)-1/24*d*a/x^3*exp(d*x+c)-1/48*d^2*a/x^2*exp(d*x+c)-1/48
*d^3*a/x*exp(d*x+c)
```

Maxima [A]
time = 0.32, size = 76, normalized size = 0.51

$$\frac{1}{8}\left(ad^3e^{(-c)}\Gamma(-3,dx)+ad^3e^c\Gamma(-3,-dx)+2\,bde^{(-c)}\Gamma(-1,dx)+2\,bde^c\Gamma(-1,-dx)\right)d-\frac{(2\,bx^2+a)\cosh{(dx+c)}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*(a*d^3*e^(-c)*gamma(-3, d*x) + a*d^3*e^c*gamma(-3, -d*x) + 2*b*d*e^(-c)
*gamma(-1, d*x) + 2*b*d*e^c*gamma(-1, -d*x))*d - 1/4*(2*b*x^2 + a)*cosh(d*x
 + c)/x^4
```

Fricas [A]
time = 0.37, size = 152, normalized size = 1.02

$$-\frac{2\left((ad^2+12\,b)x^2+6\,a\right)\cosh{(dx+c)}-\left((ad^4+12\,bd^2)x^4\mathrm{Ei}(dx)+(ad^4+12\,bd^2)x^4\mathrm{Ei}(-dx)\right)\cosh{(c)}+2\left((ad^3+12\,bd)x^3+2\,adx\right)\sinh{(dx+c)}-\left((ad^4+12\,bd^2)x^4\mathrm{Ei}(dx)-(ad^4+12\,bd^2)x^4\mathrm{Ei}(-dx)\right)\sinh{(c)}}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(2*((a*d^2 + 12*b)*x^2 + 6*a)*cosh(d*x + c) - ((a*d^4 + 12*b*d^2)*x^4
*Ei(d*x) + (a*d^4 + 12*b*d^2)*x^4*Ei(-d*x))*cosh(c) + 2*((a*d^3 + 12*b*d)*x
```

```
^3 + 2*a*d*x)*sinh(d*x + c) - ((a*d^4 + 12*b*d^2)*x^4*Ei(d*x) - (a*d^4 + 12
*b*d^2)*x^4*Ei(-d*x))*sinh(c))/x^4
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*cosh(d*x+c)/x**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 237, normalized size = 1.59

$$\frac{ad^4x^4\mathrm{Ei}(-dx)\,e^{(-c)} + ad^4x^4\mathrm{Ei}(dx)\,e^c + 12\,bd^2x^4\mathrm{Ei}(-dx)\,e^{(-c)} + 12\,bd^2x^4\mathrm{Ei}(dx)\,e^c - ad^3x^3e^{(dx+c)} + ad^3x^3e^{(-dx-c)} - ad^2x^2e^{(dx+c)} - 12\,bdx^3e^{(dx+c)} - ad^2x^2e^{(-dx-c)} + 12\,bdx^3e^{(-dx-c)} - 2\,adxe^{(dx+c)} - 12\,bx^2e^{(dx+c)} + 2\,adxe^{(-dx-c)} - 12\,bx^2e^{(-dx-c)} - 6\,ae^{(dx+c)} - 6\,ae^{(-dx-c)}}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] 1/48*(a*d^4*x^4*Ei(-d*x)*e^(-c) + a*d^4*x^4*Ei(d*x)*e^c + 12*b*d^2*x^4*Ei(-
d*x)*e^(-c) + 12*b*d^2*x^4*Ei(d*x)*e^c - a*d^3*x^3*e^(d*x + c) + a*d^3*x^3*
e^(-d*x - c) - a*d^2*x^2*e^(d*x + c) - 12*b*d*x^3*e^(d*x + c) - a*d^2*x^2*e
^(-d*x - c) + 12*b*d*x^3*e^(-d*x - c) - 2*a*d*x*e^(d*x + c) - 12*b*x^2*e^(d
*x + c) + 2*a*d*x*e^(-d*x - c) - 12*b*x^2*e^(-d*x - c) - 6*a*e^(d*x + c) -
6*a*e^(-d*x - c))/x^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^2))/x^5,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^2))/x^5, x)
```

3.49 $\int x^2 (a+bx^2)^2 \cosh(c+dx)\,dx$

Optimal. Leaf size=234

$$-\frac{720b^2x\cosh(c+dx)}{d^6}-\frac{48abx\cosh(c+dx)}{d^4}-\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{120b^2x^3\cosh(c+dx)}{d^4}-\frac{8abx^3\cosh(c+dx)}{d^2}$$

```
[Out] -720*b^2*x*cosh(d*x+c)/d^6-48*a*b*x*cosh(d*x+c)/d^4-2*a^2*x*cosh(d*x+c)/d^2
-120*b^2*x^3*cosh(d*x+c)/d^4-8*a*b*x^3*cosh(d*x+c)/d^2-6*b^2*x^5*cosh(d*x+c
)/d^2+720*b^2*sinh(d*x+c)/d^7+48*a*b*sinh(d*x+c)/d^5+2*a^2*sinh(d*x+c)/d^3+
360*b^2*x^2*sinh(d*x+c)/d^5+24*a*b*x^2*sinh(d*x+c)/d^3+a^2*x^2*sinh(d*x+c)/
d+30*b^2*x^4*sinh(d*x+c)/d^3+2*a*b*x^4*sinh(d*x+c)/d+b^2*x^6*sinh(d*x+c)/d
```

Rubi [A]

time = 0.27, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5395, 3377, 2717}

$$\frac{2a^2\sinh(c+dx)}{d^3}-\frac{2a^2x\cosh(c+dx)}{d^2}+\frac{a^2x^2\sinh(c+dx)}{d}+\frac{48ab\sinh(c+dx)}{d^5}-\frac{48abx\cosh(c+dx)}{d^4}+\frac{24abx^2\sinh(c+dx)}{d^3}-\frac{8abx^3\cosh(c+dx)}{d^2}+\frac{2abx^4\sinh(c+dx)}{d}+\frac{720b^2\sinh(c+dx)}{d^7}-\frac{720b^2x\cosh(c+dx)}{d^6}+\frac{360b^2x^2\sinh(c+dx)}{d^5}-\frac{120b^2x^3\cosh(c+dx)}{d^4}+\frac{30b^2x^4\sinh(c+dx)}{d^3}-\frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{b^2x^6\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*Cosh[c + d*x],x]
```

```
[Out] (-720*b^2*x*Cosh[c + d*x])/d^6 - (48*a*b*x*Cosh[c + d*x])/d^4 - (2*a^2*x*Co
sh[c + d*x])/d^2 - (120*b^2*x^3*Cosh[c + d*x])/d^4 - (8*a*b*x^3*Cosh[c + d*
x])/d^2 - (6*b^2*x^5*Cosh[c + d*x])/d^2 + (720*b^2*Sinh[c + d*x])/d^7 + (48
*a*b*Sinh[c + d*x])/d^5 + (2*a^2*Sinh[c + d*x])/d^3 + (360*b^2*x^2*Sinh[c +
 d*x])/d^5 + (24*a*b*x^2*Sinh[c + d*x])/d^3 + (a^2*x^2*Sinh[c + d*x])/d + (
30*b^2*x^4*Sinh[c + d*x])/d^3 + (2*a*b*x^4*Sinh[c + d*x])/d + (b^2*x^6*Sinh
[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
```

```
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(a+bx^2\right)^2 \cosh(c+dx)\, dx &= \int \left(a^2x^2\cosh(c+dx)+2abx^4\cosh(c+dx)+b^2x^6\cosh(c+dx)\right)\, dx\\
&= a^2\int x^2\cosh(c+dx)\,dx+(2ab)\int x^4\cosh(c+dx)\,dx+b^2\int x^6\cosh(c+\\
&= \frac{a^2x^2\sinh(c+dx)}{d}+\frac{2abx^4\sinh(c+dx)}{d}+\frac{b^2x^6\sinh(c+dx)}{d}-\frac{(2a^2)\int x\,\mathrm{s}}{}\\
&= -\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{8abx^3\cosh(c+dx)}{d^2}-\frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{a^2x^2\,\mathrm{s}}{}\\
&= -\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{8abx^3\cosh(c+dx)}{d^2}-\frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{2a^2\sin}{}\\
&= -\frac{48abx\cosh(c+dx)}{d^4}-\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{120b^2x^3\cosh(c+dx)}{d^4}-\frac{8ab}{}\\
&= -\frac{48abx\cosh(c+dx)}{d^4}-\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{120b^2x^3\cosh(c+dx)}{d^4}-\frac{8ab}{}\\
&= -\frac{720b^2x\cosh(c+dx)}{d^6}-\frac{48abx\cosh(c+dx)}{d^4}-\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{120b}{}\\
&= -\frac{720b^2x\cosh(c+dx)}{d^6}-\frac{48abx\cosh(c+dx)}{d^4}-\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{120b}{}
\end{aligned}$$

Mathematica [A]
time = 0.22, size = 138, normalized size = 0.59

$$\frac{-2dx(a^2d^4+4abd^2(6+d^2x^2)+3b^2(120+20d^2x^2+d^4x^4))\cosh(c+dx)+(a^2d^4(2+d^2x^2)+2abd^2(24+12d^2x^2+d^4x^4)+b^2(720+360d^2x^2+30d^4x^4+d^6x^6))\sinh(c+dx)}{d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*Cosh[c + d*x],x]
```

```
[Out] (-2*d*x*(a^2*d^4 + 4*a*b*d^2*(6 + d^2*x^2) + 3*b^2*(120 + 20*d^2*x^2 + d^4*
x^4))*Cosh[c + d*x] + (a^2*d^4*(2 + d^2*x^2) + 2*a*b*d^2*(24 + 12*d^2*x^2 +
 d^4*x^4) + b^2*(720 + 360*d^2*x^2 + 30*d^4*x^4 + d^6*x^6))*Sinh[c + d*x])/
d^7
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 737 vs. 2(234) = 468.
time = 0.62, size = 738, normalized size = 3.15 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(-2*a^2*c*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+1/d^4*b^2*c^6*sinh(d*x+c)
+1/d^4*b^2*((d*x+c)^6*sinh(d*x+c)-6*(d*x+c)^5*cosh(d*x+c)+30*(d*x+c)^4*sinh
(d*x+c)-120*(d*x+c)^3*cosh(d*x+c)+360*(d*x+c)^2*sinh(d*x+c)-720*(d*x+c)*cos
h(d*x+c)+720*sinh(d*x+c))-8/d^2*b*c*a*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*co
sh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))-8/d^2*b*c^3*a*((d*x+c)*sinh(
d*x+c)-cosh(d*x+c))+12/d^2*b*c^2*a*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*
x+c)+2*sinh(d*x+c))-6/d^4*b^2*c^5*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+15/d^4*
b^2*c^4*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-20/d^4*
b^2*c^3*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c
)-6*cosh(d*x+c))+15/d^4*b^2*c^2*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x
+c)+12*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))+2/d^2*b
*c^4*a*sinh(d*x+c)+2/d^2*b*a*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)
+12*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))-6/d^4*b^2*
c*((d*x+c)^5*sinh(d*x+c)-5*(d*x+c)^4*cosh(d*x+c)+20*(d*x+c)^3*sinh(d*x+c)-6
0*(d*x+c)^2*cosh(d*x+c)+120*(d*x+c)*sinh(d*x+c)-120*cosh(d*x+c))+a^2*((d*x+
c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+a^2*c^2*sinh(d*x+c))
```

Maxima [**A**]
time = 0.28, size = 383, normalized size = 1.64

$$-\frac{1}{210}\,d\left(\frac{35\,(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c)a^2e^{(dx)}}{d^4}+\frac{35\,(d^3x^3+3\,d^2x^2+6\,dx+6)a^2e^{(-dx-c)}}{d^4}+\frac{42\,(d^5x^5e^c-5\,d^4x^4e^c+20\,d^3x^3e^c-60\,d^2x^2e^c+120\,dxe^c-120\,e^c)abe^{(dx)}}{d^6}+\frac{42\,(d^5x^5+5\,d^4x^4+20\,d^3x^3+60\,d^2x^2+120\,dx+120)abe^{(-dx-c)}}{d^6}+\frac{15\,(d^7x^7e^c-7\,d^6x^6e^c+42\,d^5x^5e^c-210\,d^4x^4e^c+840\,d^3x^3e^c-2520\,d^2x^2e^c+5040\,dxe^c-5040\,e^c)b^2e^{(dx)}}{d^8}+\frac{15\,(d^7x^7+7\,d^6x^6+42\,d^5x^5+210\,d^4x^4+840\,d^3x^3+2520\,d^2x^2+5040\,dx+5040)b^2e^{(-dx-c)}}{d^8}\right)+\frac{1}{105}\,(15\,b^2x^7+42\,abx^5+35\,a^2x^3)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/210*d*(35*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a^2*e^(d*x)/
d^4 + 35*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a^2*e^(-d*x - c)/d^4 + 42*(d^5*x
^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 12
0*e^c)*a*b*e^(d*x)/d^6 + 42*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2
+ 120*d*x + 120)*a*b*e^(-d*x - c)/d^6 + 15*(d^7*x^7*e^c - 7*d^6*x^6*e^c + 4
2*d^5*x^5*e^c - 210*d^4*x^4*e^c + 840*d^3*x^3*e^c - 2520*d^2*x^2*e^c + 5040
*d*x*e^c - 5040*e^c)*b^2*e^(d*x)/d^8 + 15*(d^7*x^7 + 7*d^6*x^6 + 42*d^5*x^5
 + 210*d^4*x^4 + 840*d^3*x^3 + 2520*d^2*x^2 + 5040*d*x + 5040)*b^2*e^(-d*x
- c)/d^8) + 1/105*(15*b^2*x^7 + 42*a*b*x^5 + 35*a^2*x^3)*cosh(d*x + c)
```

Fricas [**A**]
time = 0.40, size = 155, normalized size = 0.66

$$-\frac{2\,(3\,b^2d^5x^5+4\,(abd^5+15\,b^2d^3)x^3+(a^2d^5+24\,abd^3+360\,b^2d)x)\cosh\left(dx+c\right)-(b^2d^6x^6+2\,a^2d^4+2\,(abd^6+15\,b^2d^4)x^4+48\,abd^2+(a^2d^6+24\,abd^4+360\,b^2d^2)x^2+720\,b^2)\sinh\left(dx+c\right)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(2*(3*b^2*d^5*x^5 + 4*(a*b*d^5 + 15*b^2*d^3)*x^3 + (a^2*d^5 + 24*a*b*d^3 +
 360*b^2*d)*x)*cosh(d*x + c) - (b^2*d^6*x^6 + 2*a^2*d^4 + 2*(a*b*d^6 + 15*b
```

```
^2*d^4)*x^4 + 48*a*b*d^2 + (a^2*d^6 + 24*a*b*d^4 + 360*b^2*d^2)*x^2 + 720*b
^2)*sinh(d*x + c))/d^7
```

Sympy [A]
time = 0.70, size = 286, normalized size = 1.22

$$\begin{cases} \frac{a^2x^2\sinh(c+dx)}{d} - \frac{2a^2x\cosh(c+dx)}{d^2} + \frac{2a^2\sinh(c+dx)}{d^3} + \frac{2abx^4\sinh(c+dx)}{d} - \frac{8abx^3\cosh(c+dx)}{d^2} + \frac{24abx^2\sinh(c+dx)}{d^3} - \frac{48abx\cosh(c+dx)}{d^4} + \frac{48ab\sinh(c+dx)}{d^5} + \frac{b^2x^6\sinh(c+dx)}{d} - \frac{6b^2x^5\cosh(c+dx)}{d^2} + \frac{30b^2x^4\sinh(c+dx)}{d^3} - \frac{120b^2x^3\cosh(c+dx)}{d^4} + \frac{360b^2x^2\sinh(c+dx)}{d^5} - \frac{720b^2x\cosh(c+dx)}{d^6} + \frac{720b^2\sinh(c+dx)}{d^7} & \text{for } d \neq 0 \\ \left(\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{b^2x^7}{7}\right)\cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*x**2*sinh(c + d*x)/d - 2*a**2*x*cosh(c + d*x)/d**2 + 2*a**2
*sinh(c + d*x)/d**3 + 2*a*b*x**4*sinh(c + d*x)/d - 8*a*b*x**3*cosh(c + d*x)
/d**2 + 24*a*b*x**2*sinh(c + d*x)/d**3 - 48*a*b*x*cosh(c + d*x)/d**4 + 48*a
*b*sinh(c + d*x)/d**5 + b**2*x**6*sinh(c + d*x)/d - 6*b**2*x**5*cosh(c + d*
x)/d**2 + 30*b**2*x**4*sinh(c + d*x)/d**3 - 120*b**2*x**3*cosh(c + d*x)/d**
4 + 360*b**2*x**2*sinh(c + d*x)/d**5 - 720*b**2*x*cosh(c + d*x)/d**6 + 720*
b**2*sinh(c + d*x)/d**7, Ne(d, 0)), ((a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**
7/7)*cosh(c), True))
```

Giac [A]
time = 0.41, size = 304, normalized size = 1.30

$$\frac{(b^2d^6x^6 + 2\,abd^6x^4 - 6\,b^2d^5x^5 + a^2d^6x^2 - 8\,abd^5x^3 + 30\,b^2d^4x^4 - 2\,a^2d^5x + 24\,abd^4x^2 - 120\,b^2d^3x^3 + 2\,a^2d^4 - 48\,abd^3x + 360\,b^2d^2x^2 + 48\,abd^2 - 720\,b^2dx + 720\,b^2)e^{(dx+c)}}{2\,d^7} - \frac{(b^2d^6x^6 + 2\,abd^6x^4 + 6\,b^2d^5x^5 + a^2d^6x^2 + 8\,abd^5x^3 + 30\,b^2d^4x^4 + 2\,a^2d^5x + 24\,abd^4x^2 + 120\,b^2d^3x^3 + 2\,a^2d^4 + 48\,abd^3x + 360\,b^2d^2x^2 + 48\,abd^2 + 720\,b^2dx + 720\,b^2)e^{(-dx-c)}}{2\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^6*x^6 + 2*a*b*d^6*x^4 - 6*b^2*d^5*x^5 + a^2*d^6*x^2 - 8*a*b*d^5*
x^3 + 30*b^2*d^4*x^4 - 2*a^2*d^5*x + 24*a*b*d^4*x^2 - 120*b^2*d^3*x^3 + 2*a
^2*d^4 - 48*a*b*d^3*x + 360*b^2*d^2*x^2 + 48*a*b*d^2 - 720*b^2*d*x + 720*b^
2)*e^(d*x + c)/d^7 - 1/2*(b^2*d^6*x^6 + 2*a*b*d^6*x^4 + 6*b^2*d^5*x^5 + a^2
*d^6*x^2 + 8*a*b*d^5*x^3 + 30*b^2*d^4*x^4 + 2*a^2*d^5*x + 24*a*b*d^4*x^2 +
120*b^2*d^3*x^3 + 2*a^2*d^4 + 48*a*b*d^3*x + 360*b^2*d^2*x^2 + 48*a*b*d^2 +
 720*b^2*d*x + 720*b^2)*e^(-d*x - c)/d^7
```

Mupad [B]
time = 1.03, size = 182, normalized size = 0.78

$$\frac{2\sinh(c+d\,x)\,(a^2\,d^4+24\,a\,b\,d^2+360\,b^2)}{d^7} - \frac{6\,b^2\,x^5\cosh(c+d\,x)}{d^2} + \frac{b^2\,x^6\sinh(c+d\,x)}{d} - \frac{2\,x\cosh(c+d\,x)\,(a^2\,d^4+24\,a\,b\,d^2+360\,b^2)}{d^6} + \frac{x^2\sinh(c+d\,x)\,(a^2\,d^4+24\,a\,b\,d^2+360\,b^2)}{d^5} - \frac{8\,x^3\cosh(c+d\,x)\,(15\,b^2+a\,b\,d^2)}{d^4} + \frac{2\,x^4\sinh(c+d\,x)\,(15\,b^2+a\,b\,d^2)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(c + d*x)*(a + b*x^2)^2,x)
```

```
[Out] (2*sinh(c + d*x)*(360*b^2 + a^2*d^4 + 24*a*b*d^2))/d^7 - (6*b^2*x^5*cosh(c
+ d*x))/d^2 + (b^2*x^6*sinh(c + d*x))/d - (2*x*cosh(c + d*x)*(360*b^2 + a^2
*d^4 + 24*a*b*d^2))/d^6 + (x^2*sinh(c + d*x)*(360*b^2 + a^2*d^4 + 24*a*b*d^
2))/d^5 - (8*x^3*cosh(c + d*x)*(15*b^2 + a*b*d^2))/d^4 + (2*x^4*sinh(c + d*
x)*(15*b^2 + a*b*d^2))/d^3
```

3.50 $\int x(a+bx^2)^2 \cosh(c+dx)\,dx$

Optimal. Leaf size=184

$$-\frac{120b^2\cosh(c+dx)}{d^6}-\frac{12ab\cosh(c+dx)}{d^4}-\frac{a^2\cosh(c+dx)}{d^2}-\frac{60b^2x^2\cosh(c+dx)}{d^4}-\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{5b^2}{}$$

```
[Out] -120*b^2*cosh(d*x+c)/d^6-12*a*b*cosh(d*x+c)/d^4-a^2*cosh(d*x+c)/d^2-60*b^2*
x^2*cosh(d*x+c)/d^4-6*a*b*x^2*cosh(d*x+c)/d^2-5*b^2*x^4*cosh(d*x+c)/d^2+120
*b^2*x*sinh(d*x+c)/d^5+12*a*b*x*sinh(d*x+c)/d^3+a^2*x*sinh(d*x+c)/d+20*b^2*
x^3*sinh(d*x+c)/d^3+2*a*b*x^3*sinh(d*x+c)/d+b^2*x^5*sinh(d*x+c)/d
```

Rubi [A]
time = 0.20, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5395, 3377, 2718}

$$-\frac{a^2\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(c+dx)}{d}-\frac{12ab\cosh(c+dx)}{d^4}+\frac{12abx\sinh(c+dx)}{d^3}-\frac{6abx^2\cosh(c+dx)}{d^2}+\frac{2abx^3\sinh(c+dx)}{d}-\frac{120b^2\cosh(c+dx)}{d^6}+\frac{120b^2x\sinh(c+dx)}{d^5}-\frac{60b^2x^2\cosh(c+dx)}{d^4}+\frac{20b^2x^3\sinh(c+dx)}{d^3}-\frac{5b^2x^4\cosh(c+dx)}{d^2}+\frac{b^2x^5\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*Cosh[c + d*x],x]
```

```
[Out] (-120*b^2*Cosh[c + d*x])/d^6 - (12*a*b*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d
*x])/d^2 - (60*b^2*x^2*Cosh[c + d*x])/d^4 - (6*a*b*x^2*Cosh[c + d*x])/d^2 -
 (5*b^2*x^4*Cosh[c + d*x])/d^2 + (120*b^2*x*Sinh[c + d*x])/d^5 + (12*a*b*x*
Sinh[c + d*x])/d^3 + (a^2*x*Sinh[c + d*x])/d + (20*b^2*x^3*Sinh[c + d*x])/d
^3 + (2*a*b*x^3*Sinh[c + d*x])/d + (b^2*x^5*Sinh[c + d*x])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x(a+bx^2)^2\cosh(c+dx)\,dx = \int \left(a^2x\cosh(c+dx)+2abx^3\cosh(c+dx)+b^2x^5\cosh(c+dx)\right)\,dx$$
$$= a^2\int x\cosh(c+dx)\,dx + (2ab)\int x^3\cosh(c+dx)\,dx + b^2\int x^5\cosh(c+d$$
$$= \frac{a^2x\sinh(c+dx)}{d} + \frac{2abx^3\sinh(c+dx)}{d} + \frac{b^2x^5\sinh(c+dx)}{d} - \frac{a^2\int\sinh(c-}{d}$$
$$= -\frac{a^2\cosh(c+dx)}{d^2} - \frac{6abx^2\cosh(c+dx)}{d^2} - \frac{5b^2x^4\cosh(c+dx)}{d^2} + \frac{a^2x\sinh(}{d}$$
$$= -\frac{a^2\cosh(c+dx)}{d^2} - \frac{6abx^2\cosh(c+dx)}{d^2} - \frac{5b^2x^4\cosh(c+dx)}{d^2} + \frac{12abx\sin}{d}$$
$$= -\frac{12ab\cosh(c+dx)}{d^4} - \frac{a^2\cosh(c+dx)}{d^2} - \frac{60b^2x^2\cosh(c+dx)}{d^4} - \frac{6abx^2\cos}{d}$$
$$= -\frac{12ab\cosh(c+dx)}{d^4} - \frac{a^2\cosh(c+dx)}{d^2} - \frac{60b^2x^2\cosh(c+dx)}{d^4} - \frac{6abx^2\cos}{d}$$
$$= -\frac{120b^2\cosh(c+dx)}{d^6} - \frac{12ab\cosh(c+dx)}{d^4} - \frac{a^2\cosh(c+dx)}{d^2} - \frac{60b^2x^2\cosh}{d^4}$$

Mathematica [A]
time = 0.15, size = 113, normalized size = 0.61

$$\frac{-((a^2d^4+6abd^2(2+d^2x^2)+5b^2(24+12d^2x^2+d^4x^4))\cosh(c+dx))+dx(a^2d^4+2abd^2(6+d^2x^2)+b^2(120+20d^2x^2+d^4x^4))\sinh(c+dx)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*Cosh[c + d*x],x]
```

```
[Out] (-((a^2*d^4 + 6*a*b*d^2*(2 + d^2*x^2) + 5*b^2*(24 + 12*d^2*x^2 + d^4*x^4))*
Cosh[c + d*x]) + d*x*(a^2*d^4 + 2*a*b*d^2*(6 + d^2*x^2) + b^2*(120 + 20*d^2
*x^2 + d^4*x^4))*Sinh[c + d*x])/d^6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(184) = 368.
time = 0.61, size = 513, normalized size = 2.79

method	result
risch	$\frac{(b^2x^5d^5+2ab\,d^5x^3-5x^4b^2d^4+a^2d^5x-6ab\,d^4x^2+20b^2d^3x^3-a^2d^4+12ab\,d^3x-60b^2d^2x^2-12a\,d^2b+120b^2dx-120b^2)\mathrm{e}^{dx+c}}{2d^6}$
derivativedivides	$\frac{\frac{5b^2c^4((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^4}-\frac{10b^2c^3\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^4}+\frac{10b^2c^2\left((dx+c)^3\sinh(dx\right.}{}}{}$
default	$\frac{\frac{5b^2c^4((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^4}-\frac{10b^2c^3\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^4}+\frac{10b^2c^2\left((dx+c)^3\sinh(dx\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(5/d^4*b^2*c^4*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-10/d^4*b^2*c^3*((d*x
+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+10/d^4*b^2*c^2*((d*x
+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+
c))+6/d^2*b*c^2*a*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-5/d^4*b^2*c*((d*x+c)^4*
sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)*cos
h(d*x+c)+24*sinh(d*x+c))-6/d^2*b*c*a*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(
d*x+c)+2*sinh(d*x+c))+1/d^4*b^2*((d*x+c)^5*sinh(d*x+c)-5*(d*x+c)^4*cosh(d*x
+c)+20*(d*x+c)^3*sinh(d*x+c)-60*(d*x+c)^2*cosh(d*x+c)+120*(d*x+c)*sinh(d*x+
c)-120*cosh(d*x+c))+2/d^2*b*a*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c
)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))+a^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c)
)-1/d^4*b^2*c^5*sinh(d*x+c)-2/d^2*b*c^3*a*sinh(d*x+c)-c*a^2*sinh(d*x+c))
```

Maxima [A]
time = 0.28, size = 353, normalized size = 1.92

$$\frac{(bx^2+a)^3\cosh(dx+c)}{6\,b} - \frac{\left(\frac{a^3e^{(dx+c)}}{d} + \frac{a^3e^{(-dx-c)}}{d} + \frac{3\,(d^2x^2e^c-2\,dxe^c+2\,e^c)a^2be^{(dx)}}{d^3} + \frac{3\,(d^2x^2+2\,dx+2)a^2be^{(-dx-c)}}{d^3} + \frac{3\,(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2e^c-24\,dxe^c+24\,e^c)ab^2e^{(dx)}}{d^5} + \frac{3\,(d^4x^4+4\,d^3x^3+12\,d^2x^2+24\,dx+24)ab^2e^{(-dx-c)}}{d^5} + \frac{(d^6x^6e^c-6\,d^5x^5e^c+30\,d^4x^4e^c-120\,d^3x^3e^c+360\,d^2x^2e^c-720\,dxe^c+720\,e^c)b^3e^{(dx)}}{d^7} + \frac{(d^6x^6+6\,d^5x^5+30\,d^4x^4+120\,d^3x^3+360\,d^2x^2+720\,dx+720)b^3e^{(-dx-c)}}{d^7}\right)d}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^3*cosh(d*x + c)/b - 1/12*(a^3*e^(d*x + c)/d + a^3*e^(-d*x -
 c)/d + 3*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a^2*b*e^(d*x)/d^3 + 3*(d^2*x^2
+ 2*d*x + 2)*a^2*b*e^(-d*x - c)/d^3 + 3*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d
^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*a*b^2*e^(d*x)/d^5 + 3*(d^4*x^4 + 4*d^3*x^
3 + 12*d^2*x^2 + 24*d*x + 24)*a*b^2*e^(-d*x - c)/d^5 + (d^6*x^6*e^c - 6*d^5
*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*d*x*e^c
 + 720*e^c)*b^3*e^(d*x)/d^7 + (d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 120*d^3*x
^3 + 360*d^2*x^2 + 720*d*x + 720)*b^3*e^(-d*x - c)/d^7)*d/b
```

Fricas [A]
time = 0.38, size = 126, normalized size = 0.68

$$-\frac{(5\,b^2d^4x^4 + a^2d^4 + 12\,abd^2 + 6\,(abd^4 + 10\,b^2d^2)x^2 + 120\,b^2)\cosh(dx+c) - (b^2d^5x^5 + 2\,(abd^5 + 10\,b^2d^3)x^3 + (a^2d^5 + 12\,abd^3 + 120\,b^2d)x)\sinh(dx+c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((5*b^2*d^4*x^4 + a^2*d^4 + 12*a*b*d^2 + 6*(a*b*d^4 + 10*b^2*d^2)*x^2 + 12
0*b^2)*cosh(d*x + c) - (b^2*d^5*x^5 + 2*(a*b*d^5 + 10*b^2*d^3)*x^3 + (a^2*d
^5 + 12*a*b*d^3 + 120*b^2*d)*x)*sinh(d*x + c))/d^6
```

Sympy [A]
time = 0.47, size = 226, normalized size = 1.23

$$\begin{cases} \frac{a^2x\sinh(c+dx)}{d} - \frac{a^2\cosh(c+dx)}{d^2} + \frac{2abx^3\sinh(c+dx)}{d} - \frac{6abx^2\cosh(c+dx)}{d^2} + \frac{12abx\sinh(c+dx)}{d^3} - \frac{12ab\cosh(c+dx)}{d^4} + \frac{b^2x^5\sinh(c+dx)}{d} - \frac{5b^2x^4\cosh(c+dx)}{d^2} + \frac{20b^2x^3\sinh(c+dx)}{d^3} - \frac{60b^2x^2\cosh(c+dx)}{d^4} + \frac{120b^2x\sinh(c+dx)}{d^5} - \frac{120b^2\cosh(c+dx)}{d^6} & \text{for}\ d \neq 0 \\ \left(\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}\right)\cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*x*sinh(c + d*x)/d - a**2*cosh(c + d*x)/d**2 + 2*a*b*x**3*si
nh(c + d*x)/d - 6*a*b*x**2*cosh(c + d*x)/d**2 + 12*a*b*x*sinh(c + d*x)/d**3
 - 12*a*b*cosh(c + d*x)/d**4 + b**2*x**5*sinh(c + d*x)/d - 5*b**2*x**4*cosh
(c + d*x)/d**2 + 20*b**2*x**3*sinh(c + d*x)/d**3 - 60*b**2*x**2*cosh(c + d*
x)/d**4 + 120*b**2*x*sinh(c + d*x)/d**5 - 120*b**2*cosh(c + d*x)/d**6, Ne(d
, 0)), ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)*cosh(c), True))
```

Giac [A]
time = 0.41, size = 239, normalized size = 1.30

$$\frac{(b^2d^5x^5 + 2\,abd^5x^3 - 5\,b^2d^4x^4 + a^2d^5x - 6\,abd^4x^2 + 20\,b^2d^3x^3 - a^2d^4 + 12\,abd^3x - 60\,b^2d^2x^2 - 12\,abd^2 + 120\,b^2dx - 120\,b^2)e^{(dx+c)}}{2\,d^6} - \frac{(b^2d^5x^5 + 2\,abd^5x^3 + 5\,b^2d^4x^4 + a^2d^5x + 6\,abd^4x^2 + 20\,b^2d^3x^3 + a^2d^4 + 12\,abd^3x + 60\,b^2d^2x^2 + 12\,abd^2 + 120\,b^2dx + 120\,b^2)e^{(-dx-c)}}{2\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^5*x^5 + 2*a*b*d^5*x^3 - 5*b^2*d^4*x^4 + a^2*d^5*x - 6*a*b*d^4*x^
2 + 20*b^2*d^3*x^3 - a^2*d^4 + 12*a*b*d^3*x - 60*b^2*d^2*x^2 - 12*a*b*d^2 +
 120*b^2*d*x - 120*b^2)*e^(d*x + c)/d^6 - 1/2*(b^2*d^5*x^5 + 2*a*b*d^5*x^3
+ 5*b^2*d^4*x^4 + a^2*d^5*x + 6*a*b*d^4*x^2 + 20*b^2*d^3*x^3 + a^2*d^4 + 12
*a*b*d^3*x + 60*b^2*d^2*x^2 + 12*a*b*d^2 + 120*b^2*d*x + 120*b^2)*e^(-d*x -
 c)/d^6
```

Mupad [B]
time = 0.98, size = 148, normalized size = 0.80

$$\frac{b^2\,x^5\,\sinh(c+d\,x)}{d} - \frac{5\,b^2\,x^4\,\cosh(c+d\,x)}{d^2} - \frac{\cosh(c+d\,x)\,\left(a^2\,d^4+12\,a\,b\,d^2+120\,b^2\right)}{d^6} + \frac{x\,\sinh(c+d\,x)\,\left(a^2\,d^4+12\,a\,b\,d^2+120\,b^2\right)}{d^5} - \frac{6\,x^2\,\cosh(c+d\,x)\,\left(10\,b^2+a\,b\,d^2\right)}{d^4} + \frac{2\,x^3\,\sinh(c+d\,x)\,\left(10\,b^2+a\,b\,d^2\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(c + d*x)*(a + b*x^2)^2,x)
```

```
[Out] (b^2*x^5*sinh(c + d*x))/d - (5*b^2*x^4*cosh(c + d*x))/d^2 - (cosh(c + d*x)*
(120*b^2 + a^2*d^4 + 12*a*b*d^2))/d^6 + (x*sinh(c + d*x)*(120*b^2 + a^2*d^4
 + 12*a*b*d^2))/d^5 - (6*x^2*cosh(c + d*x)*(10*b^2 + a*b*d^2))/d^4 + (2*x^3
*sinh(c + d*x)*(10*b^2 + a*b*d^2))/d^3
```

3.51 $\int (a+bx^2)^2 \cosh(c+dx)\,dx$

Optimal. Leaf size=136

$$-\frac{24b^2x\cosh(c+dx)}{d^4}-\frac{4abx\cosh(c+dx)}{d^2}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+\frac{24b^2\sinh(c+dx)}{d^5}+\frac{4ab\sinh(c+dx)}{d^3}+$$ a^2 si

```
[Out] -24*b^2*x*cosh(d*x+c)/d^4-4*a*b*x*cosh(d*x+c)/d^2-4*b^2*x^3*cosh(d*x+c)/d^2
+24*b^2*sinh(d*x+c)/d^5+4*a*b*sinh(d*x+c)/d^3+a^2*sinh(d*x+c)/d+12*b^2*x^2*
sinh(d*x+c)/d^3+2*a*b*x^2*sinh(d*x+c)/d+b^2*x^4*sinh(d*x+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5385, 2717, 3377}

$$\frac{a^2\sinh(c+dx)}{d}+\frac{4ab\sinh(c+dx)}{d^3}-\frac{4abx\cosh(c+dx)}{d^2}+\frac{2abx^2\sinh(c+dx)}{d}+\frac{24b^2\sinh(c+dx)}{d^5}-\frac{24b^2x\cosh(c+dx)}{d^4}+\frac{12b^2x^2\sinh(c+dx)}{d^3}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+\frac{b^2x^4\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*Cosh[c + d*x],x]
```

```
[Out] (-24*b^2*x*Cosh[c + d*x])/d^4 - (4*a*b*x*Cosh[c + d*x])/d^2 - (4*b^2*x^3*Co
sh[c + d*x])/d^2 + (24*b^2*Sinh[c + d*x])/d^5 + (4*a*b*Sinh[c + d*x])/d^3 +
 (a^2*Sinh[c + d*x])/d + (12*b^2*x^2*Sinh[c + d*x])/d^3 + (2*a*b*x^2*Sinh[c
 + d*x])/d + (b^2*x^4*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5385

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c,
d, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int (a+bx^2)^2 \cosh(c+dx)\,dx = \int \left(a^2\cosh(c+dx)+2abx^2\cosh(c+dx)+b^2x^4\cosh(c+dx)\right)\,dx$$
$$= a^2\int \cosh(c+dx)\,dx + (2ab)\int x^2\cosh(c+dx)\,dx + b^2\int x^4\cosh(c+dx)\,d$$
$$= \frac{a^2\sinh(c+dx)}{d} + \frac{2abx^2\sinh(c+dx)}{d} + \frac{b^2x^4\sinh(c+dx)}{d} - \frac{(4ab)\int x\sinh(}{d}$$
$$= -\frac{4abx\cosh(c+dx)}{d^2} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + \frac{a^2\sinh(c+dx)}{d} + \frac{2abx^2\sinh(c}{d}$$
$$= -\frac{4abx\cosh(c+dx)}{d^2} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + \frac{4ab\sinh(c+dx)}{d^3} + \frac{a^2\sinh(c+}{d}$$
$$= -\frac{24b^2x\cosh(c+dx)}{d^4} - \frac{4abx\cosh(c+dx)}{d^2} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + \frac{4ab\sinh}{d}$$
$$= -\frac{24b^2x\cosh(c+dx)}{d^4} - \frac{4abx\cosh(c+dx)}{d^2} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + \frac{24b^2\sinh}{c}$$

Mathematica [**A**]
time = 0.12, size = 85, normalized size = 0.62

$$\frac{-4bdx(ad^2+b(6+d^2x^2))\cosh(c+dx)+(a^2d^4+2abd^2(2+d^2x^2)+b^2(24+12d^2x^2+d^4x^4))\sinh(c+dx)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*Cosh[c + d*x],x]
```

```
[Out] (-4*b*d*x*(a*d^2 + b*(6 + d^2*x^2))*Cosh[c + d*x] + (a^2*d^4 + 2*a*b*d^2*(2
 + d^2*x^2) + b^2*(24 + 12*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(136) = 272$.
time = 0.58, size = 332, normalized size = 2.44

method	result
risch	$\frac{(x^4b^2d^4+2ab\,d^4x^2-4b^2d^3x^3+a^2d^4-4ab\,d^3x+12b^2d^2x^2+4a\,d^2b-24b^2dx+24b^2)\mathrm{e}^{dx+c}}{2d^5} - \frac{(x^4b^2d^4+2ab\,d^4x^2+4b^2d^3x^3+}{}$
derivativedivides	$\frac{\frac{b^2c^4\sinh(dx+c)}{d^4} - \frac{4b^2c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^4} + \frac{6b^2c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^4} + \frac{2b\,c^2a}{}}{}$
default	$\frac{\frac{b^2c^4\sinh(dx+c)}{d^4} - \frac{4b^2c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^4} + \frac{6b^2c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^4} + \frac{2b\,c^2a}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/d^4*b^2*c^4*sinh(d*x+c)-4/d^4*b^2*c^3*((d*x+c)*sinh(d*x+c)-cosh(d*x+
c))+6/d^4*b^2*c^2*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c
))+2/d^2*b*c^2*a*sinh(d*x+c)-4/d^4*b^2*c*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2
*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))-4/d^2*b*c*a*((d*x+c)*sinh
(d*x+c)-cosh(d*x+c))+1/d^4*b^2*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+
c)+12*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))+2/d^2*b*
a*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+a^2*sinh(d*x+
c))
```

Maxima [A]
time = 0.27, size = 189, normalized size = 1.39

$$\frac{a^2e^{(dx+c)}}{2\,d}-\frac{a^2e^{(-dx-c)}}{2\,d}+\frac{(d^2x^2e^c-2\,dxe^c+2\,e^c)abe^{(dx)}}{d^3}-\frac{(d^2x^2+2\,dx+2)abe^{(-dx-c)}}{d^3}+\frac{(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2e^c-24\,dxe^c+24\,e^c)b^2e^{(dx)}}{2\,d^5}-\frac{(d^4x^4+4\,d^3x^3+12\,d^2x^2+24\,dx+24)b^2e^{(-dx-c)}}{2\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*e^(d*x + c)/d - 1/2*a^2*e^(-d*x - c)/d + (d^2*x^2*e^c - 2*d*x*e^c +
 2*e^c)*a*b*e^(d*x)/d^3 - (d^2*x^2 + 2*d*x + 2)*a*b*e^(-d*x - c)/d^3 + 1/2*
(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*b^2*e^
(d*x)/d^5 - 1/2*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*b^2*e^(-d*
x - c)/d^5
```

Fricas [A]
time = 0.39, size = 98, normalized size = 0.72

$$-\frac{4\,(b^2d^3x^3+(abd^3+6\,b^2d)x)\cosh\left(dx+c\right)-(b^2d^4x^4+a^2d^4+4\,abd^2+2\,(abd^4+6\,b^2d^2)x^2+24\,b^2)\sinh\left(dx+c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(4*(b^2*d^3*x^3 + (a*b*d^3 + 6*b^2*d)*x)*cosh(d*x + c) - (b^2*d^4*x^4 + a^
2*d^4 + 4*a*b*d^2 + 2*(a*b*d^4 + 6*b^2*d^2)*x^2 + 24*b^2)*sinh(d*x + c))/d^
5
```

Sympy [A]
time = 0.31, size = 172, normalized size = 1.26

$$\begin{cases}\frac{a^2\sinh\left(c+dx\right)}{d}+\frac{2abx^2\sinh\left(c+dx\right)}{d}-\frac{4abx\cosh\left(c+dx\right)}{d^2}+\frac{4ab\sinh\left(c+dx\right)}{d^3}+\frac{b^2x^4\sinh\left(c+dx\right)}{d}-\frac{4b^2x^3\cosh\left(c+dx\right)}{d^2}+\frac{12b^2x^2\sinh\left(c+dx\right)}{d^3}-\frac{24b^2x\cosh\left(c+dx\right)}{d^4}+\frac{24b^2\sinh\left(c+dx\right)}{d^5} & \text{for}\ d\neq 0\\ \left(a^2x+\frac{2abx^3}{3}+\frac{b^2x^5}{5}\right)\cosh\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*x**2*sinh(c + d*x)/d - 4*a*b*x*cosh
(c + d*x)/d**2 + 4*a*b*sinh(c + d*x)/d**3 + b**2*x**4*sinh(c + d*x)/d - 4*b
```

```
**2*x**3*cosh(c + d*x)/d**2 + 12*b**2*x**2*sinh(c + d*x)/d**3 - 24*b**2*x*c
osh(c + d*x)/d**4 + 24*b**2*sinh(c + d*x)/d**5, Ne(d, 0)), ((a**2*x + 2*a*b
*x**3/3 + b**2*x**5/5)*cosh(c), True))
```

Giac [A]
time = 0.42, size = 180, normalized size = 1.32

$$\frac{(b^2d^4x^4 + 2\,abd^4x^2 - 4\,b^2d^3x^3 + a^2d^4 - 4\,abd^3x + 12\,b^2d^2x^2 + 4\,abd^2 - 24\,b^2dx + 24\,b^2)e^{(dx+c)}}{2\,d^5} - \frac{(b^2d^4x^4 + 2\,abd^4x^2 + 4\,b^2d^3x^3 + a^2d^4 + 4\,abd^3x + 12\,b^2d^2x^2 + 4\,abd^2 + 24\,b^2dx + 24\,b^2)e^{(-dx-c)}}{2\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^4*x^4 + 2*a*b*d^4*x^2 - 4*b^2*d^3*x^3 + a^2*d^4 - 4*a*b*d^3*x +
12*b^2*d^2*x^2 + 4*a*b*d^2 - 24*b^2*d*x + 24*b^2)*e^(d*x + c)/d^5 - 1/2*(b^
2*d^4*x^4 + 2*a*b*d^4*x^2 + 4*b^2*d^3*x^3 + a^2*d^4 + 4*a*b*d^3*x + 12*b^2*
d^2*x^2 + 4*a*b*d^2 + 24*b^2*d*x + 24*b^2)*e^(-d*x - c)/d^5
```

Mupad [B]
time = 0.12, size = 114, normalized size = 0.84

$$\frac{\sinh(c+d\,x)\,\left(a^2\,d^4+4\,a\,b\,d^2+24\,b^2\right)}{d^5} - \frac{4\,b^2\,x^3\,\cosh(c+d\,x)}{d^2} + \frac{b^2\,x^4\,\sinh(c+d\,x)}{d} - \frac{4\,x\,\cosh(c+d\,x)\,\left(6\,b^2+a\,b\,d^2\right)}{d^4} + \frac{2\,x^2\,\sinh(c+d\,x)\,\left(6\,b^2+a\,b\,d^2\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b*x^2)^2,x)
```

```
[Out] (sinh(c + d*x)*(24*b^2 + a^2*d^4 + 4*a*b*d^2))/d^5 - (4*b^2*x^3*cosh(c + d*
x))/d^2 + (b^2*x^4*sinh(c + d*x))/d - (4*x*cosh(c + d*x)*(6*b^2 + a*b*d^2))
/d^4 + (2*x^2*sinh(c + d*x)*(6*b^2 + a*b*d^2))/d^3
```

3.52 $\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x} \, dx$

Optimal. Leaf size=110

$$-\frac{6b^2\cosh(c+dx)}{d^4}-\frac{2ab\cosh(c+dx)}{d^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+a^2\cosh(c)\text{Chi}(dx)+\frac{6b^2x\sinh(c+dx)}{d^3}+\frac{2abx\sin}{}$$

```
[Out] a^2*Chi(d*x)*cosh(c)-6*b^2*cosh(d*x+c)/d^4-2*a*b*cosh(d*x+c)/d^2-3*b^2*x^2*
cosh(d*x+c)/d^2+a^2*Shi(d*x)*sinh(c)+6*b^2*x*sinh(d*x+c)/d^3+2*a*b*x*sinh(d
*x+c)/d+b^2*x^3*sinh(d*x+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5395, 3384, 3379, 3382, 3377, 2718}

$$a^2\cosh(c)\text{Chi}(dx)+a^2\sinh(c)\text{Shi}(dx)-\frac{2ab\cosh(c+dx)}{d^2}+\frac{2abx\sinh(c+dx)}{d}-\frac{6b^2\cosh(c+dx)}{d^4}+\frac{6b^2x\sinh(c+dx)}{d^3}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{b^2x^3\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Cosh[c + d*x])/x,x]
```

```
[Out] (-6*b^2*Cosh[c + d*x])/d^4 - (2*a*b*Cosh[c + d*x])/d^2 - (3*b^2*x^2*Cosh[c
+ d*x])/d^2 + a^2*Cosh[c]*CoshIntegral[d*x] + (6*b^2*x*Sinh[c + d*x])/d^3 +
 (2*a*b*x*Sinh[c + d*x])/d + (b^2*x^3*Sinh[c + d*x])/d + a^2*Sinh[c]*SinhIn
tegral[d*x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2\cosh(c+dx)}{x}\,dx &= \int \left(\frac{a^2\cosh(c+dx)}{x}+2abx\cosh(c+dx)+b^2x^3\cosh(c+dx)\right)dx\\ &= a^2\int \frac{\cosh(c+dx)}{x}\,dx+(2ab)\int x\cosh(c+dx)\,dx+b^2\int x^3\cosh(c+dx)\,d\\ &= \frac{2abx\sinh(c+dx)}{d}+\frac{b^2x^3\sinh(c+dx)}{d}-\frac{(2ab)\int\sinh(c+dx)\,dx}{d}-\frac{(3b^2)\int\\ &= -\frac{2ab\cosh(c+dx)}{d^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+a^2\cosh(c)\text{Chi}(dx)+\frac{2abx\sinh(}{d}\\ &= -\frac{2ab\cosh(c+dx)}{d^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+a^2\cosh(c)\text{Chi}(dx)+\frac{6b^2x\sinh(}{d^3}\\ &= -\frac{6b^2\cosh(c+dx)}{d^4}-\frac{2ab\cosh(c+dx)}{d^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+a^2\cosh(c)\text{Ch}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 82, normalized size = 0.75

$$-\frac{b(2ad^2+3b(2+d^2x^2))\cosh(c+dx)}{d^4}+a^2\cosh(c)\text{Chi}(dx)+\frac{bx(2ad^2+b(6+d^2x^2))\sinh(c+dx)}{d^3}+a^2\sinh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x,x]
```

```
[Out] -((b*(2*a*d^2 + 3*b*(2 + d^2*x^2))*Cosh[c + d*x])/d^4) + a^2*Cosh[c]*CoshIn
tegral[d*x] + (b*x*(2*a*d^2 + b*(6 + d^2*x^2))*Sinh[c + d*x])/d^3 + a^2*Sin
h[c]*SinhIntegral[d*x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(110) = 220$.
time = 0.84, size = 226, normalized size = 2.05

method	result
risch	$-\frac{3b^2 e^{-dx-c}}{d^4} - \frac{b^2 e^{-dx-c} x^3}{2d} - \frac{3b^2 e^{-dx-c} x^2}{2d^2} + \frac{ab\, e^{dx+c} x}{d} - \frac{a^2 e^{-c} \operatorname{expIntegral}(1,dx)}{2} - \frac{a^2 e^{c} \operatorname{expIntegral}(1,-dx)}{2} - \frac{3b^2 e^{d}}{d^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*cosh(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -3/d^4*b^2*exp(-d*x-c)-1/2/d*b^2*exp(-d*x-c)*x^3-3/2/d^2*b^2*exp(-d*x-c)*x^
2+1/d*a*b*exp(d*x+c)*x-1/2*a^2*exp(-c)*Ei(1,d*x)-1/2*a^2*exp(c)*Ei(1,-d*x)-
3/d^4*b^2*exp(d*x+c)-1/d*a*b*exp(-d*x-c)*x-1/d^2*a*b*exp(-d*x-c)+1/2/d*b^2*
exp(d*x+c)*x^3-3/2/d^2*b^2*exp(d*x+c)*x^2+3/d^3*b^2*exp(d*x+c)*x-1/d^2*a*b*
exp(d*x+c)-3/d^3*b^2*exp(-d*x-c)*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(110) = 220$.
time = 0.32, size = 235, normalized size = 2.14

$$-\frac{1}{8}\left(4\,ab\left(\frac{(d^2x^2e^c - 2\,dxe^c + 2\,e^c)e^{(dx)}}{d^3} + \frac{(d^2x^2 + 2\,dx + 2)e^{(-dx-c)}}{d^3}\right) + b^2\left(\frac{(d^4x^4e^c - 4\,d^3x^3e^c + 12\,d^2x^2e^c - 24\,dxe^c + 24\,e^c)e^{(dx)}}{d^5} + \frac{(d^4x^4 + 4\,d^3x^3 + 12\,d^2x^2 + 24\,dx + 24)e^{(-dx-c)}}{d^5}\right) + \frac{4\,a^2\cosh(dx+c)\log(x^2)}{d} - \frac{4\left(\operatorname{Ei}(-dx)\,e^{(-c)} + \operatorname{Ei}(dx)\,e^c\right)a^2}{d}\right)d + \frac{1}{4}\left(b^2x^4 + 4\,abx^2 + 2\,a^2\log(x^2)\right)\cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/8*(4*a*b*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d
*x + 2)*e^(-d*x - c)/d^3) + b^2*((d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*
e^c - 24*d*x*e^c + 24*e^c)*e^(d*x)/d^5 + (d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2
+ 24*d*x + 24)*e^(-d*x - c)/d^5) + 4*a^2*cosh(d*x + c)*log(x^2)/d - 4*(Ei(-
d*x)*e^(-c) + Ei(d*x)*e^c)*a^2/d)*d + 1/4*(b^2*x^4 + 4*a*b*x^2 + 2*a^2*log(
x^2))*cosh(d*x + c)
```

Fricas [A]
time = 0.42, size = 130, normalized size = 1.18

$$-\frac{2\,(3\,b^2d^2x^2 + 2\,abd^2 + 6\,b^2)\cosh(dx+c) - (a^2d^4\operatorname{Ei}(dx) + a^2d^4\operatorname{Ei}(-dx))\cosh(c) - 2\,(b^2d^3x^3 + 2\,(abd^3 + 3\,b^2d)x)\sinh(dx+c) - (a^2d^4\operatorname{Ei}(dx) - a^2d^4\operatorname{Ei}(-dx))\sinh(c)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(3*b^2*d^2*x^2 + 2*a*b*d^2 + 6*b^2)*cosh(d*x + c) - (a^2*d^4*Ei(d*x
) + a^2*d^4*Ei(-d*x))*cosh(c) - 2*(b^2*d^3*x^3 + 2*(a*b*d^3 + 3*b^2*d)*x)*s
inh(d*x + c) - (a^2*d^4*Ei(d*x) - a^2*d^4*Ei(-d*x))*sinh(c))/d^4
```

Sympy [A]
time = 2.47, size = 121, normalized size = 1.10

$$a^{2}\sinh(c)\operatorname{Shi}(dx) + a^{2}\cosh(c)\operatorname{Chi}(dx) + 2ab\left(\begin{cases} \frac{x\sinh(c+dx)}{d} - \frac{\cosh(c+dx)}{d^{2}} & \text{for}\ d \neq 0 \\ \frac{x^{2}\cosh(c)}{2} & \text{otherwise} \end{cases}\right) + b^{2}\left(\begin{cases} \frac{x^{3}\sinh(c+dx)}{d} - \frac{3x^{2}\cosh(c+dx)}{d^{2}} + \frac{6x\sinh(c+dx)}{d^{3}} - \frac{6\cosh(c+dx)}{d^{4}} & \text{for}\ d \neq 0 \\ \frac{x^{4}\cosh(c)}{4} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x,x)
```

```
[Out] a**2*sinh(c)*Shi(d*x) + a**2*cosh(c)*Chi(d*x) + 2*a*b*Piecewise((x*sinh(c +
 d*x)/d - cosh(c + d*x)/d**2, Ne(d, 0)), (x**2*cosh(c)/2, True)) + b**2*Pie
cewise((x**3*sinh(c + d*x)/d - 3*x**2*cosh(c + d*x)/d**2 + 6*x*sinh(c + d*x
)/d**3 - 6*cosh(c + d*x)/d**4, Ne(d, 0)), (x**4*cosh(c)/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(110) = 220.
time = 0.41, size = 222, normalized size = 2.02

$$\frac{b^{2}d^{3}x^{3}e^{(dx+c)} - b^{2}d^{3}x^{3}e^{(-dx-c)} + a^{2}d^{4}\operatorname{Ei}(-dx)\,e^{(-c)} + a^{2}d^{4}\operatorname{Ei}(dx)\,e^{c} + 2\,abd^{3}xe^{(dx+c)} - 3\,b^{2}d^{2}x^{2}e^{(dx+c)} - 2\,abd^{3}xe^{(-dx-c)} - 3\,b^{2}d^{2}x^{2}e^{(-dx-c)} - 2\,abd^{2}e^{(dx+c)} + 6\,b^{2}dxe^{(dx+c)} - 2\,abd^{2}e^{(-dx-c)} - 6\,b^{2}dxe^{(-dx-c)} - 6\,b^{2}e^{(dx+c)} - 6\,b^{2}e^{(-dx-c)}}{2\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^3*x^3*e^(d*x + c) - b^2*d^3*x^3*e^(-d*x - c) + a^2*d^4*Ei(-d*x)*
e^(-c) + a^2*d^4*Ei(d*x)*e^c + 2*a*b*d^3*x*e^(d*x + c) - 3*b^2*d^2*x^2*e^(d
*x + c) - 2*a*b*d^3*x*e^(-d*x - c) - 3*b^2*d^2*x^2*e^(-d*x - c) - 2*a*b*d^2
*e^(d*x + c) + 6*b^2*d*x*e^(d*x + c) - 2*a*b*d^2*e^(-d*x - c) - 6*b^2*d*x*e
^(-d*x - c) - 6*b^2*e^(d*x + c) - 6*b^2*e^(-d*x - c))/d^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,{\left(b\,x^2+a\right)}^2}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^2)^2)/x,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^2)^2)/x, x)
```

3.53 $\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^2} \, dx$

Optimal. Leaf size=95

$$-\frac{a^2\cosh(c+dx)}{x}-\frac{2b^2x\cosh(c+dx)}{d^2}+a^2d\text{Chi}(dx)\sinh(c)+\frac{2b^2\sinh(c+dx)}{d^3}+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2x^2\sinh(}{d}$$

```
[Out] -a^2*cosh(d*x+c)/x-2*b^2*x*cosh(d*x+c)/d^2+a^2*d*cosh(c)*Shi(d*x)+a^2*d*Chi
(d*x)*sinh(c)+2*b^2*sinh(d*x+c)/d^3+2*a*b*sinh(d*x+c)/d+b^2*x^2*sinh(d*x+c)
/d
```

Rubi [A]
time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5395, 2717, 3378, 3384, 3379, 3382, 3377}

$$a^2d\sinh(c)\text{Chi}(dx)+a^2d\cosh(c)\text{Shi}(dx)-\frac{a^2\cosh(c+dx)}{x}+\frac{2ab\sinh(c+dx)}{d}+\frac{2b^2\sinh(c+dx)}{d^3}-\frac{2b^2x\cosh(c+dx)}{d^2}+\frac{b^2x^2\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a^2*Cosh[c + d*x])/x) - (2*b^2*x*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral
[d*x]*Sinh[c] + (2*b^2*Sinh[c + d*x])/d^3 + (2*a*b*Sinh[c + d*x])/d + (b^2*
x^2*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^2}\,dx &= \int \left(2ab\cosh(c+dx)+\frac{a^2\cosh(c+dx)}{x^2}+b^2x^2\cosh(c+dx)\right)dx\\ &= a^2\int \frac{\cosh(c+dx)}{x^2}\,dx+(2ab)\int \cosh(c+dx)\,dx+b^2\int x^2\cosh(c+dx)\,dx\\ &= -\frac{a^2\cosh(c+dx)}{x}+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2x^2\sinh(c+dx)}{d}-\frac{(2b^2)\int x\sinh(c\\ &= -\frac{a^2\cosh(c+dx)}{x}-\frac{2b^2x\cosh(c+dx)}{d^2}+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2x^2\sinh(c+\\ &= -\frac{a^2\cosh(c+dx)}{x}-\frac{2b^2x\cosh(c+dx)}{d^2}+a^2d\text{Chi}(dx)\sinh(c)+\frac{2b^2\sinh(c+}{d^3}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 95, normalized size = 1.00

$$-\frac{a^2\cosh(c+dx)}{x}-\frac{2b^2x\cosh(c+dx)}{d^2}+a^2d\text{Chi}(dx)\sinh(c)+\frac{2b^2\sinh(c+dx)}{d^3}+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2x^2\sinh(c+dx)}{d}+a^2d\cosh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a^2*Cosh[c + d*x])/x) - (2*b^2*x*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral
[d*x]*Sinh[c] + (2*b^2*Sinh[c + d*x])/d^3 + (2*a*b*Sinh[c + d*x])/d + (b^2*
x^2*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
```

Maple [A]
time = 0.88, size = 190, normalized size = 2.00

method	result
risch	$-\frac{b^{2} \mathrm{e}^{-d x -c}}{d^{3}}-\frac{b^{2} \mathrm{e}^{-d x -c} x^{2}}{2 d}-\frac{a^{2} \mathrm{e}^{-d x -c}}{2 x}+\frac{d \,a^{2} \mathrm{e}^{-c} \operatorname{expIntegral}(1,d x)}{2}-\frac{a b \,\mathrm{e}^{-d x -c}}{d}-\frac{b^{2} \mathrm{e}^{-d x -c} x}{d^{2}}-\frac{d \,a^{2} \mathrm{e}^{c} \operatorname{expIntegral}(1,-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*cosh(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^3*b^2*exp(-d*x-c)-1/2/d*b^2*exp(-d*x-c)*x^2-1/2*a^2*exp(-d*x-c)/x+1/2*
d*a^2*exp(-c)*Ei(1,d*x)-1/d*a*b*exp(-d*x-c)-1/d^2*b^2*exp(-d*x-c)*x-1/2*d*a
^2*exp(c)*Ei(1,-d*x)+1/d^3*b^2*exp(d*x+c)+1/2/d*b^2*exp(d*x+c)*x^2-1/d^2*b^
2*exp(d*x+c)*x+a*b/d*exp(d*x+c)-1/2*a^2/x*exp(d*x+c)
```

Maxima [A]
time = 0.32, size = 179, normalized size = 1.88

$$-\frac{1}{6}\left(3\,a^{2}\mathrm{Ei}(-dx)\,e^{(-c)}-3\,a^{2}\mathrm{Ei}(dx)\,e^{c}+\frac{6\,(dxe^{c}-e^{c})abe^{(dx)}}{d^{2}}+\frac{6\,(dx+1)abe^{(-dx-c)}}{d^{2}}+\frac{(d^{3}x^{3}e^{c}-3\,d^{2}x^{2}e^{c}+6\,dxe^{c}-6\,e^{c})b^{2}e^{(dx)}}{d^{4}}+\frac{(d^{3}x^{3}+3\,d^{2}x^{2}+6\,dx+6)b^{2}e^{(-dx-c)}}{d^{4}}\right)d+\frac{1}{3}\left(b^{2}x^{3}+6\,abx-\frac{3\,a^{2}}{x}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*a^2*Ei(-d*x)*e^(-c) - 3*a^2*Ei(d*x)*e^c + 6*(d*x*e^c - e^c)*a*b*e^(
d*x)/d^2 + 6*(d*x + 1)*a*b*e^(-d*x - c)/d^2 + (d^3*x^3*e^c - 3*d^2*x^2*e^c
+ 6*d*x*e^c - 6*e^c)*b^2*e^(d*x)/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*b^
2*e^(-d*x - c)/d^4)*d + 1/3*(b^2*x^3 + 6*a*b*x - 3*a^2/x)*cosh(d*x + c)
```

Fricas [A]
time = 0.39, size = 127, normalized size = 1.34

$$-\frac{2\,(a^{2}d^{3}+2\,b^{2}dx^{2})\cosh\left(dx+c\right)-(a^{2}d^{4}x\mathrm{Ei}(dx)-a^{2}d^{4}x\mathrm{Ei}(-dx))\cosh\left(c\right)-2\,(b^{2}d^{2}x^{3}+2\,(abd^{2}+b^{2})x)\sinh\left(dx+c\right)-(a^{2}d^{4}x\mathrm{Ei}(dx)+a^{2}d^{4}x\mathrm{Ei}(-dx))\sinh\left(c\right)}{2\,d^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^2*d^3 + 2*b^2*d*x^2)*cosh(d*x + c) - (a^2*d^4*x*Ei(d*x) - a^2*d^
4*x*Ei(-d*x))*cosh(c) - 2*(b^2*d^2*x^3 + 2*(a*b*d^2 + b^2)*x)*sinh(d*x + c)
 - (a^2*d^4*x*Ei(d*x) + a^2*d^4*x*Ei(-d*x))*sinh(c))/(d^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{2}\right)^{2}\cosh\left(c+dx\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**2)**2*cosh(c + d*x)/x**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(95) = 190.
time = 0.41, size = 197, normalized size = 2.07

$$-\frac{a^2d^4x\mathrm{Ei}(-dx)\,e^{(-c)}-a^2d^4x\mathrm{Ei}(dx)\,e^c-b^2d^2x^3e^{(dx+c)}+b^2d^2x^3e^{(-dx-c)}+a^2d^3e^{(dx+c)}-2\,abd^2xe^{(dx+c)}+2\,b^2dx^2e^{(dx+c)}+a^2d^3e^{(-dx-c)}+2\,abd^2xe^{(-dx-c)}+2\,b^2dx^2e^{(-dx-c)}-2\,b^2xe^{(dx+c)}+2\,b^2xe^{(-dx-c)}}{2\,d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*d^4*x*Ei(-d*x)*e^(-c) - a^2*d^4*x*Ei(d*x)*e^c - b^2*d^2*x^3*e^(d*
x + c) + b^2*d^2*x^3*e^(-d*x - c) + a^2*d^3*e^(d*x + c) - 2*a*b*d^2*x*e^(d*
x + c) + 2*b^2*d*x^2*e^(d*x + c) + a^2*d^3*e^(-d*x - c) + 2*a*b*d^2*x*e^(-d
*x - c) + 2*b^2*d*x^2*e^(-d*x - c) - 2*b^2*x*e^(d*x + c) + 2*b^2*x*e^(-d*x
- c))/(d^3*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^2)^2)/x^2,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^2)^2)/x^2, x)
```

3.54 $\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3} \, dx$

Optimal. Leaf size=114

$$-\frac{b^2 \cosh(c+dx)}{d^2}-\frac{a^2 \cosh(c+dx)}{2x^2}+2ab\cosh(c)\text{Chi}(dx)+\frac{1}{2}a^2d^2\cosh(c)\text{Chi}(dx)-\frac{a^2 d\sinh(c+dx)}{2x}+\frac{b^2 x \sin$$

```
[Out] 2*a*b*Chi(d*x)*cosh(c)+1/2*a^2*d^2*Chi(d*x)*cosh(c)-b^2*cosh(d*x+c)/d^2-1/2
*a^2*cosh(d*x+c)/x^2+2*a*b*Shi(d*x)*sinh(c)+1/2*a^2*d^2*Shi(d*x)*sinh(c)-1/
2*a^2*d*sinh(d*x+c)/x+b^2*x*sinh(d*x+c)/d
```

Rubi [A]
time = 0.15, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5395, 3378, 3384, 3379, 3382, 3377, 2718}

$$\frac{1}{2}a^2d^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}a^2d^2\sinh(c)\text{Shi}(dx)-\frac{a^2\cosh(c+dx)}{2x^2}-\frac{a^2 d\sinh(c+dx)}{2x}+2ab\cosh(c)\text{Chi}(dx)+2ab\sinh(c)\text{Shi}(dx)-\frac{b^2\cosh(c+dx)}{d^2}+\frac{b^2 x\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Cosh[c + d*x])/x^3,x]
```

```
[Out] -((b^2*Cosh[c + d*x])/d^2) - (a^2*Cosh[c + d*x])/(2*x^2) + 2*a*b*Cosh[c]*Co
shIntegral[d*x] + (a^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 - (a^2*d*Sinh[c + d
*x])/(2*x) + (b^2*x*Sinh[c + d*x])/d + 2*a*b*Sinh[c]*SinhIntegral[d*x] + (a
^2*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3}\,dx &= \int \left(\frac{a^2\cosh(c+dx)}{x^3} + \frac{2ab\cosh(c+dx)}{x} + b^2x\cosh(c+dx)\right)dx \\
&= a^2\int \frac{\cosh(c+dx)}{x^3}\,dx + (2ab)\int \frac{\cosh(c+dx)}{x}\,dx + b^2\int x\cosh(c+dx)\,dx \\
&= -\frac{a^2\cosh(c+dx)}{2x^2} + \frac{b^2x\sinh(c+dx)}{d} - \frac{b^2\int \sinh(c+dx)\,dx}{d} + \frac{1}{2}(a^2d)\int \text{si} \\
&= -\frac{b^2\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{2x^2} + 2ab\cosh(c)\text{Chi}(dx) - \frac{a^2d\sinh(c+d}{2x} \\
&= -\frac{b^2\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{2x^2} + 2ab\cosh(c)\text{Chi}(dx) - \frac{a^2d\sinh(c+d}{2x} \\
&= -\frac{b^2\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{2x^2} + 2ab\cosh(c)\text{Chi}(dx) + \frac{1}{2}a^2d^2\cosh(c)\text{C}
\end{aligned}$$

Mathematica [A]
time = 0.27, size = 97, normalized size = 0.85

$$\frac{1}{2}\left(-\frac{2b^2\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{x^2} + a(4b+ad^2)\cosh(c)\text{Chi}(dx) - \frac{a^2d\sinh(c+dx)}{x} + \frac{2b^2x\sinh(c+dx)}{d} + a(4b+ad^2)\sinh(c)\text{Shi}(dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^3,x]
```

```
[Out] ((-2*b^2*Cosh[c + d*x])/d^2 - (a^2*Cosh[c + d*x])/x^2 + a*(4*b + a*d^2)*Cos
h[c]*CoshIntegral[d*x] - (a^2*d*Sinh[c + d*x])/x + (2*b^2*x*Sinh[c + d*x])/
d + a*(4*b + a*d^2)*Sinh[c]*SinhIntegral[d*x])/2
```

Maple [A]
time = 0.86, size = 188, normalized size = 1.65

method	result
risch	$-\frac{b^2 e^{-dx-c}}{2d^2} - ab\,e^{-c}\,\text{expIntegral}\left(1, dx\right) - \frac{d^2 a^2 e^{-c}\,\text{expIntegral}(1,dx)}{4} + \frac{d\,a^2 e^{-dx-c}}{4x} - \frac{a^2 e^{-dx-c}}{4x^2} - \frac{b^2 e^{-dx-c} x}{2d} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*cosh(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d^2*b^2*exp(-d*x-c)-a*b*exp(-c)*Ei(1,d*x)-1/4*d^2*a^2*exp(-c)*Ei(1,d*x
)+1/4*d*a^2*exp(-d*x-c)/x-1/4*a^2*exp(-d*x-c)/x^2-1/2/d*b^2*exp(-d*x-c)*x-1
/4*d^2*a^2*exp(c)*Ei(1,-d*x)-1/2/d^2*b^2*exp(d*x+c)-1/4*a^2/x^2*exp(d*x+c)+
1/2/d*b^2*exp(d*x+c)*x-a*b*exp(c)*Ei(1,-d*x)-1/4*d*a^2/x*exp(d*x+c)
```

Maxima [A]
time = 0.34, size = 165, normalized size = 1.45

$$\frac{1}{4}\left(\left(de^{(-c)}\Gamma(-1,dx) + de^{c}\Gamma(-1,-dx)\right)a^2 - b^2\left(\frac{(d^2x^2e^c - 2\,dxe^c + 2\,e^c)e^{(dx)}}{d^3} + \frac{(d^2x^2 + 2\,dx + 2)e^{(-dx-c)}}{d^3}\right) - \frac{4\,ab\cosh\left(dx+c\right)\log\left(x^2\right)}{d} + \frac{4\left(\text{Ei}(-dx)\,e^{(-c)} + \text{Ei}(dx)\,e^{c}\right)ab}{d}\right)d + \frac{1}{2}\left(b^2x^2 + 2\,ab\log\left(x^2\right) - \frac{a^2}{x^2}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*((d*e^(-c)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))*a^2 - b^2*((d^2*x^2*
e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^(-d*x - c)/d
^3) - 4*a*b*cosh(d*x + c)*log(x^2)/d + 4*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*a*
b/d)*d + 1/2*(b^2*x^2 + 2*a*b*log(x^2) - a^2/x^2)*cosh(d*x + c)
```

Fricas [A]
time = 0.36, size = 164, normalized size = 1.44

$$-\frac{2\left(a^2d^2 + 2\,b^2x^2\right)\cosh\left(dx+c\right) - \left((a^2d^4 + 4\,abd^2)x^2\text{Ei}(dx) + (a^2d^4 + 4\,abd^2)x^2\text{Ei}(-dx)\right)\cosh\left(c\right) + 2\left(a^2d^3x - 2\,b^2dx^3\right)\sinh\left(dx+c\right) - \left((a^2d^4 + 4\,abd^2)x^2\text{Ei}(dx) - (a^2d^4 + 4\,abd^2)x^2\text{Ei}(-dx)\right)\sinh\left(c\right)}{4\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a^2*d^2 + 2*b^2*x^2)*cosh(d*x + c) - ((a^2*d^4 + 4*a*b*d^2)*x^2*Ei
(d*x) + (a^2*d^4 + 4*a*b*d^2)*x^2*Ei(-d*x))*cosh(c) + 2*(a^2*d^3*x - 2*b^2*
```

```
d*x^3)*sinh(d*x + c) - ((a^2*d^4 + 4*a*b*d^2)*x^2*Ei(d*x) - (a^2*d^4 + 4*a*
b*d^2)*x^2*Ei(-d*x))*sinh(c))/(d^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**2)**2*cosh(c + d*x)/x**3, x)
```

Giac [A]
time = 0.41, size = 206, normalized size = 1.81

$$\frac{a^2d^4x^2\mathrm{Ei}(-dx)\,e^{(-c)} + a^2d^4x^2\mathrm{Ei}(dx)\,e^c + 4\,abd^2x^2\mathrm{Ei}(-dx)\,e^{(-c)} + 4\,abd^2x^2\mathrm{Ei}(dx)\,e^c - a^2d^3xe^{(dx+c)} + 2\,b^2dx^3e^{(dx+c)} + a^2d^3xe^{(-dx-c)} - 2\,b^2dx^3e^{(-dx-c)} - a^2d^2e^{(dx+c)} - 2\,b^2x^2e^{(dx+c)} - a^2d^2e^{(-dx-c)} - 2\,b^2x^2e^{(-dx-c)}}{4\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^4*x^2*Ei(-d*x)*e^(-c) + a^2*d^4*x^2*Ei(d*x)*e^c + 4*a*b*d^2*x^2*
Ei(-d*x)*e^(-c) + 4*a*b*d^2*x^2*Ei(d*x)*e^c - a^2*d^3*x*e^(d*x + c) + 2*b^2
*d*x^3*e^(d*x + c) + a^2*d^3*x*e^(-d*x - c) - 2*b^2*d*x^3*e^(-d*x - c) - a^
2*d^2*e^(d*x + c) - 2*b^2*x^2*e^(d*x + c) - a^2*d^2*e^(-d*x - c) - 2*b^2*x^
2*e^(-d*x - c))/(d^2*x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,{\left(b\,x^2+a\right)}^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^2)^2)/x^3,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^2)^2)/x^3, x)
```

3.55 $\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^4} \, dx$

Optimal. Leaf size=133

$$-\frac{a^2 \cosh(c+dx)}{3x^3} - \frac{2ab\cosh(c+dx)}{x} - \frac{a^2 d^2 \cosh(c+dx)}{6x} + 2abd\text{Chi}(dx)\sinh(c) + \frac{1}{6}a^2 d^3 \text{Chi}(dx)\sinh(c) +$$

```
[Out] -1/3*a^2*cosh(d*x+c)/x^3-2*a*b*cosh(d*x+c)/x-1/6*a^2*d^2*cosh(d*x+c)/x+2*a*
b*d*cosh(c)*Shi(d*x)+1/6*a^2*d^3*cosh(c)*Shi(d*x)+2*a*b*d*Chi(d*x)*sinh(c)+
1/6*a^2*d^3*Chi(d*x)*sinh(c)+b^2*sinh(d*x+c)/d-1/6*a^2*d*sinh(d*x+c)/x^2
```

Rubi [A]

time = 0.18, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5395, 2717, 3378, 3384, 3379, 3382}

$$\frac{1}{6}a^2 d^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}a^2 d^3 \cosh(c)\text{Shi}(dx) - \frac{a^2 d^2 \cosh(c+dx)}{6x} - \frac{a^2 \cosh(c+dx)}{3x^3} - \frac{a^2 d \sinh(c+dx)}{6x^2} + 2abd\sinh(c)\text{Chi}(dx) + 2abd\cosh(c)\text{Shi}(dx) - \frac{2ab\cosh(c+dx)}{x} + \frac{b^2 \sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Cosh[c + d*x])/x^4,x]
```

```
[Out] -1/3*(a^2*Cosh[c + d*x])/x^3 - (2*a*b*Cosh[c + d*x])/x - (a^2*d^2*Cosh[c +
d*x])/(6*x) + 2*a*b*d*CoshIntegral[d*x]*Sinh[c] + (a^2*d^3*CoshIntegral[d*x
]*Sinh[c])/6 + (b^2*Sinh[c + d*x])/d - (a^2*d*Sinh[c + d*x])/(6*x^2) + 2*a*
b*d*Cosh[c]*SinhIntegral[d*x] + (a^2*d^3*Cosh[c]*SinhIntegral[d*x])/6
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^4}\,dx &= \int \left(b^2\cosh(c+dx)+\frac{a^2\cosh(c+dx)}{x^4}+\frac{2ab\cosh(c+dx)}{x^2}\right)dx \\
&= a^2\int \frac{\cosh(c+dx)}{x^4}\,dx+(2ab)\int \frac{\cosh(c+dx)}{x^2}\,dx+b^2\int \cosh(c+dx)\,dx \\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{2ab\cosh(c+dx)}{x}+\frac{b^2\sinh(c+dx)}{d}+\frac{1}{3}\left(a^2d\right)\int \frac{\sinh(c}{x} \\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{2ab\cosh(c+dx)}{x}+\frac{b^2\sinh(c+dx)}{d}-\frac{a^2d\sinh(c+dx)}{6x^2} \\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{2ab\cosh(c+dx)}{x}-\frac{a^2d^2\cosh(c+dx)}{6x}+2abd\mathrm{Chi}(dx)\,\mathrm{s} \\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{2ab\cosh(c+dx)}{x}-\frac{a^2d^2\cosh(c+dx)}{6x}+2abd\mathrm{Chi}(dx)\,\mathrm{s} \\
&= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{2ab\cosh(c+dx)}{x}-\frac{a^2d^2\cosh(c+dx)}{6x}+2abd\mathrm{Chi}(dx)\,\mathrm{s}
\end{aligned}
$$

Mathematica [A]
time = 0.28, size = 114, normalized size = 0.86

$$\frac{1}{6}\left(-\frac{2a^2\cosh(c+dx)}{x^3}-\frac{12ab\cosh(c+dx)}{x}-\frac{a^2d^2\cosh(c+dx)}{x}+ad\left(12b+ad^2\right)\mathrm{Chi}(dx)\sinh(c)+\frac{6b^2\sinh(c+dx)}{d}-\frac{a^2d\sinh(c+dx)}{x^2}+ad\left(12b+ad^2\right)\cosh(c)\mathrm{Shi}(dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^4,x]
```

```
[Out] ((-2*a^2*Cosh[c + d*x])/x^3 - (12*a*b*Cosh[c + d*x])/x - (a^2*d^2*Cosh[c +
d*x])/x + a*d*(12*b + a*d^2)*CoshIntegral[d*x]*Sinh[c] + (6*b^2*Sinh[c + d*
x])/d - (a^2*d*Sinh[c + d*x])/x^2 + a*d*(12*b + a*d^2)*Cosh[c]*SinhIntegral
[d*x])/6
```

Maple [A]
time = 0.83, size = 222, normalized size = 1.67

method	result
risch	$-\frac{b^2 \mathrm{e}^{-dx-c}}{2d}+\frac{d^3 a^2 \mathrm{e}^{-c} \operatorname{expIntegral}(1,dx)}{12}-\frac{d^2 a^2 \mathrm{e}^{-dx-c}}{12x}+\frac{d\,a^2 \mathrm{e}^{-dx-c}}{12x^2}-\frac{a^2 \mathrm{e}^{-dx-c}}{6x^3}-\frac{ab\,\mathrm{e}^{-dx-c}}{x}+dab\,\mathrm{e}^{-c} \operatorname{expInte}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*cosh(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*b^2*exp(-d*x-c)+1/12*d^3*a^2*exp(-c)*Ei(1,d*x)-1/12*d^2*a^2*exp(-d*x
-c)/x+1/12*d*a^2*exp(-d*x-c)/x^2-1/6*a^2*exp(-d*x-c)/x^3-a*b*exp(-d*x-c)/x+
d*a*b*exp(-c)*Ei(1,d*x)-1/12*d^3*a^2*exp(c)*Ei(1,-d*x)-1/6*a^2/x^3*exp(d*x+
c)+1/2/d*exp(d*x+c)*b^2-a*b/x*exp(d*x+c)-d*a*b*exp(c)*Ei(1,-d*x)-1/12*d*a^2
/x^2*exp(d*x+c)-1/12*d^2*a^2/x*exp(d*x+c)
```

Maxima [A]
time = 0.33, size = 135, normalized size = 1.02

$$\frac{1}{6}\left(a^2d^2e^{(-c)}\Gamma(-2,dx)-a^2d^2e^c\Gamma(-2,-dx)-6\,ab\mathrm{Ei}(-dx)\,e^{(-c)}+6\,ab\mathrm{Ei}(dx)\,e^c-\frac{3\,(dxe^c-e^c)b^2e^{(dx)}}{d^2}-\frac{3\,(dx+1)b^2e^{(-dx-c)}}{d^2}\right)d+\frac{1}{3}\left(3\,b^2x-\frac{6\,abx^2+a^2}{x^3}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*d^2*e^(-c)*gamma(-2, d*x) - a^2*d^2*e^c*gamma(-2, -d*x) - 6*a*b*Ei
(-d*x)*e^(-c) + 6*a*b*Ei(d*x)*e^c - 3*(d*x*e^c - e^c)*b^2*e^(d*x)/d^2 - 3*(
d*x + 1)*b^2*e^(-d*x - c)/d^2)*d + 1/3*(3*b^2*x - (6*a*b*x^2 + a^2)/x^3)*co
sh(d*x + c)
```

Fricas [A]
time = 0.37, size = 171, normalized size = 1.29

$$-\frac{2\,(2\,a^2d+(a^2d^3+12\,abd)x^2)\cosh\left(dx+c\right)-((a^2d^4+12\,abd^2)x^3\mathrm{Ei}(dx)-(a^2d^4+12\,abd^2)x^3\mathrm{Ei}(-dx))\cosh\left(c\right)+2\,(a^2d^2x-6\,b^2x^3)\sinh\left(dx+c\right)-((a^2d^4+12\,abd^2)x^3\mathrm{Ei}(dx)+(a^2d^4+12\,abd^2)x^3\mathrm{Ei}(-dx))\sinh\left(c\right)}{12\,dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(2*a^2*d + (a^2*d^3 + 12*a*b*d)*x^2)*cosh(d*x + c) - ((a^2*d^4 + 1
2*a*b*d^2)*x^3*Ei(d*x) - (a^2*d^4 + 12*a*b*d^2)*x^3*Ei(-d*x))*cosh(c) + 2*(
a^2*d^2*x - 6*b^2*x^3)*sinh(d*x + c) - ((a^2*d^4 + 12*a*b*d^2)*x^3*Ei(d*x)
+ (a^2*d^4 + 12*a*b*d^2)*x^3*Ei(-d*x))*sinh(c))/(d*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2\right)^2 \cosh\left(c + dx\right)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x**2)**2*cosh(c + d*x)/x**4, x)
```

Giac [A]
time = 0.42, size = 236, normalized size = 1.77

$$-\frac{a^2 d^4 x^3 \mathrm{Ei}\left(-dx\right) e^{(-c)} - a^2 d^4 x^3 \mathrm{Ei}\left(dx\right) e^c + 12\, abd^2 x^3 \mathrm{Ei}\left(-dx\right) e^{(-c)} - 12\, abd^2 x^3 \mathrm{Ei}\left(dx\right) e^c + a^2 d^3 x^2 e^{(dx+c)} + a^2 d^3 x^2 e^{(-dx-c)} + a^2 d^2 x e^{(dx+c)} + 12\, abdx^2 e^{(dx+c)} - 6\, b^2 x^3 e^{(dx+c)} - a^2 d^2 x e^{(-dx-c)} + 12\, abdx^2 e^{(-dx-c)} + 6\, b^2 x^3 e^{(-dx-c)} + 2\, a^2 de^{(dx+c)} + 2\, a^2 de^{(-dx-c)}}{12\, dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(a^2*d^4*x^3*Ei(-d*x)*e^(-c) - a^2*d^4*x^3*Ei(d*x)*e^c + 12*a*b*d^2*x
^3*Ei(-d*x)*e^(-c) - 12*a*b*d^2*x^3*Ei(d*x)*e^c + a^2*d^3*x^2*e^(d*x + c) +
 a^2*d^3*x^2*e^(-d*x - c) + a^2*d^2*x*e^(d*x + c) + 12*a*b*d*x^2*e^(d*x + c
) - 6*b^2*x^3*e^(d*x + c) - a^2*d^2*x*e^(-d*x - c) + 12*a*b*d*x^2*e^(-d*x -
 c) + 6*b^2*x^3*e^(-d*x - c) + 2*a^2*d*e^(d*x + c) + 2*a^2*d*e^(-d*x - c))/
(d*x^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{cosh}\left(c+d\,x\right)\,{\left(b\,x^2+a\right)}^2}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^2)^2)/x^4,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^2)^2)/x^4, x)
```

3.56 $\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^5} \, dx$

Optimal. Leaf size=175

$$-\frac{a^2\cosh(c+dx)}{4x^4}-\frac{ab\cosh(c+dx)}{x^2}-\frac{a^2d^2\cosh(c+dx)}{24x^2}+b^2\cosh(c)\text{Chi}(dx)+abd^2\cosh(c)\text{Chi}(dx)+\frac{1}{24}a^2d^4\text{ c}$$

```
[Out] b^2*Chi(d*x)*cosh(c)+a*b*d^2*Chi(d*x)*cosh(c)+1/24*a^2*d^4*Chi(d*x)*cosh(c)
-1/4*a^2*cosh(d*x+c)/x^4-a*b*cosh(d*x+c)/x^2-1/24*a^2*d^2*cosh(d*x+c)/x^2+b
^2*Shi(d*x)*sinh(c)+a*b*d^2*Shi(d*x)*sinh(c)+1/24*a^2*d^4*Shi(d*x)*sinh(c)-
1/12*a^2*d*sinh(d*x+c)/x^3-a*b*d*sinh(d*x+c)/x-1/24*a^2*d^3*sinh(d*x+c)/x
```

Rubi [A]
time = 0.24, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5395, 3378, 3384, 3379, 3382}

$$\frac{1}{24}a^2d^4\cosh(c)\text{Chi}(dx)+\frac{1}{24}a^2d^4\sinh(c)\text{Shi}(dx)-\frac{a^2d^3\sinh(c+dx)}{24x}-\frac{a^2d^2\cosh(c+dx)}{24x^2}-\frac{a^2\cosh(c+dx)}{4x^4}-\frac{a^2d\sinh(c+dx)}{12x^3}+abd^2\cosh(c)\text{Chi}(dx)+abd^2\sinh(c)\text{Shi}(dx)-\frac{ab\cosh(c+dx)}{x^2}-\frac{abd\sinh(c+dx)}{x}+b^2\cosh(c)\text{Chi}(dx)+b^2\sinh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Cosh[c + d*x])/x^5,x]
```

```
[Out] -1/4*(a^2*Cosh[c + d*x])/x^4 - (a*b*Cosh[c + d*x])/x^2 - (a^2*d^2*Cosh[c +
d*x])/(24*x^2) + b^2*Cosh[c]*CoshIntegral[d*x] + a*b*d^2*Cosh[c]*CoshIntegr
al[d*x] + (a^2*d^4*Cosh[c]*CoshIntegral[d*x])/24 - (a^2*d*Sinh[c + d*x])/(1
2*x^3) - (a*b*d*Sinh[c + d*x])/x - (a^2*d^3*Sinh[c + d*x])/(24*x) + b^2*Sin
h[c]*SinhIntegral[d*x] + a*b*d^2*Sinh[c]*SinhIntegral[d*x] + (a^2*d^4*Sinh[
c]*SinhIntegral[d*x])/24
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^5}\,dx &= \int \left(\frac{a^2\cosh(c+dx)}{x^5}+\frac{2ab\cosh(c+dx)}{x^3}+\frac{b^2\cosh(c+dx)}{x}\right)dx \\
&= a^2\int \frac{\cosh(c+dx)}{x^5}\,dx+(2ab)\int \frac{\cosh(c+dx)}{x^3}\,dx+b^2\int \frac{\cosh(c+dx)}{x}\,dx \\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{ab\cosh(c+dx)}{x^2}+\frac{1}{4}(a^2d)\int \frac{\sinh(c+dx)}{x^4}\,dx+(abd)\int \\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{ab\cosh(c+dx)}{x^2}+b^2\cosh(c)\text{Chi}(dx)-\frac{a^2d\sinh(c+dx}{12x^3} \\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{ab\cosh(c+dx)}{x^2}-\frac{a^2d^2\cosh(c+dx)}{24x^2}+b^2\cosh(c)\text{Chi}(d \\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{ab\cosh(c+dx)}{x^2}-\frac{a^2d^2\cosh(c+dx)}{24x^2}+b^2\cosh(c)\text{Chi}(d \\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{ab\cosh(c+dx)}{x^2}-\frac{a^2d^2\cosh(c+dx)}{24x^2}+b^2\cosh(c)\text{Chi}(d \\
&= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{ab\cosh(c+dx)}{x^2}-\frac{a^2d^2\cosh(c+dx)}{24x^2}+b^2\cosh(c)\text{Chi}(d
\end{aligned}$$

Mathematica [A]
time = 0.30, size = 124, normalized size = 0.71

$$\frac{(24b^2+24abd^2+a^2d^4)\,x^4\cosh(c)\text{Chi}(dx)-a\left((6a+24bx^2+ad^2x^2)\cosh(c+dx)+dx(2a+24bx^2+ad^2x^2)\sinh(c+dx)\right)+(24b^2+24abd^2+a^2d^4)\,x^4\sinh(c)\text{Shi}(dx)}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^5,x]
```

```
[Out] ((24*b^2 + 24*a*b*d^2 + a^2*d^4)*x^4*Cosh[c]*CoshIntegral[d*x] - a*((6*a +
24*b*x^2 + a*d^2*x^2)*Cosh[c + d*x] + d*x*(2*a + 24*b*x^2 + a*d^2*x^2)*Sinh
[c + d*x]) + (24*b^2 + 24*a*b*d^2 + a^2*d^4)*x^4*Sinh[c]*SinhIntegral[d*x])
/(24*x^4)
```

Maple [A]
time = 0.79, size = 291, normalized size = 1.66

method	result
risch	$-\frac{b^{2} \mathrm{e}^{-c} \mathrm{expIntegral}(1, d x)}{2}-\frac{d^{4} a^{2} \mathrm{e}^{-c} \mathrm{expIntegral}(1, d x)}{48}-\frac{a b\, \mathrm{e}^{-d x-c}}{2 x^{2}}-\frac{d^{2} a b\, \mathrm{e}^{-c} \mathrm{expIntegral}(1, d x)}{2}+\frac{d a b\, \mathrm{e}^{-d x-c}}{2 x}+\frac{d^{3} a^{2} \mathrm{e}^{-}}{48}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*cosh(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^2*exp(-c)*Ei(1,d*x)-1/48*d^4*a^2*exp(-c)*Ei(1,d*x)-1/2*a*b*exp(-d*x-
c)/x^2-1/2*d^2*a*b*exp(-c)*Ei(1,d*x)+1/2*d*a*b*exp(-d*x-c)/x+1/48*d^3*a^2*e
xp(-d*x-c)/x-1/48*d^2*a^2*exp(-d*x-c)/x^2+1/24*d*a^2*exp(-d*x-c)/x^3-1/8*a^
2*exp(-d*x-c)/x^4-1/48*d^4*a^2*exp(c)*Ei(1,-d*x)-1/8*a^2/x^4*exp(d*x+c)-1/2
4*d*a^2/x^3*exp(d*x+c)-1/48*d^2*a^2/x^2*exp(d*x+c)-1/48*d^3*a^2/x*exp(d*x+c
)-1/2*a*b/x^2*exp(d*x+c)-1/2*d*a*b/x*exp(d*x+c)-1/2*d^2*a*b*exp(c)*Ei(1,-d*
x)-1/2*b^2*exp(c)*Ei(1,-d*x)
```

Maxima [A]
time = 0.35, size = 139, normalized size = 0.79

$$\frac{1}{8}\left(\left(d^{3} e^{(-c)} \Gamma(-3, d x)+d^{3} e^{c} \Gamma(-3,-d x)\right) a^{2}+4\left(d e^{(-c)} \Gamma(-1, d x)+d e^{c} \Gamma(-1,-d x)\right) a b-\frac{4 b^{2} \cosh (d x+c) \log \left(x^{2}\right)}{d}+\frac{4\left(\mathrm{Ei}(-d x) e^{(-c)}+\mathrm{Ei}(d x) e^{c}\right) b^{2}}{d}\right) d+\frac{1}{4}\left(2 b^{2} \log \left(x^{2}\right)-\frac{4 a b x^{2}+a^{2}}{x^{4}}\right) \cosh (d x+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*((d^3*e^(-c)*gamma(-3, d*x) + d^3*e^c*gamma(-3, -d*x))*a^2 + 4*(d*e^(-c
)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))*a*b - 4*b^2*cosh(d*x + c)*log(x^2
)/d + 4*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b^2/d)*d + 1/4*(2*b^2*log(x^2) - (4
*a*b*x^2 + a^2)/x^4)*cosh(d*x + c)
```

Fricas [A]
time = 0.36, size = 194, normalized size = 1.11

$$-\frac{2\left(\left(a^{2} d^{2}+24 a b\right) x^{2}+6 a^{2}\right) \cosh (d x+c)-\left(\left(a^{2} d^{4}+24 a b d^{2}+24 b^{2}\right) x^{4} \mathrm{Ei}(d x)+\left(a^{2} d^{4}+24 a b d^{2}+24 b^{2}\right) x^{4} \mathrm{Ei}(-d x)\right) \cosh (c)+2\left(2 a^{2} d x+\left(a^{2} d^{3}+24 a b d\right) x^{3}\right) \sinh (d x+c)-\left(\left(a^{2} d^{4}+24 a b d^{2}+24 b^{2}\right) x^{4} \mathrm{Ei}(d x)-\left(a^{2} d^{4}+24 a b d^{2}+24 b^{2}\right) x^{4} \mathrm{Ei}(-d x)\right) \sinh (c)}{48 x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(2*((a^2*d^2 + 24*a*b)*x^2 + 6*a^2)*cosh(d*x + c) - ((a^2*d^4 + 24*a*
b*d^2 + 24*b^2)*x^4*Ei(d*x) + (a^2*d^4 + 24*a*b*d^2 + 24*b^2)*x^4*Ei(-d*x))
```

```
*cosh(c) + 2*(2*a^2*d*x + (a^2*d^3 + 24*a*b*d)*x^3)*sinh(d*x + c) - ((a^2*d
^4 + 24*a*b*d^2 + 24*b^2)*x^4*Ei(d*x) - (a^2*d^4 + 24*a*b*d^2 + 24*b^2)*x^4
*Ei(-d*x))*sinh(c))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{2}\right)^{2} \cosh\left(c + dx\right)}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x**2)**2*cosh(c + d*x)/x**5, x)
```

Giac [A]
time = 0.41, size = 294, normalized size = 1.68

$$\frac{a^{2} d^{4} x^{4} \mathrm{Ei}\left(-dx\right) e^{\left(-c\right)} + a^{2} d^{4} x^{4} \mathrm{Ei}\left(dx\right) e^{c} + 24\, abd^{2} x^{4} \mathrm{Ei}\left(-dx\right) e^{\left(-c\right)} + 24\, abd^{2} x^{4} \mathrm{Ei}\left(dx\right) e^{c} - a^{2} d^{3} x^{3} e^{\left(dx+c\right)} + a^{2} d^{3} x^{3} e^{\left(-dx-c\right)} + 24\, b^{2} x^{4} \mathrm{Ei}\left(-dx\right) e^{\left(-c\right)} + 24\, b^{2} x^{4} \mathrm{Ei}\left(dx\right) e^{c} - a^{2} d^{2} x^{2} e^{\left(dx+c\right)} - 24\, abdx^{3} e^{\left(dx+c\right)} - a^{2} d^{2} x^{2} e^{\left(-dx-c\right)} + 24\, abdx^{3} e^{\left(-dx-c\right)} - 2\, a^{2} dx e^{\left(dx+c\right)} - 24\, abx^{2} e^{\left(dx+c\right)} + 2\, a^{2} dx e^{\left(-dx-c\right)} - 24\, abx^{2} e^{\left(-dx-c\right)} - 6\, a^{2} e^{\left(dx+c\right)} - 6\, a^{2} e^{\left(-dx-c\right)}}{48\, x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] 1/48*(a^2*d^4*x^4*Ei(-d*x)*e^(-c) + a^2*d^4*x^4*Ei(d*x)*e^c + 24*a*b*d^2*x^
4*Ei(-d*x)*e^(-c) + 24*a*b*d^2*x^4*Ei(d*x)*e^c - a^2*d^3*x^3*e^(d*x + c) +
a^2*d^3*x^3*e^(-d*x - c) + 24*b^2*x^4*Ei(-d*x)*e^(-c) + 24*b^2*x^4*Ei(d*x)*
e^c - a^2*d^2*x^2*e^(d*x + c) - 24*a*b*d*x^3*e^(d*x + c) - a^2*d^2*x^2*e^(-
d*x - c) + 24*a*b*d*x^3*e^(-d*x - c) - 2*a^2*d*x*e^(d*x + c) - 24*a*b*x^2*e
^(d*x + c) + 2*a^2*d*x*e^(-d*x - c) - 24*a*b*x^2*e^(-d*x - c) - 6*a^2*e^(d*
x + c) - 6*a^2*e^(-d*x - c))/x^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + d\,x)\,{\left(b\,x^2 + a\right)}^2}{x^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^2)^2)/x^5,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^2)^2)/x^5, x)
```

3.57 $\int \frac{x^4 \cosh(c+dx)}{a+bx^2} \, dx$

Optimal. Leaf size=273

$$-\frac{2x\cosh(c+dx)}{bd^2}+\frac{(-a)^{3/2}\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^{5/2}}-\frac{(-a)^{3/2}\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-}}{\sqrt{}}\right.}{2b^{5/2}}$$

```
[Out] -2*x*cosh(d*x+c)/b/d^2-1/2*(-a)^(3/2)*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-
d*(-a)^(1/2)/b^(1/2))/b^(5/2)+1/2*(-a)^(3/2)*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))
*cosh(c+d*(-a)^(1/2)/b^(1/2))/b^(5/2)+2*sinh(d*x+c)/b/d^3-a*sinh(d*x+c)/b^2
/d+x^2*sinh(d*x+c)/b/d-1/2*(-a)^(3/2)*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-
d*(-a)^(1/2)/b^(1/2))/b^(5/2)+1/2*(-a)^(3/2)*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*
sinh(c+d*(-a)^(1/2)/b^(1/2))/b^(5/2)
```

Rubi [A]
time = 0.51, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5401, 2717, 3377, 5389, 3384, 3379, 3382}

$$\frac{(-a)^{3/2}\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^{5/2}}-\frac{(-a)^{3/2}\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{(-a)^{3/2}\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^{5/2}}-\frac{(-a)^{3/2}\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{a\sinh(c+dx)}{b^2d}+\frac{2\sinh(c+dx)}{bd^3}-\frac{2x\cosh(c+dx)}{bd^2}+\frac{x^2\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] (-2*x*Cosh[c + d*x])/(b*d^2) + ((-a)^(3/2)*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*C
oshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^(5/2)) - ((-a)^(3/2)*Cosh[c -
 (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(5/2)
) + (2*Sinh[c + d*x])/(b*d^3) - (a*Sinh[c + d*x])/(b^2*d) + (x^2*Sinh[c + d
*x])/(b*d) - ((-a)^(3/2)*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[
-a]*d)/Sqrt[b] - d*x])/(2*b^(5/2)) - ((-a)^(3/2)*Sinh[c - (Sqrt[-a]*d)/Sqrt
[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(5/2))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^2}\,dx = \int \left(-\frac{a\cosh(c+dx)}{b^2} + \frac{x^2\cosh(c+dx)}{b} + \frac{a^2\cosh(c+dx)}{b^2(a+bx^2)}\right)dx$$

$$= -\frac{a\int \cosh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{\cosh(c+dx)}{a+bx^2}\,dx}{b^2} + \frac{\int x^2\cosh(c+dx)\,dx}{b}$$

$$= -\frac{a\sinh(c+dx)}{b^2 d} + \frac{x^2\sinh(c+dx)}{bd} + \frac{a^2\int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right.}{b^2}$$

$$= -\frac{2x\cosh(c+dx)}{bd^2} - \frac{a\sinh(c+dx)}{b^2 d} + \frac{x^2\sinh(c+dx)}{bd} - \frac{(-a)^{3/2}\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2b^2}$$

$$= -\frac{2x\cosh(c+dx)}{bd^2} + \frac{2\sinh(c+dx)}{bd^3} - \frac{a\sinh(c+dx)}{b^2 d} + \frac{x^2\sinh(c+dx)}{bd} - \frac{\left((-a)^3\right.}{}$$

$$= -\frac{2x\cosh(c+dx)}{bd^2} + \frac{(-a)^{3/2}\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^{5/2}} - \frac{(-a)^{3/2}\,\mathrm{co}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.33, size = 274, normalized size = 1.00

$$\frac{-4b^{3/2}dx\cosh(c+dx) + ia^{3/2}d^3\cosh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(-\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right) - ia^{3/2}d^3\cosh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right) + 4b^{3/2}\sinh(c+dx) - 2a\sqrt{b}\,d^2\sinh(c+dx) + 2b^{3/2}d^2x^2\sinh(c+dx) - a^{3/2}d^3\sinh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}-idx\right) - a^{3/2}d^3\sinh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)}{2b^{5/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] (-4*b^(3/2)*d*x*Cosh[c + d*x] + I*a^(3/2)*d^3*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b
]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - I*a^(3/2)*d^3*Cosh[c + (I*
Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + 4*b^(3/2)*Si
nh[c + d*x] - 2*a*Sqrt[b]*d^2*Sinh[c + d*x] + 2*b^(3/2)*d^2*x^2*Sinh[c + d*
x] - a^(3/2)*d^3*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sq
rt[b] - I*d*x] - a^(3/2)*d^3*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(S
qrt[a]*d)/Sqrt[b] + I*d*x])/(2*b^(5/2)*d^3)
```

Maple [A]
time = 1.17, size = 369, normalized size = 1.35

method	result

risch	$-\frac{\mathrm{e}^{-dx-c}x^{2}}{2db}+\frac{\mathrm{e}^{-dx-c}a}{2b^{2}d}-\frac{\mathrm{e}^{-dx-c}x}{d^{2}b}-\frac{\mathrm{e}^{-dx-c}}{d^{3}b}+\frac{\mathrm{e}^{-\frac{-d\sqrt{-ab}+bc}{b}}\operatorname{expIntegral}\left(1,\frac{d\sqrt{-ab}+b(dx+c)-bc}{b}\right)a^{2}}{4b^{2}\sqrt{-ab}}-\frac{\mathrm{e}^{-\frac{d\sqrt{-ab}+}{b}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cosh(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*exp(-d*x-c)/b*x^2+1/2/b^2/d*exp(-d*x-c)*a-1/d^2*exp(-d*x-c)/b*x-1/d^
3*exp(-d*x-c)/b+1/4/b^2/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*
(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)*a^2-1/4/b^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2
)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)*a^2+1/2/d*exp(d*x+c)/b*x^
2-1/2*a/b^2/d*exp(d*x+c)-1/d^2/b*exp(d*x+c)*x+1/d^3/b*exp(d*x+c)-1/4/b^2/(-
a*b)^(1/2)*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/
b)*a^2+1/4/b^2/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1
/2)+b*(d*x+c)-b*c)/b)*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*((b*d^2*x^4*e^(2*c) - 2*b*d*x^3*e^(2*c) - 2*a*d*x*e^(2*c) + 2*b*x^2*e^(
2*c))*e^(d*x) - (b*d^2*x^4 + 2*b*d*x^3 + 2*a*d*x + 2*b*x^2)*e^(-d*x))/(b^2*
d^3*x^2*e^c + a*b*d^3*e^c) + 1/2*integrate(2*(a*b*d*x^2*e^c + a^2*d*e^c + (
a^2*d^2*e^c - 2*a*b*e^c)*x)*e^(d*x)/(b^3*d^3*x^4 + 2*a*b^2*d^3*x^2 + a^2*b*
d^3), x) + 1/2*integrate(2*(a*b*d*x^2 + a^2*d - (a^2*d^2 - 2*a*b)*x)*e^(-d*
x)/(b^3*d^3*x^4*e^c + 2*a*b^2*d^3*x^2*e^c + a^2*b*d^3*e^c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(217) = 434$.
time = 0.36, size = 605, normalized size = 2.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(8*b*d*x*cosh(d*x + c) + ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)
^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + (a*d^2*cosh(d*x + c)^2 - a*d^
2*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(
-a*d^2/b)) - ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*sqrt(-a*d^2/b
```

```
)*Ei(d*x + sqrt(-a*d^2/b)) + (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2
)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - 4*(
b*d^2*x^2 - a*d^2 + 2*b)*sinh(d*x + c) + ((a*d^2*cosh(d*x + c)^2 - a*d^2*si
nh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - (a*d^2*cosh(d*x +
c)^2 - a*d^2*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b)))*sin
h(c + sqrt(-a*d^2/b)) + ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*sq
rt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) - (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh
(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^
2/b)))/(b^2*d^3*cosh(d*x + c)^2 - b^2*d^3*sinh(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \cosh(c + dx)}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cosh(d*x+c)/(b*x**2+a),x)
[Out] Integral(x**4*cosh(c + d*x)/(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(x^4*cosh(d*x + c)/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \cosh(c + d\,x)}{b\,x^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*cosh(c + d*x))/(a + b*x^2),x)
[Out] int((x^4*cosh(c + d*x))/(a + b*x^2), x)
```

3.58 $\int \frac{x^3 \cosh(c+dx)}{a+bx^2}\, dx$

Optimal. Leaf size=209

$$-\frac{\cosh(c+dx)}{bd^2}-\frac{a\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^2}-\frac{a\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2b^2}+\frac{x\,\text{si}}{}$$

```
[Out] -cosh(d*x+c)/b/d^2-1/2*a*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/
b^(1/2))/b^2-1/2*a*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/
2))/b^2+x*sinh(d*x+c)/b/d-1/2*a*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)
^(1/2)/b^(1/2))/b^2-1/2*a*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)
/b^(1/2))/b^2
```

Rubi [A]
time = 0.26, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5401, 3377, 2718, 3384, 3379, 3382}

$$-\frac{a\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^2}-\frac{a\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^2}+\frac{a\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^2}-\frac{a\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^2}-\frac{\cosh(c+dx)}{bd^2}+\frac{x\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] -(Cosh[c + d*x]/(b*d^2)) - (a*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(
Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2) - (a*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*Cos
hIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^2) + (x*Sinh[c + d*x])/(b*d) +
(a*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])
/(2*b^2) - (a*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt
[b] + d*x])/(2*b^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\cosh(c+dx)}{a+bx^2}\,dx &= \int \left(\frac{x\cosh(c+dx)}{b}-\frac{ax\cosh(c+dx)}{b\left(a+bx^2\right)}\right)dx \\
&= \frac{\int x\cosh(c+dx)\,dx}{b}-\frac{a\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{b} \\
&= \frac{x\sinh(c+dx)}{bd}-\frac{a\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx}{b}-\frac{\int \sinh
\\
&= -\frac{\cosh(c+dx)}{bd^2}+\frac{x\sinh(c+dx)}{bd}+\frac{a\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2b^{3/2}}-\frac{a\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2b^{3/2}} \\
&= -\frac{\cosh(c+dx)}{bd^2}+\frac{x\sinh(c+dx)}{bd}-\frac{\left(a\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2b^{3/2}} \\
&= -\frac{\cosh(c+dx)}{bd^2}-\frac{a\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^2}-\frac{a\cosh\left(c-\frac{\sqrt{-a}}{\sqrt{b}}\right.}{}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.30, size = 210, normalized size = 1.00

$$-\frac{2b\cosh(c+dx)+ad^2\cosh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(-\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)+ad^2\cosh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)-2bdx\sinh(c+dx)+iad^2\sinh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}-idx\right)-iad^2\sinh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)}{2b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] -1/2*(2*b*Cosh[c + d*x] + a*d^2*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral
[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + a*d^2*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*Co
sIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - 2*b*d*x*Sinh[c + d*x] + I*a*d^2*Si
nh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - I*
a*d^2*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d
*x])/(b^2*d^2)
```

Maple [A]
time = 0.99, size = 268, normalized size = 1.28

method	result
risch	$-\frac{e^{-dx-c}x}{2db}-\frac{e^{-dx-c}}{2d^2b}+\frac{e^{-\frac{d\sqrt{-ab}+bc}{b}}\text{expIntegral}\left(1,-\frac{d\sqrt{-ab}-b(dx+c)+bc}{b}\right)a}{4b^2}+\frac{e^{-\frac{-d\sqrt{-ab}+bc}{b}}\text{expIntegral}\left(1,\frac{d\sqrt{-ab}}{}\right.}{4b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*exp(-d*x-c)/b*x-1/2/d^2*exp(-d*x-c)/b+1/4/b^2*exp(-(d*(-a*b)^(1/2)+b
*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)*a+1/4/b^2*exp(-(-d*(-a*b)^(1
/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)*a+1/2/d/b*exp(d*x+c)*x-1
/2/d^2/b*exp(d*x+c)+1/4/b^2*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2
)-b*(d*x+c)+b*c)/b)*a+1/4/b^2*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^
(1/2)+b*(d*x+c)-b*c)/b)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*((d*x^3*e^(2*c) - x^2*e^(2*c))*e^(d*x) - (d*x^3 + x^2)*e^(-d*x))/(b*d^2
*x^2*e^c + a*d^2*e^c) - 1/2*integrate(2*(a*d*x^2*e^c - a*x*e^c)*e^(d*x)/(b^
2*d^2*x^4 + 2*a*b*d^2*x^2 + a^2*d^2), x) + 1/2*integrate(2*(a*d*x^2 + a*x)*
e^(-d*x)/(b^2*d^2*x^4*e^c + 2*a*b*d^2*x^2*e^c + a^2*d^2*e^c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(169) = 338$.
time = 0.39, size = 502, normalized size = 2.40

$$\frac{4\,bdx\sinh\left(dx+c\right)-4\,b\cosh\left(dx+c\right)-\left(\left(ad^{2}\cosh\left(dx+c\right)^{2}-ad^{2}\sinh\left(dx+c\right)^{2}\right){\rm Ei}\left(dx-\sqrt{-\frac{ad^{2}}{b}}\right)+\left(ad^{2}\cosh\left(dx+c\right)^{2}-ad^{2}\sinh\left(dx+c\right)^{2}\right){\rm Ei}\left(-dx+\sqrt{-\frac{ad^{2}}{b}}\right)\right)\cosh\left(c+\sqrt{-\frac{ad^{2}}{b}}\right)-\left(\left(ad^{2}\cosh\left(dx+c\right)^{2}-ad^{2}\sinh\left(dx+c\right)^{2}\right){\rm Ei}\left(dx+\sqrt{-\frac{ad^{2}}{b}}\right)+\left(ad^{2}\cosh\left(dx+c\right)^{2}-ad^{2}\sinh\left(dx+c\right)^{2}\right){\rm Ei}\left(-dx-\sqrt{-\frac{ad^{2}}{b}}\right)\right)\cosh\left(-c+\sqrt{-\frac{ad^{2}}{b}}\right)-\left(\left(ad^{2}\cosh\left(dx+c\right)^{2}-ad^{2}\sinh\left(dx+c\right)^{2}\right){\rm Ei}\left(dx-\sqrt{-\frac{ad^{2}}{b}}\right)-\left(ad^{2}\cosh\left(dx+c\right)^{2}-ad^{2}\sinh\left(dx+c\right)^{2}\right){\rm Ei}\left(-dx+\sqrt{-\frac{ad^{2}}{b}}\right)\right)\sinh\left(c+\sqrt{-\frac{ad^{2}}{b}}\right)+\left(\left(ad^{2}\cosh\left(dx+c\right)^{2}-ad^{2}\sinh\left(dx+c\right)^{2}\right){\rm Ei}\left(dx+\sqrt{-\frac{ad^{2}}{b}}\right)-\left(ad^{2}\cosh\left(dx+c\right)^{2}-ad^{2}\sinh\left(dx+c\right)^{2}\right){\rm Ei}\left(-dx-\sqrt{-\frac{ad^{2}}{b}}\right)\right)\sinh\left(-c+\sqrt{-\frac{ad^{2}}{b}}\right)}{4\left(b^{2}d^{2}\cosh\left(dx+c\right)^{2}-b^{2}d^{2}\sinh\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*d*x*sinh(d*x + c) - 4*b*cosh(d*x + c) - ((a*d^2*cosh(d*x + c)^2 -
a*d^2*sinh(d*x + c)^2)*Ei(d*x - sqrt(-a*d^2/b)) + (a*d^2*cosh(d*x + c)^2 -
a*d^2*sinh(d*x + c)^2)*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b))
- ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b))
 + (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)
))*cosh(-c + sqrt(-a*d^2/b)) - ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c
)^2)*Ei(d*x - sqrt(-a*d^2/b)) - (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c
)^2)*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + ((a*d^2*cosh(d*x
 + c)^2 - a*d^2*sinh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) - (a*d^2*cosh(d*x
 + c)^2 - a*d^2*sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(
-a*d^2/b)))/(b^2*d^2*cosh(d*x + c)^2 - b^2*d^2*sinh(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \cosh{\left(c + d x \right)}}{a + b x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(d*x + c)/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\mathrm{cosh}\left(c+d\,x\right)}{b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*cosh(c + d*x))/(a + b*x^2),x)
[Out] int((x^3*cosh(c + d*x))/(a + b*x^2), x)
```

3.59 $\int \frac{x^2 \cosh(c+dx)}{a+bx^2} dx$

Optimal. Leaf size=226

$$\frac{\sqrt{-a}\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2b^{3/2}}+\frac{\sinh(c+}{bd}$$

```
[Out] sinh(d*x+c)/b/d-1/2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/
2))*(-a)^(1/2)/b^(3/2)+1/2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/
2)/b^(1/2))*(-a)^(1/2)/b^(3/2)-1/2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(
-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(3/2)+1/2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sin
h(c+d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(3/2)
```

Rubi [A]
time = 0.27, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5401, 2717, 5389, 3384, 3379, 3382}

$$\frac{\sqrt{-a}\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{\sqrt{-a}\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] (Sqrt[-a]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b]
- d*x])/(2*b^(3/2)) - (Sqrt[-a]*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral
[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(3/2)) + Sinh[c + d*x]/(b*d) - (Sqrt[-a]
*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
2*b^(3/2)) - (Sqrt[-a]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x])/(2*b^(3/2))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\cosh(c+dx)}{a+bx^2}\,dx &= \int \left(\frac{\cosh(c+dx)}{b}-\frac{a\cosh(c+dx)}{b\left(a+bx^2\right)}\right) dx\\
&= \frac{\int \cosh(c+dx)\,dx}{b}-\frac{a\int \frac{\cosh(c+dx)}{a+bx^2}\,dx}{b}\\
&= \frac{\sinh(c+dx)}{bd}-\frac{a\int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right) dx}{b}\\
&= \frac{\sinh(c+dx)}{bd}-\frac{\sqrt{-a}\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2b}-\frac{\sqrt{-a}\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2b}\\
&= \frac{\sinh(c+dx)}{bd}-\frac{\left(\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2b}-\frac{\left(\sqrt{-a}\cosh\right.}{}\\
&= \frac{\sqrt{-a}\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\right.}{2b^{3/2}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 213, normalized size = 0.94

$$\frac{-i\sqrt{a}\,d\cosh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(-\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)+i\sqrt{a}\,d\cosh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)+2\sqrt{b}\,\sinh(c+dx)+\sqrt{a}\,d\sinh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}-idx\right)+\sqrt{a}\,d\sinh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)}{2b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] ((-I)*Sqrt[a]*d*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/S
qrt[b]) + I*d*x] + I*Sqrt[a]*d*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[
(Sqrt[a]*d)/Sqrt[b] + I*d*x] + 2*Sqrt[b]*Sinh[c + d*x] + Sqrt[a]*d*Sinh[c -
 (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + Sqrt[a]*
d*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])
/(2*b^(3/2)*d)
```

Maple [A]
time = 0.89, size = 259, normalized size = 1.15

method	result
risch	$-\frac{e^{-dx-c}}{2bd}+\frac{e^{-\frac{d\sqrt{-ab}+bc}{b}}\,\text{expIntegral}\left(1,-\frac{d\sqrt{-ab}-b(dx+c)+bc}{b}\right)a}{4b\sqrt{-ab}}-\frac{e^{-\frac{-d\sqrt{-ab}+bc}{b}}\,\text{expIntegral}\left(1,\frac{d\sqrt{-ab}+b(dx+c)-}{b}\right.}{4b\sqrt{-ab}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/d*exp(-d*x-c)+1/4/b/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-
(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)*a-1/4/b/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2
)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)*a+1/2/b/d*exp(d*x+c)+1/4/b
/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*
c)/b)*a-1/4/b/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/
2)+b*(d*x+c)-b*c)/b)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -a*integrate(x*e^(d*x + c)/(b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d), x) + a*integr
ate(x*e^(-d*x)/(b^2*d*x^4*e^c + 2*a*b*d*x^2*e^c + a^2*d*e^c), x) + 1/2*(x^2
*e^(d*x + 2*c) - x^2*e^(-d*x))/(b*d*x^2*e^c + a*d*e^c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(170) = 340$.
time = 0.37, size = 496, normalized size = 2.19

$$\frac{\left(\sqrt{-\frac{ad^2}{b}}\left(\cosh\left(dx+c\right)^2-\sinh\left(dx+c\right)^2\right)\operatorname{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\sqrt{-\frac{ad^2}{b}}\left(\cosh\left(dx+c\right)^2-\sinh\left(dx+c\right)^2\right)\operatorname{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(c+\sqrt{-\frac{ad^2}{b}}\right)-\left(\sqrt{-\frac{ad^2}{b}}\left(\cosh\left(dx+c\right)^2-\sinh\left(dx+c\right)^2\right)\operatorname{Ei}\left(dx+\sqrt{-\frac{ad^2}{b}}\right)+\sqrt{-\frac{ad^2}{b}}\left(\cosh\left(dx+c\right)^2-\sinh\left(dx+c\right)^2\right)\operatorname{Ei}\left(-dx-\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(-c+\sqrt{-\frac{ad^2}{b}}\right)+\left(\sqrt{-\frac{ad^2}{b}}\left(\cosh\left(dx+c\right)^2-\sinh\left(dx+c\right)^2\right)\operatorname{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)-\sqrt{-\frac{ad^2}{b}}\left(\cosh\left(dx+c\right)^2-\sinh\left(dx+c\right)^2\right)\operatorname{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)\sinh\left(c+\sqrt{-\frac{ad^2}{b}}\right)+\left(\sqrt{-\frac{ad^2}{b}}\left(\cosh\left(dx+c\right)^2-\sinh\left(dx+c\right)^2\right)\operatorname{Ei}\left(dx+\sqrt{-\frac{ad^2}{b}}\right)-\sqrt{-\frac{ad^2}{b}}\left(\cosh\left(dx+c\right)^2-\sinh\left(dx+c\right)^2\right)\operatorname{Ei}\left(-dx-\sqrt{-\frac{ad^2}{b}}\right)\right)\sinh\left(-c+\sqrt{-\frac{ad^2}{b}}\right)+4\sinh\left(dx+c\right)}{4\left(bd\cosh\left(dx+c\right)^2-bd\sinh\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x - sqrt(-a*d
^2/b)) + sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(-d*x + sqrt(
-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - (sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - s
inh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) + sqrt(-a*d^2/b)*(cosh(d*x + c)^2
- sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + (
sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x - sqrt(-a*d^2/b))
 - sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(-d*x + sqrt(-a*d^2
/b)))*sinh(c + sqrt(-a*d^2/b)) + (sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*
x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) - sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh
(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)) + 4*sinh(
d*x + c))/(b*d*cosh(d*x + c)^2 - b*d*sinh(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cosh\left(c + dx\right)}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(d*x + c)/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\mathrm{cosh}\left(c+d\,x\right)}{b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(c + d*x))/(a + b*x^2),x)
[Out] int((x^2*cosh(c + d*x))/(a + b*x^2), x)
```

3.60 $\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx$

Optimal. Leaf size=177

$$\frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b}+\frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2b}-\frac{\sinh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}}{2b}$$

```
[Out] 1/2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/b+1/2*Chi(-d
*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/b+1/2*Shi(d*x+d*(-a)^
(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/b+1/2*Shi(d*x-d*(-a)^(1/2)/b^(1
/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/b
```

Rubi [A]
time = 0.18, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5401, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b}+\frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b}-\frac{\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b}+\frac{\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
2*b) + (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] +
d*x])/(2*b) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqr
t[b] - d*x])/(2*b) + (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]
*d)/Sqrt[b] + d*x])/(2*b)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx &= \int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx\\ &= -\frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2\sqrt{b}}+\frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2\sqrt{b}}\\ &= \frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2\sqrt{b}}-\frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}\,x}}{2\sqrt{b}}\\ &= \frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b}+\frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2b}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.15, size = 171, normalized size = 0.97

$$\frac{\cosh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(-\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)+\cosh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)+i\left(\sinh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}-idx\right)-\sinh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] (Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x
] + Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x
] + I*(Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*
d*x] - Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*
d*x]))/(2*b)
```

Maple [A]
time = 0.73, size = 200, normalized size = 1.13

method	result
risch	$-\frac{e^{-\frac{d\sqrt{-ab}+bc}{b}} \operatorname{expIntegral}\left(1,-\frac{d\sqrt{-ab}-b(dx+c)+bc}{b}\right)}{4b} - \frac{e^{-\frac{-d\sqrt{-ab}+bc}{b}} \operatorname{expIntegral}\left(1,\frac{d\sqrt{-ab}+b(dx+c)-bc}{b}\right)}{4b} - \frac{e^{\frac{d\sqrt{-}}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)
-1/4/b*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)
-1/4/b*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)-1
/4/b*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b*d*x^2*e^c + a*d*e^c) + 1/2*integrate(
(b*x^2*e^c - a*e^c)*e^(d*x)/(b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d), x) - 1/2*int
egrate((b*x^2 - a)*e^(-d*x)/(b^2*d*x^4*e^c + 2*a*b*d*x^2*e^c + a^2*d*e^c),
x)
```

Fricas [A]
time = 0.39, size = 219, normalized size = 1.24

$$\frac{\left(\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\mathrm{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(c+\sqrt{-\frac{ad^2}{b}}\right)+\left(\mathrm{Ei}\left(dx+\sqrt{-\frac{ad^2}{b}}\right)+\mathrm{Ei}\left(-dx-\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(-c+\sqrt{-\frac{ad^2}{b}}\right)+\left(\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)-\mathrm{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)\sinh\left(c+\sqrt{-\frac{ad^2}{b}}\right)-\left(\mathrm{Ei}\left(dx+\sqrt{-\frac{ad^2}{b}}\right)-\mathrm{Ei}\left(-dx-\sqrt{-\frac{ad^2}{b}}\right)\right)\sinh\left(-c+\sqrt{-\frac{ad^2}{b}}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*((Ei(d*x - sqrt(-a*d^2/b)) + Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-
a*d^2/b)) + (Ei(d*x + sqrt(-a*d^2/b)) + Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c
+ sqrt(-a*d^2/b)) + (Ei(d*x - sqrt(-a*d^2/b)) - Ei(-d*x + sqrt(-a*d^2/b)))*
sinh(c + sqrt(-a*d^2/b)) - (Ei(d*x + sqrt(-a*d^2/b)) - Ei(-d*x - sqrt(-a*d^
2/b)))*sinh(-c + sqrt(-a*d^2/b)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x*cosh(c + d*x)/(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(d*x + c)/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \cosh(c + d\,x)}{b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(c + d*x))/(a + b*x^2),x)
```

```
[Out] int((x*cosh(c + d*x))/(a + b*x^2), x)
```

3.61 $\int \frac{\cosh(c+dx)}{a+bx^2}\,dx$

Optimal. Leaf size=213

$$\frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\,\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2\sqrt{-a}\,\sqrt{b}}-\frac{\sinh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}}{2\sqrt{-a}\,\sqrt{}}$$

```
[Out] -1/2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(1/2)/
b^(1/2)+1/2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/(-a
)^(1/2)/b^(1/2)-1/2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/
2))/(-a)^(1/2)/b^(1/2)+1/2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2
)/b^(1/2))/(-a)^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5389, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\,\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2\sqrt{-a}\,\sqrt{b}}-\frac{\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\,\sqrt{b}}-\frac{\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2\sqrt{-a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*x^2),x]
```

```
[Out] (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
2*Sqrt[-a]*Sqrt[b]) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]
*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Sinh[c -
 (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a
]*Sqrt[b])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{a+bx^2}\,dx &= \int \left(\frac{\sqrt{-a}\,\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\sqrt{-a}\,\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx\\
&= -\frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2\sqrt{-a}}-\frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2\sqrt{-a}}\\
&= -\frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2\sqrt{-a}}-\frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}\,x}}{2\sqrt{-a}}\\
&= \frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\,\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2\sqrt{-a}\,\sqrt{b}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.17, size = 180, normalized size = 0.85

$$\frac{i\left(\cosh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\operatorname{CosIntegral}\left(-\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)-\cosh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)+i\left(\sinh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}-idx\right)+\sinh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)\right)\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x^2),x]
```

```
[Out] ((I/2)*(Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b])
+ I*d*x] - Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b]
+ I*d*x] + I*(Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[
b] - I*d*x] + Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[
b] + I*d*x])))/(Sqrt[a]*Sqrt[b])
```

Maple [A]
time = 0.71, size = 212, normalized size = 1.00

method	result
risch	$\frac{e^{-\frac{-d\sqrt{-ab}+bc}{b}}\operatorname{expIntegral}\left(1,\frac{d\sqrt{-ab}+b(dx+c)-bc}{b}\right)}{4\sqrt{-ab}}-\frac{e^{-\frac{d\sqrt{-ab}+bc}{b}}\operatorname{expIntegral}\left(1,-\frac{d\sqrt{-ab}-b(dx+c)+bc}{b}\right)}{4\sqrt{-ab}}-\frac{e^{\frac{d\sqrt{-ab}}{b}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+
c)-b*c)/b)-1/4/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1
/2)-b*(d*x+c)+b*c)/b)-1/4/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*
(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/4/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+b*c)/b
)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/(b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(157) = 314$.
time = 0.42, size = 316, normalized size = 1.48

$$-\frac{\left(\sqrt{-\frac{ad^2}{b}}\,\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\sqrt{-\frac{ad^2}{b}}\,\mathrm{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(c+\sqrt{-\frac{ad^2}{b}}\right)-\left(\sqrt{-\frac{ad^2}{b}}\,\mathrm{Ei}\left(dx+\sqrt{-\frac{ad^2}{b}}\right)+\sqrt{-\frac{ad^2}{b}}\,\mathrm{Ei}\left(-dx-\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(-c+\sqrt{-\frac{ad^2}{b}}\right)+\left(\sqrt{-\frac{ad^2}{b}}\,\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)-\sqrt{-\frac{ad^2}{b}}\,\mathrm{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)\sinh\left(c+\sqrt{-\frac{ad^2}{b}}\right)+\left(\sqrt{-\frac{ad^2}{b}}\,\mathrm{Ei}\left(dx+\sqrt{-\frac{ad^2}{b}}\right)-\sqrt{-\frac{ad^2}{b}}\,\mathrm{Ei}\left(-dx-\sqrt{-\frac{ad^2}{b}}\right)\right)\sinh\left(-c+\sqrt{-\frac{ad^2}{b}}\right)}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + sqrt(-a*d^2/b)*Ei(-d*x + s
qrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - (sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a
*d^2/b)) + sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/
b)) + (sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - sqrt(-a*d^2/b)*Ei(-d*x + s
qrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a
*d^2/b)) - sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/
b)))/(a*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh{(c+dx)}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x**2+a),x)
[Out] Integral(cosh(c + d*x)/(a + b*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/(b*x^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*x^2),x)
[Out] int(cosh(c + d*x)/(a + b*x^2), x)
```

3.62 $\int \frac{\cosh(c+dx)}{x(a+bx^2)}\,dx$

Optimal. Leaf size=197

$$\frac{\cosh(c)\text{Chi}(dx)}{a}-\frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a}-\frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2a}+\frac{\sinh(c}{}$$

```
[Out] Chi(d*x)*cosh(c)/a-1/2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^
(1/2))/a-1/2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/a+
Shi(d*x)*sinh(c)/a-1/2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^
(1/2))/a-1/2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a
```

Rubi [A]
time = 0.27, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5401, 3384, 3379, 3382}

$$-\frac{\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a}-\frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a}+\frac{\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a}-\frac{\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a}+\frac{\cosh(c)\text{Chi}(dx)}{a}+\frac{\sinh(c)\text{Shi}(dx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x^2)),x]
```

```
[Out] (Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*Cosh
Integral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a) + (Sinh[c]*SinhIntegral[d*x])/a
 + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]
)/(2*a) - (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b]
 + d*x])/(2*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)}\,dx &= \int \left(\frac{\cosh(c+dx)}{ax}-\frac{bx\cosh(c+dx)}{a\left(a+bx^2\right)}\right)dx\\
&= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a}-\frac{b\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{a}\\
&= -\frac{b\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx}{a}+\frac{\cosh(c)\int \frac{\cosh(dx)}{x}\,dx}{a}+\frac{\mathrm{si}}{}\\
&= \frac{\cosh(c)\mathrm{Chi}(dx)}{a}+\frac{\sinh(c)\mathrm{Shi}(dx)}{a}+\frac{\sqrt{b}\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2a}-\frac{\sqrt{b}\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2a}\\
&= \frac{\cosh(c)\mathrm{Chi}(dx)}{a}+\frac{\sinh(c)\mathrm{Shi}(dx)}{a}-\frac{\left(\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}\,x}\,d}{2a}\\
&= \frac{\cosh(c)\mathrm{Chi}(dx)}{a}-\frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a}-\frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Ch}}{2a}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 187, normalized size = 0.95

$$-\frac{-2\cosh(c)\mathrm{Chi}(dx)+\cosh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(-\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)+\cosh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)-2\sinh(c)\mathrm{Shi}(dx)+i\sinh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}-idx\right)-i\sinh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^2)),x]
```

```
[Out] -1/2*(-2*Cosh[c]*CoshIntegral[d*x] + Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosInt
egral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*Cos
```

```
Integral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - 2*Sinh[c]*SinhIntegral[d*x] + I*Sin
h[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - I*S
inh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/a
```

Maple [A]
time = 0.71, size = 227, normalized size = 1.15

method	result
risch	$-\frac{e^{-c}\,\text{expIntegral}(1,dx)}{2a}+\frac{e^{-\frac{d\sqrt{-ab}+bc}{b}}\,\text{expIntegral}\left(1,-\frac{d\sqrt{-ab}-b(dx+c)+bc}{b}\right)}{4a}+\frac{e^{-\frac{-d\sqrt{-ab}+bc}{b}}\,\text{expIntegral}\left(1,\frac{d\sqrt{-ab}}{}\right.}{4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*exp(-c)*Ei(1,d*x)+1/4/a*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)
^(1/2)-b*(d*x+c)+b*c)/b)+1/4/a*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)
^(1/2)+b*(d*x+c)-b*c)/b)-1/2/a*exp(c)*Ei(1,-d*x)+1/4/a*exp((d*(-a*b)^(1/2)+
b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/4/a*exp((-d*(-a*b)^(1/2)+b
*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x), x)
```

Fricas [A]
time = 0.43, size = 249, normalized size = 1.26

$$-\frac{\left(\text{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\text{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(c+\sqrt{-\frac{ad^2}{b}}\right)-2\left(\text{Ei}(dx)+\text{Ei}(-dx)\right)\cosh(c)+\left(\text{Ei}\left(dx+\sqrt{-\frac{ad^2}{b}}\right)+\text{Ei}\left(-dx-\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(-c+\sqrt{-\frac{ad^2}{b}}\right)+\left(\text{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)-\text{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)\sinh\left(c+\sqrt{-\frac{ad^2}{b}}\right)-2\left(\text{Ei}(dx)-\text{Ei}(-dx)\right)\sinh(c)-\left(\text{Ei}\left(dx+\sqrt{-\frac{ad^2}{b}}\right)-\text{Ei}\left(-dx-\sqrt{-\frac{ad^2}{b}}\right)\right)\sinh\left(-c+\sqrt{-\frac{ad^2}{b}}\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*((Ei(d*x - sqrt(-a*d^2/b)) + Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(
-a*d^2/b)) - 2*(Ei(d*x) + Ei(-d*x))*cosh(c) + (Ei(d*x + sqrt(-a*d^2/b)) + E
i(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + (Ei(d*x - sqrt(-a*d^2
/b)) - Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - 2*(Ei(d*x) - E
i(-d*x))*sinh(c) - (Ei(d*x + sqrt(-a*d^2/b)) - Ei(-d*x - sqrt(-a*d^2/b)))*s
inh(-c + sqrt(-a*d^2/b)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh (c + dx)}{x (a + bx^2)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x**2+a),x)
[Out] Integral(cosh(c + d*x)/(x*(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a),x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + d\,x)}{x\,\left(b\,x^2 + a\right)} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x*(a + b*x^2)),x)
[Out] int(cosh(c + d*x)/(x*(a + b*x^2)), x)
```

3.63 $\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=249

$$-\frac{\cosh(c+dx)}{ax}+\frac{\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}}-\frac{\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2(-a)^{3/2}}$$

```
[Out] -cosh(d*x+c)/a/x+d*cosh(c)*Shi(d*x)/a+d*Chi(d*x)*sinh(c)/a-1/2*Chi(d*x+d*(-
a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(3/2)+1/2*Chi(-
d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(3/2)-1
/2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^
(3/2)+1/2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))*b^(1/2
)/(-a)^(3/2)
```

Rubi [A]
time = 0.37, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5401, 3378, 3384, 3379, 3382, 5389}

$$\frac{\sqrt{b}\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}}-\frac{\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2(-a)^{3/2}}-\frac{\sqrt{b}\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}}-\frac{\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2(-a)^{3/2}}+\frac{d\sinh(c)\mathrm{Chi}(dx)}{a}+\frac{d\cosh(c)\mathrm{Shi}(dx)}{a}-\frac{\cosh(c+dx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x^2)),x]
```

```
[Out] -(Cosh[c + d*x]/(a*x)) + (Sqrt[b]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegr
al[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*(-a)^(3/2)) - (Sqrt[b]*Cosh[c - (Sqrt[-a
]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*(-a)^(3/2)) + (d
*CoshIntegral[d*x]*Sinh[c])/a + (d*Cosh[c]*SinhIntegral[d*x])/a - (Sqrt[b]*
Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2
*(-a)^(3/2)) - (Sqrt[b]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-
a]*d)/Sqrt[b] + d*x])/(2*(-a)^(3/2))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx = \int \left(\frac{\cosh(c+dx)}{ax^2} - \frac{b\cosh(c+dx)}{a(a+bx^2)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a} - \frac{b\int \frac{\cosh(c+dx)}{a+bx^2}\,dx}{a}$$

$$= -\frac{\cosh(c+dx)}{ax} - \frac{b\int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx}{a} + \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{a}$$

$$= -\frac{\cosh(c+dx)}{ax} - \frac{b\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2(-a)^{3/2}} - \frac{b\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2(-a)^{3/2}} + \frac{(d\cosh(c))\int \frac{\sinh(dx)}{x}\,dx}{a}$$

$$= -\frac{\cosh(c+dx)}{ax} + \frac{d\text{Chi}(dx)\sinh(c)}{a} + \frac{d\cosh(c)\text{Shi}(dx)}{a} - \frac{\left(b\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\right)\int}{2(-a)^{3/2}}$$

$$= -\frac{\cosh(c+dx)}{ax} + \frac{\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}}{\sqrt{b}}\right.}{2(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.29, size = 243, normalized size = 0.98

$$\frac{-2\sqrt{a}\cosh(c+dx) - i\sqrt{b}\,x\cosh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(-\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right) + i\sqrt{b}\,x\cosh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right) + 2\sqrt{a}\,dx\text{Chi}(dx)\sinh(c) + 2\sqrt{a}\,dx\cosh(c)\text{Shi}(dx) + \sqrt{b}\,x\sinh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}-idx\right) + \sqrt{b}\,x\sinh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)}{2a^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x^2)),x]
```

```
[Out] (-2*Sqrt[a]*Cosh[c + d*x] - I*Sqrt[b]*x*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*Cos
Integral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + I*Sqrt[b]*x*Cosh[c + (I*Sqrt[a]*
d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + 2*Sqrt[a]*d*x*CoshIn
tegral[d*x]*Sinh[c] + 2*Sqrt[a]*d*x*Cosh[c]*SinhIntegral[d*x] + Sqrt[b]*x*S
inh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + S
qrt[b]*x*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] +
I*d*x])/(2*a^(3/2)*x)
```

Maple [A]
time = 0.72, size = 288, normalized size = 1.16

method	result

risch	$-\frac{\mathrm{e}^{-dx-c}}{2ax}+\frac{d\,\mathrm{e}^{-c}\,\mathrm{expIntegral}(1,dx)}{2a}+\frac{b\,\mathrm{e}^{-\frac{d\sqrt{-ab}+bc}{b}}\,\mathrm{expIntegral}\left(1,-\frac{d\sqrt{-ab}-b(dx+c)+bc}{b}\right)}{4a\sqrt{-ab}}-\frac{b\,\mathrm{e}^{-\frac{-d\sqrt{-ab}+bc}{b}}\,\mathrm{expInt}}{4}$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*exp(-d*x-c)/a/x+1/2*d/a*exp(-c)*Ei(1,d*x)+1/4*b/a/(-a*b)^(1/2)*exp(-(d
*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)-1/4*b/a/(-a*b
)^(1/2)*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b
)-1/2*exp(d*x+c)/a/x-1/2*d/a*exp(c)*Ei(1,-d*x)+1/4*b/a/(-a*b)^(1/2)*exp((d*
(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)-1/4*b/a/(-a*b)^
(1/2)*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)
```

<u>Maxima</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x^2), x)
```

<u>Fricas</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 599 vs. 2(193) = 386.
time = 0.50, size = 599, normalized size = 2.41

$$-\frac{4\,ad\cosh(dx+c)-\left(\left(bx\cosh(dx+c)^2-bx\sinh(dx+c)^2\right)\sqrt{-\frac{ad^2}{b}}\,\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\left(bx\cosh(dx+c)^2-bx\sinh(dx+c)^2\right)\sqrt{-\frac{ad^2}{b}}\,\mathrm{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(c+\sqrt{-\frac{ad^2}{b}}\right)-2\left(ad^2x\mathrm{Ei}(dx)-ad^2x\mathrm{Ei}(-dx)\right)\cosh(c)+\cdots}{\cdots}$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*d*cosh(d*x + c) - ((b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*sq
rt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + (b*x*cosh(d*x + c)^2 - b*x*sinh(d*x
 + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b))
 - 2*(a*d^2*x*Ei(d*x) - a*d^2*x*Ei(-d*x))*cosh(c) + ((b*x*cosh(d*x + c)^2 -
 b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) + (b*x*cosh(d
*x + c)^2 - b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*
cosh(-c + sqrt(-a*d^2/b)) - ((b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*sq
rt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - (b*x*cosh(d*x + c)^2 - b*x*sinh(d*x
 + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b))
 - 2*(a*d^2*x*Ei(d*x) + a*d^2*x*Ei(-d*x))*sinh(c) - ((b*x*cosh(d*x + c)^2 -
 b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) - (b*x*cosh(d
```

```
*x + c)^2 - b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b))*
sinh(-c + sqrt(-a*d^2/b)))/(a^2*d*x*cosh(d*x + c)^2 - a^2*d*x*sinh(d*x + c)
^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c + dx)}{x^2 (a + bx^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**2/(b*x**2+a),x)
[Out] Integral(cosh(c + d*x)/(x**2*(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a),x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c + d\,x)}{x^2\,\left(b\,x^2 + a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x^2*(a + b*x^2)),x)
[Out] int(cosh(c + d*x)/(x^2*(a + b*x^2)), x)
```

3.64 $\int \frac{\cosh(c+dx)}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=270

$$-\frac{\cosh(c+dx)}{2ax^2}-\frac{b\cosh(c)\text{Chi}(dx)}{a^2}+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a}+\frac{b\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^2}+\frac{b\cosh}{}$$

```
[Out] -b*Chi(d*x)*cosh(c)/a^2+1/2*d^2*Chi(d*x)*cosh(c)/a-1/2*cosh(d*x+c)/a/x^2+1/
2*b*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/a^2+1/2*b*Ch
i(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/a^2-b*Shi(d*x)*si
nh(c)/a^2+1/2*d^2*Shi(d*x)*sinh(c)/a-1/2*d*sinh(d*x+c)/a/x+1/2*b*Shi(d*x+d*
(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a^2+1/2*b*Shi(d*x-d*(-a)^(
1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^2
```

Rubi [A]
time = 0.35, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5401, 3378, 3384, 3379, 3382}

$$-\frac{b\cosh(c)\text{Chi}(dx)}{a^2}+\frac{b\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^2}+\frac{b\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a^2}-\frac{b\sinh(c)\text{Shi}(dx)}{a^2}-\frac{b\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^2}+\frac{b\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a^2}+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a}+\frac{d^2\sinh(c)\text{Shi}(dx)}{2a}-\frac{\cosh(c+dx)}{2ax^2}-\frac{d\sinh(c+dx)}{2ax}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*Cosh[c + d*x]/(a*x^2) - (b*Cosh[c]*CoshIntegral[d*x])/a^2 + (d^2*Cosh[
c]*CoshIntegral[d*x])/(2*a) + (b*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) + (b*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*
CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^2) - (d*Sinh[c + d*x])/(2*a*
x) - (b*Sinh[c]*SinhIntegral[d*x])/a^2 + (d^2*Sinh[c]*SinhIntegral[d*x])/(2
*a) - (b*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x])/(2*a^2) + (b*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d
)/Sqrt[b] + d*x])/(2*a^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^2)}\,dx = \int \left(\frac{\cosh(c+dx)}{ax^3} - \frac{b\cosh(c+dx)}{a^2x} + \frac{b^2x\cosh(c+dx)}{a^2(a+bx^2)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^3}\,dx}{a} - \frac{b\int \frac{\cosh(c+dx)}{x}\,dx}{a^2} + \frac{b^2\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{a^2}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b^2\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx}{a^2} + \frac{d\int \frac{\sinh(c}{x^2}}{2a}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b\cosh(c)\mathrm{Chi}(dx)}{a^2} - \frac{d\sinh(c+dx)}{2ax} - \frac{b\sinh(c)\mathrm{Shi}(dx)}{a^2} - \frac{b^{3/2}\int \frac{\cosh}{\sqrt{-a}}}{2a}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b\cosh(c)\mathrm{Chi}(dx)}{a^2} - \frac{d\sinh(c+dx)}{2ax} - \frac{b\sinh(c)\mathrm{Shi}(dx)}{a^2} + \frac{(d^2\cosh(c)}{}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b\cosh(c)\mathrm{Chi}(dx)}{a^2} + \frac{d^2\cosh(c)\mathrm{Chi}(dx)}{2a} + \frac{b\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\Big(}{2a^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.42, size = 257, normalized size = 0.95

$$\frac{-a\cosh(c+dx)-(2b-ad^2)\,x^2\cosh(c)\mathrm{Chi}(dx)+bx^2\cosh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(-\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)+bx^2\cosh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)-adx\sinh(c+dx)-2bx^2\sinh(c)\mathrm{Shi}(dx)+ad^2x^2\sinh(c)\mathrm{Shi}(dx)+ibx^2\sinh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}-idx\right)-ibx^2\sinh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x^2)),x]
```

```
[Out] (-(a*Cosh[c + d*x]) - (2*b - a*d^2)*x^2*Cosh[c]*CoshIntegral[d*x] + b*x^2*C
osh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]
+ b*x^2*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I
*d*x] - a*d*x*Sinh[c + d*x] - 2*b*x^2*Sinh[c]*SinhIntegral[d*x] + a*d^2*x^2
*Sinh[c]*SinhIntegral[d*x] + I*b*x^2*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinInt
egral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - I*b*x^2*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]
]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/(2*a^2*x^2)
```

Maple [A]
time = 0.74, size = 330, normalized size = 1.22

method	result
risch	$\frac{d\,e^{-dx-c}}{4ax}-\frac{e^{-dx-c}}{4a\,x^2}-\frac{d^2e^{-c}\,\mathrm{expIntegral}(1,dx)}{4a}+\frac{e^{-c}\,\mathrm{expIntegral}(1,dx)b}{2a^2}-\frac{b\,e^{-\frac{d\sqrt{-ab}+bc}{b}}\,\mathrm{expIntegral}\left(1,-\frac{d\sqrt{-ab}-b(d}{b}\right.}{4a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d*exp(-d*x-c)/a/x-1/4*exp(-d*x-c)/a/x^2-1/4*d^2/a*exp(-c)*Ei(1,d*x)+1/2
/a^2*exp(-c)*Ei(1,d*x)*b-1/4*b/a^2*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-
a*b)^(1/2)-b*(d*x+c)+b*c)/b)-1/4*b/a^2*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(
d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)-1/4*d*exp(d*x+c)/a/x-1/4*exp(d*x+c)/a/x^2-
1/4*d^2/a*exp(c)*Ei(1,-d*x)+1/2/a^2*exp(c)*Ei(1,-d*x)*b-1/4*b/a^2*exp((d*(-
a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)-1/4*b/a^2*exp((-d
*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(222) = 444$.
time = 0.39, size = 583, normalized size = 2.16

$$-\frac{2 \, a d x \sinh\left(d x + c\right) + 2 \, a \cosh\left(d x + c\right) - \left({\left(b x^{2} \cosh\left(d x + c\right)^{2} - b x^{2} \sinh\left(d x + c\right)^{2}\right)} {\rm Ei}\left(d x - \sqrt{-\frac{a d^{2}}{b}}\right) + {\left(b x^{2} \cosh\left(d x + c\right)^{2} - b x^{2} \sinh\left(d x + c\right)^{2}\right)} {\rm Ei}\left(-d x + \sqrt{-\frac{a d^{2}}{b}}\right)\right)} \cosh\left(c + \sqrt{-\frac{a d^{2}}{b}}\right) - {\left({\left(a d^{2} - 2 \, b\right)} x^{2} {\rm Ei}\left(d x\right) + {\left(a d^{2} - 2 \, b\right)} x^{2} {\rm Ei}\left(-d x\right)\right)} \cosh\left(c\right) - \left({\left(b x^{2} \cosh\left(d x + c\right)^{2} - b x^{2} \sinh\left(d x + c\right)^{2}\right)} {\rm Ei}\left(d x + \sqrt{-\frac{a d^{2}}{b}}\right) + {\left(b x^{2} \cosh\left(d x + c\right)^{2} - b x^{2} \sinh\left(d x + c\right)^{2}\right)} {\rm Ei}\left(-d x - \sqrt{-\frac{a d^{2}}{b}}\right)\right)} \cosh\left(-c + \sqrt{-\frac{a d^{2}}{b}}\right) - \left({\left(b x^{2} \cosh\left(d x + c\right)^{2} - b x^{2} \sinh\left(d x + c\right)^{2}\right)} {\rm Ei}\left(d x - \sqrt{-\frac{a d^{2}}{b}}\right) - {\left(b x^{2} \cosh\left(d x + c\right)^{2} - b x^{2} \sinh\left(d x + c\right)^{2}\right)} {\rm Ei}\left(-d x + \sqrt{-\frac{a d^{2}}{b}}\right)\right)} \sinh\left(c + \sqrt{-\frac{a d^{2}}{b}}\right) - {\left({\left(a d^{2} - 2 \, b\right)} x^{2} {\rm Ei}\left(d x\right) - {\left(a d^{2} - 2 \, b\right)} x^{2} {\rm Ei}\left(-d x\right)\right)} \sinh\left(c\right) + \left({\left(b x^{2} \cosh\left(d x + c\right)^{2} - b x^{2} \sinh\left(d x + c\right)^{2}\right)} {\rm Ei}\left(d x + \sqrt{-\frac{a d^{2}}{b}}\right) - {\left(b x^{2} \cosh\left(d x + c\right)^{2} - b x^{2} \sinh\left(d x + c\right)^{2}\right)} {\rm Ei}\left(-d x - \sqrt{-\frac{a d^{2}}{b}}\right)\right)} \sinh\left(-c + \sqrt{-\frac{a d^{2}}{b}}\right)}{4 \, {\left(a^{2} x^{2} \cosh\left(d x + c\right)^{2} - a^{2} x^{2} \sinh\left(d x + c\right)^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d*x*sinh(d*x + c) + 2*a*cosh(d*x + c) - ((b*x^2*cosh(d*x + c)^2 -
 b*x^2*sinh(d*x + c)^2)*Ei(d*x - sqrt(-a*d^2/b)) + (b*x^2*cosh(d*x + c)^2 -
 b*x^2*sinh(d*x + c)^2)*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b))
 - ((a*d^2 - 2*b)*x^2*Ei(d*x) + (a*d^2 - 2*b)*x^2*Ei(-d*x))*cosh(c) - ((b*x
^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) + (b*x
^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*cosh
(-c + sqrt(-a*d^2/b)) - ((b*x^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*Ei
(d*x - sqrt(-a*d^2/b)) - (b*x^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*Ei
(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - ((a*d^2 - 2*b)*x^2*Ei(d
*x) - (a*d^2 - 2*b)*x^2*Ei(-d*x))*sinh(c) + ((b*x^2*cosh(d*x + c)^2 - b*x^2
*sinh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) - (b*x^2*cosh(d*x + c)^2 - b*x^2
*sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/(a^
2*x^2*cosh(d*x + c)^2 - a^2*x^2*sinh(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh{\left(c + d x \right)}}{x^{3} \left(a + b x^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**3/(b*x**2+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(x**3*(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{cosh}\left(c+d\,x\right)}{x^3\,\left(b\,x^2+a\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x^3*(a + b*x^2)),x)
[Out] int(cosh(c + d*x)/(x^3*(a + b*x^2)), x)
```

3.65 $\int \frac{x^4 \cosh(c+dx)}{\left(a+bx^2\right)^2}\, dx$

Optimal. Leaf size=449

$$\frac{x\cosh(c+dx)}{2b^2}-\frac{x^3\cosh(c+dx)}{2b\,(a+bx^2)}+\frac{3\sqrt{-a}\,\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^{5/2}}-\frac{3\sqrt{-a}\,\cosh\left(c-\frac{\sqrt{-a}}{\sqrt{b}}\right.}{4b}$$

```
[Out] 1/2*x*cosh(d*x+c)/b^2-1/2*x^3*cosh(d*x+c)/b/(b*x^2+a)-1/4*a*d*cosh(c+d*(-a)
^(1/2)/b^(1/2))*Shi(d*x-d*(-a)^(1/2)/b^(1/2))/b^3-1/4*a*d*cosh(c-d*(-a)^(1/
2)/b^(1/2))*Shi(d*x+d*(-a)^(1/2)/b^(1/2))/b^3+sinh(d*x+c)/b^2/d-1/4*a*d*Chi
(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/b^3-1/4*a*d*Chi(-d*
x+d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/b^3-3/4*Chi(d*x+d*(-a)
^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)+3/4*Chi(-d*
x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)-3/4
*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5
/2)+3/4*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))*(-a)^(1/
2)/b^(5/2)
```

Rubi [A]
time = 0.60, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5399, 5401, 2717, 5389, 3384, 3379, 3382, 5400, 3377}

$$\frac{3\sqrt{-a}\,\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^{5/2}}-\frac{3\sqrt{-a}\,\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4b^{5/2}}-\frac{3\sqrt{-a}\,\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^{5/2}}-\frac{3\sqrt{-a}\,\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4b^{5/2}}-\frac{ad\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4b^3}-\frac{ad\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^3}+\frac{ad\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^3}-\frac{ad\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4b^3}-\frac{x^3\cosh(c+dx)}{2b\,(a+bx^2)}+\frac{\sinh(c+dx)}{b^2d}+\frac{x\cosh(c+dx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] (x*Cosh[c + d*x])/(2*b^2) - (x^3*Cosh[c + d*x])/(2*b*(a + b*x^2)) + (3*Sqrt
[-a]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x
])/(4*b^(5/2)) - (3*Sqrt[-a]*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(S
qrt[-a]*d)/Sqrt[b] + d*x])/(4*b^(5/2)) - (a*d*CoshIntegral[(Sqrt[-a]*d)/Sqr
t[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*b^3) - (a*d*CoshIntegral[(Sq
rt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*b^3) + Sinh[c +
 d*x]/(b^2*d) + (a*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*
d)/Sqrt[b] - d*x])/(4*b^3) - (3*Sqrt[-a]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*Sin
hIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^(5/2)) - (a*d*Cosh[c - (Sqrt[-a
]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^3) - (3*Sqrt[-
a]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])
/(4*b^(5/2))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^4\cosh(c+dx)}{(a+bx^2)^2}\,dx = -\frac{x^3\cosh(c+dx)}{2b(a+bx^2)} + \frac{3\int \frac{x^2\cosh(c+dx)}{a+bx^2}\,dx}{2b} + \frac{d\int \frac{x^3\sinh(c+dx)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^3\cosh(c+dx)}{2b(a+bx^2)} + \frac{3\int \left(\frac{\cosh(c+dx)}{b} - \frac{a\cosh(c+dx)}{b(a+bx^2)}\right)dx}{2b} + \frac{d\int \left(\frac{x\sinh(c+dx)}{b} - \frac{ax\sinh(c+dx}{b(a+bx^2)}\right.}{2b}$$

$$= -\frac{x^3\cosh(c+dx)}{2b(a+bx^2)} + \frac{3\int \cosh(c+dx)\,dx}{2b^2} - \frac{(3a)\int \frac{\cosh(c+dx)}{a+bx^2}\,dx}{2b^2} + \frac{d\int x\sinh(c+dx)}{2b^2}$$

$$= \frac{x\cosh(c+dx)}{2b^2} - \frac{x^3\cosh(c+dx)}{2b(a+bx^2)} + \frac{3\sinh(c+dx)}{2b^2d} - \frac{\int \cosh(c+dx)\,dx}{2b^2} - \frac{(3a)\int \Big(}{}$$

$$= \frac{x\cosh(c+dx)}{2b^2} - \frac{x^3\cosh(c+dx)}{2b(a+bx^2)} + \frac{\sinh(c+dx)}{b^2d} - \frac{\left(3\sqrt{-a}\right)\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{4b^2} -$$

$$= \frac{x\cosh(c+dx)}{2b^2} - \frac{x^3\cosh(c+dx)}{2b(a+bx^2)} + \frac{\sinh(c+dx)}{b^2d} - \frac{\left(3\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\right)}{4b^2}$$

$$= \frac{x\cosh(c+dx)}{2b^2} - \frac{x^3\cosh(c+dx)}{2b(a+bx^2)} + \frac{3\sqrt{-a}\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right.}{4b^{5/2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.09, size = 621, normalized size = 1.38

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] (2*Cosh[d*x]*((a*x*Cosh[c])/(a + b*x^2) + (2*Sinh[c])/d) + 2*((2*Cosh[c])/d
 + (a*x*Sinh[c])/(a + b*x^2))*Sinh[d*x] - ((3*I)*Sqrt[a]*Cosh[c]*(Cos[(Sqrt
```

```
[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - Cos[(Sqrt[a]*
d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a]*d)/Sqrt
[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - SinIntegral[(Sqrt[a]*d)/Sq
rt[b] + I*d*x])))/Sqrt[b] + (I*a*d*Cosh[c]*(CosIntegral[-((Sqrt[a]*d)/Sqrt[
b]) + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] - CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I
*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] + Cos[(Sqrt[a]*d)/Sqrt[b]]*(-SinIntegral[(Sq
rt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/b -
 (3*Sqrt[a]*Sinh[c]*(CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sin[(Sqrt[
a]*d)/Sqrt[b]] + CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqrt[a]*d)/S
qrt[b]] - Cos[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x
] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/Sqrt[b] - (a*d*Sinh[c]*(Cos
[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + Cos[(Sq
rt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a]*d
)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]
*d)/Sqrt[b] + I*d*x])))/b)/(4*b^2)
```

Maple [A]
time = 1.14, size = 532, normalized size = 1.18

method	result
risch	$-\frac{e^{-dx-c}}{2d b^2}+\frac{d^2 e^{-dx-c} a x}{4b^2(b d^2x^2+a d^2)}-\frac{3a\, e^{-\frac{-d\sqrt{-ab}+bc}{b}}\,\mathrm{expIntegral}\left(1,\frac{d\sqrt{-ab}+b(dx+c)-bc}{b}\right)}{8b^2\sqrt{-ab}}+\frac{3a\, e^{-\frac{d\sqrt{-ab}+bc}{b}}\,\mathrm{expIntegral}}{8b^2\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cosh(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*exp(-d*x-c)/b^2+1/4*d^2*exp(-d*x-c)*a/b^2/(b*d^2*x^2+a*d^2)*x-3/8/b^
2/(-a*b)^(1/2)*a*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+
c)-b*c)/b)+3/8/b^2/(-a*b)^(1/2)*a*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a
*b)^(1/2)-b*(d*x+c)+b*c)/b)-1/8*d/b^3*a*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,
(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)-1/8*d/b^3*a*exp(-(d*(-a*b)^(1/2)+b*c)/b)*
Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/4*d^2*exp(d*x+c)*a/b^2/(b*d^2*x^2
+a*d^2)*x+3/8/b^2/(-a*b)^(1/2)*a*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)
^(1/2)-b*(d*x+c)+b*c)/b)-3/8/b^2/(-a*b)^(1/2)*a*exp((-d*(-a*b)^(1/2)+b*c)/b
)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)+1/2/b^2/d*exp(d*x+c)+1/8*d/b^3*a*
exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/8*d/b^
3*a*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*d*x^4*e^(2*c) - 4*a*x*e^(2*c))*e^(d*x) - (b*d*x^4 + 4*a*x)*e^(-d*x)
)/(b^3*d^2*x^4*e^c + 2*a*b^2*d^2*x^2*e^c + a^2*b*d^2*e^c) - 1/2*integrate(-
4*(a^2*d*x*e^c - 3*a*b*x^2*e^c + a^2*e^c)*e^(d*x)/(b^4*d^2*x^6 + 3*a*b^3*d^
2*x^4 + 3*a^2*b^2*d^2*x^2 + a^3*b*d^2), x) - 1/2*integrate(4*(a^2*d*x + 3*a
*b*x^2 - a^2)*e^(-d*x)/(b^4*d^2*x^6*e^c + 3*a*b^3*d^2*x^4*e^c + 3*a^2*b^2*d
^2*x^2*e^c + a^3*b*d^2*e^c), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1179 vs. $2(349) = 698$.
time = 0.66, size = 1179, normalized size = 2.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*b*d*x*cosh(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 -
(a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - 3*((b^2*x^2 + a*b)*cosh(d*x + c)^
2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b
)) - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sin
h(d*x + c)^2 + 3*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*
x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b
)) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*si
nh(d*x + c)^2 + 3*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d
*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^2 + a^2*d
^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - 3*((b^2*x^2
 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*
Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + 8*(b^2*x^2 + a*b)*si
nh(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^
2*d^2)*sinh(d*x + c)^2 - 3*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*
b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + ((a*b*d^2*x^
2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + 3*
((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a
*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (((a*b*d^2*x
^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + 3
*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-
a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)
^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - 3*((b^2*x^2 + a*b)*cosh(d*x
+ c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a
*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((b^4*d*x^2 + a*b^3*d)*cosh(d*x + c)^2
 - (b^4*d*x^2 + a*b^3*d)*sinh(d*x + c)^2)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cosh(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**4*cosh(c + d*x)/(a + b*x**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError >> type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \cosh(c+d\,x)}{(b\,x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*cosh(c + d*x))/(a + b*x^2)^2,x)
```

```
[Out] int((x^4*cosh(c + d*x))/(a + b*x^2)^2, x)
```

3.66 $\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=431

$$\frac{\cosh(c+dx)}{2b^2} - \frac{x^2\cosh(c+dx)}{2b\,(a+bx^2)} + \frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^2} + \frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}\right.}{2b^2}$$

```
[Out] 1/2*cosh(d*x+c)/b^2-1/2*x^2*cosh(d*x+c)/b/(b*x^2+a)+1/2*Chi(d*x+d*(-a)^(1/2
)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/b^2+1/2*Chi(-d*x+d*(-a)^(1/2)/b^(1/
2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/b^2+1/2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh
(c-d*(-a)^(1/2)/b^(1/2))/b^2+1/2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a
)^(1/2)/b^(1/2))/b^2+1/4*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(d*x-d*(-a)^(1/2
)/b^(1/2))*(-a)^(1/2)/b^(5/2)-1/4*d*cosh(c-d*(-a)^(1/2)/b^(1/2))*Shi(d*x+d*
(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)-1/4*d*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*
sinh(c-d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)+1/4*d*Chi(-d*x+d*(-a)^(1/2)
/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)
```

Rubi [A]

time = 0.50, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {5399, 5401, 3384, 3379, 3382, 5400, 2718, 5388}

$$-\frac{\sqrt{-a}\,d\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4b^{5/2}} + \frac{\sqrt{-a}\,d\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^{5/2}} - \frac{\sqrt{-a}\,d\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^{5/2}} - \frac{\sqrt{-a}\,d\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4b^{5/2}} + \frac{\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^2} + \frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^2} - \frac{\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^2} + \frac{\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^2} - \frac{x^2\cosh(c+dx)}{2b\,(a+bx^2)} + \frac{\cosh(c+dx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] Cosh[c + d*x]/(2*b^2) - (x^2*Cosh[c + d*x])/(2*b*(a + b*x^2)) + (Cosh[c + (
Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2) + (C
osh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*
b^2) - (Sqrt[-a]*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[
-a]*d)/Sqrt[b]])/(4*b^(5/2)) + (Sqrt[-a]*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b
] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*b^(5/2)) - (Sqrt[-a]*d*Cosh[c +
 (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^(5/2)
) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x
])/(2*b^2) - (Sqrt[-a]*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[
-a]*d)/Sqrt[b] + d*x])/(4*b^(5/2)) + (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIn
tegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
```

```
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^2)^2}\,dx = -\frac{x^2\cosh(c+dx)}{2b(a+bx^2)} + \frac{\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{b} + \frac{d\int \frac{x^2\sinh(c+dx)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^2\cosh(c+dx)}{2b(a+bx^2)} + \frac{\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx}{b} + \frac{d\int \big(}{}$$

$$= -\frac{x^2\cosh(c+dx)}{2b(a+bx^2)} - \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2b^{3/2}} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2b^{3/2}} + \frac{d\int \sinh(c+dx)\,dx}{2b^2} -$$

$$= \frac{\cosh(c+dx)}{2b^2} - \frac{x^2\cosh(c+dx)}{2b(a+bx^2)} - \frac{(ad)\int \left(\frac{\sqrt{-a}\,\sinh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{-a}\,\sinh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)}{2b^2}$$

$$= \frac{\cosh(c+dx)}{2b^2} - \frac{x^2\cosh(c+dx)}{2b(a+bx^2)} + \frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^2} + \frac{\cosh}{}$$

$$= \frac{\cosh(c+dx)}{2b^2} - \frac{x^2\cosh(c+dx)}{2b(a+bx^2)} + \frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^2} + \frac{\cosh}{}$$

$$= \frac{\cosh(c+dx)}{2b^2} - \frac{x^2\cosh(c+dx)}{2b(a+bx^2)} + \frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^2} + \frac{\cosh}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.49, size = 582, normalized size = 1.35

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] (2*a*Sqrt[b]*Cosh[c + d*x] + (a + b*x^2)*CosIntegral[-((Sqrt[a]*d)/Sqrt[b])
 + I*d*x]*(2*Sqrt[b]*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]] - I*Sqrt[a]*d*Sinh[c -
 (I*Sqrt[a]*d)/Sqrt[b]]) + (a + b*x^2)*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*
d*x]*(2*Sqrt[b]*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]] + I*Sqrt[a]*d*Sinh[c + (I*S
```

```
qrt[a]*d)/Sqrt[b]]) + a^(3/2)*d*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral
[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + Sqrt[a]*b*d*x^2*Cosh[c - (I*Sqrt[a]*d)/Sqrt
[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + (2*I)*a*Sqrt[b]*Sinh[c - (I
*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + (2*I)*b^(3/
2)*x^2*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*
d*x] + a^(3/2)*d*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sq
rt[b] + I*d*x] + Sqrt[a]*b*d*x^2*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegra
l[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - (2*I)*a*Sqrt[b]*Sinh[c + (I*Sqrt[a]*d)/Sqr
t[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - (2*I)*b^(3/2)*x^2*Sinh[c +
 (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/(4*b^(5/2
)*(a + b*x^2))
```

Maple [A]
time = 0.90, size = 495, normalized size = 1.15

method	result
risch	$-\frac{e^{-\frac{-d\sqrt{-ab}+bc}{b}}\operatorname{expIntegral}\left(1,\frac{d\sqrt{-ab}+b(dx+c)-bc}{b}\right)}{4b^2}-\frac{e^{-\frac{d\sqrt{-ab}+bc}{b}}\operatorname{expIntegral}\left(1,-\frac{d\sqrt{-ab}-b(dx+c)+bc}{b}\right)}{4b^2}+\frac{da\,e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b^2*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/
b)-1/4/b^2*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c
)/b)+1/8*d/b^2*a/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^
(1/2)+b*(d*x+c)-b*c)/b)-1/8*d/b^2*a/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+b*c)/
b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/4*d^2*exp(-d*x-c)*a/b^2/(b*d^2
*x^2+a*d^2)+1/4*d^2*exp(d*x+c)*a/b^2/(b*d^2*x^2+a*d^2)-1/8*d/b^2*a/(-a*b)^(
1/2)*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)+1
/8*d/b^2*a/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*
(d*x+c)+b*c)/b)-1/4/b^2*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+
b*(d*x+c)-b*c)/b)-1/4/b^2*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-
b*(d*x+c)+b*c)/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((d^2*x^3*e^(2*c) + d*x^2*e^(2*c) + 2*x*e^(2*c))*e^(d*x) - (d^2*x^3 - d
*x^2 + 2*x)*e^(-d*x))/(b^2*d^3*x^4*e^c + 2*a*b*d^3*x^2*e^c + a^2*d^3*e^c) -
```

```
1/2*integrate(2*(2*a*d*x*e^c + (2*a*d^2*e^c - 3*b*e^c)*x^2 + a*e^c)*e^(d*x
)/(b^3*d^3*x^6 + 3*a*b^2*d^3*x^4 + 3*a^2*b*d^3*x^2 + a^3*d^3), x) + 1/2*int
egrate(-2*(2*a*d*x - (2*a*d^2 - 3*b)*x^2 - a)*e^(-d*x)/(b^3*d^3*x^6*e^c + 3
*a*b^2*d^3*x^4*e^c + 3*a^2*b*d^3*x^2*e^c + a^3*d^3*e^c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(331) = 662$.
time = 0.45, size = 931, normalized size = 2.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*cosh(d*x + c) + ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*si
nh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)
*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + (2*(b*x^2 + a)*cosh(d*x + c)^2
- 2*(b*x^2 + a)*sinh(d*x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a
)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt
(-a*d^2/b)) + ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)
^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^
2/b))*Ei(d*x + sqrt(-a*d^2/b)) + (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2
+ a)*sinh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x
+ c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)
) + ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 + ((b*x
^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d
*x - sqrt(-a*d^2/b)) - (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(
d*x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sq
rt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - ((2*(b*
x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 - ((b*x^2 + a)*cos
h(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-
a*d^2/b)) - (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2
+ ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b
))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((b^3*x^2 + a*b^2)
*cosh(d*x + c)^2 - (b^3*x^2 + a*b^2)*sinh(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(x^3*cosh(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \cosh(c+d\,x)}{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*cosh(c + d*x))/(a + b*x^2)^2,x)
[Out] int((x^3*cosh(c + d*x))/(a + b*x^2)^2, x)
```

3.67 $\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=416

$$-\frac{x\cosh(c+dx)}{2b(a+bx^2)}+\frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}\,b^{3/2}}-\frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}\,b^{3/2}}+\frac{d\mathrm{Chi}}{}$$

```
[Out] -1/2*x*cosh(d*x+c)/b/(b*x^2+a)+1/4*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(d*x-d
*(-a)^(1/2)/b^(1/2))/b^2+1/4*d*cosh(c-d*(-a)^(1/2)/b^(1/2))*Shi(d*x+d*(-a)^
(1/2)/b^(1/2))/b^2+1/4*d*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/
b^(1/2))/b^2+1/4*d*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/
2))/b^2-1/4*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/b^(3
/2)/(-a)^(1/2)+1/4*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/
2))/b^(3/2)/(-a)^(1/2)-1/4*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2
)/b^(1/2))/b^(3/2)/(-a)^(1/2)+1/4*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-
a)^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5399, 5389, 3384, 3379, 3382, 5400}

$$\frac{\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}\,b^{3/2}}-\frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4\sqrt{-a}\,b^{3/2}}-\frac{\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}\,b^{3/2}}-\frac{\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4\sqrt{-a}\,b^{3/2}}+\frac{d\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4b^2}+\frac{d\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^2}-\frac{d\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^2}+\frac{d\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4b^2}-\frac{x\cosh(c+dx)}{2b(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] -1/2*(x*Cosh[c + d*x])/(b*(a + b*x^2)) + (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*Co
shIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*Sqrt[-a]*b^(3/2)) - (Cosh[c - (S
qrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b
^(3/2)) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)
/Sqrt[b]])/(4*b^2) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (
Sqrt[-a]*d)/Sqrt[b]])/(4*b^2) - (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhInteg
ral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^2) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*
SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*Sqrt[-a]*b^(3/2)) + (d*Cosh[c
- (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^2) -
 (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(4*Sqrt[-a]*b^(3/2))
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^2}\,dx = -\frac{x\cosh(c+dx)}{2b\,(a+bx^2)} + \frac{\int \frac{\cosh(c+dx)}{a+bx^2}\,dx}{2b} + \frac{d\int \frac{x\sinh(c+dx)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x\cosh(c+dx)}{2b\,(a+bx^2)} + \frac{\int \left(\frac{\sqrt{-a}\,\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{-a}\,\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx}{2b} + \frac{d\int \left(-\frac{\text{s}}{2\sqrt{b}\,(\ldots}\right.}{\ldots}$$

$$= -\frac{x\cosh(c+dx)}{2b\,(a+bx^2)} - \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{4\sqrt{-a}\,b} - \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{4\sqrt{-a}\,b} - \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{4b^{3/2}} + \ldots$$

$$= -\frac{x\cosh(c+dx)}{2b\,(a+bx^2)} - \frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{4\sqrt{-a}\,b} + \frac{\left(d\cosh\left(c-\frac{\sqrt{-\ldots}}{\sqrt{b}}\right.\right.}{\ldots}$$

$$= -\frac{x\cosh(c+dx)}{2b\,(a+bx^2)} + \frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}\,b^{3/2}} - \frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\ldots}{4\sqrt{-\ldots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.70, size = 364, normalized size = 0.88

$$\frac{-2\sqrt{a}\,bx\cosh(c+dx)+(a+bx^2)\,\text{CosIntegral}\left(-\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)\left(i\sqrt{b}\,\cosh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)+\sqrt{a}\,d\sinh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\right)+(a+bx^2)\,\text{CosIntegral}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)\left(-i\sqrt{b}\,\cosh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)+\sqrt{a}\,d\sinh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\right)+(a+bx^2)\left(i\sqrt{a}\,d\cosh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)-\sqrt{b}\,\sinh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\right)\text{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}-idx\right)-(a+bx^2)\left(i\sqrt{a}\,d\cosh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)+\sqrt{b}\,\sinh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\right)\text{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)}{4\sqrt{a}\,b^2\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] (-2*Sqrt[a]*b*x*Cosh[c + d*x] + (a + b*x^2)*CosIntegral[-((Sqrt[a]*d)/Sqrt[
b]) + I*d*x]*(I*Sqrt[b]*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[a]*d*Sinh[c
- (I*Sqrt[a]*d)/Sqrt[b]]) + (a + b*x^2)*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I
*d*x]*((-I)*Sqrt[b]*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[a]*d*Sinh[c + (I
*Sqrt[a]*d)/Sqrt[b]]) + (a + b*x^2)*(I*Sqrt[a]*d*Cosh[c - (I*Sqrt[a]*d)/Sqr
t[b]] - Sqrt[b]*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]])*SinIntegral[(Sqrt[a]*d)/Sq
rt[b] - I*d*x] - (a + b*x^2)*(I*Sqrt[a]*d*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]] +
 Sqrt[b]*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]])*SinIntegral[(Sqrt[a]*d)/Sqrt[b] +
 I*d*x])/(4*Sqrt[a]*b^2*(a + b*x^2))
```

Maple [A]
time = 0.83, size = 491, normalized size = 1.18

method	result

risch	$-\frac{d^2 \mathrm{e}^{-dx-c} x}{4b(b\,d^2x^2+a\,d^2)}+\frac{d\,\mathrm{e}^{-\frac{-d\sqrt{-ab}+bc}{b}}\,\mathrm{expIntegral}\left(1,\frac{d\sqrt{-ab}+b(dx+c)-bc}{b}\right)}{8b^2}+\frac{d\,\mathrm{e}^{-\frac{d\sqrt{-ab}+bc}{b}}\,\mathrm{expIntegral}\left(1,-\frac{d\sqrt{-ab}}{}\right.}{8b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d^2*exp(-d*x-c)/b/(b*d^2*x^2+a*d^2)*x+1/8*d/b^2*exp(-(-d*(-a*b)^(1/2)+
b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)+1/8*d/b^2*exp(-(d*(-a*b)^(1/
2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/8/b/(-a*b)^(1/2)*exp(-
(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)-1/8/b/(-a*b
)^(1/2)*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b
)-1/4*d^2*exp(d*x+c)/b/(b*d^2*x^2+a*d^2)*x-1/8*d/b^2*exp((d*(-a*b)^(1/2)+b*
c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)-1/8*d/b^2*exp((-d*(-a*b)^(1/2)
+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)-1/8/b/(-a*b)^(1/2)*exp((d*
(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/8/b/(-a*b)^(1
/2)*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((d*x^2*e^(2*c) + 2*x*e^(2*c))*e^(d*x) - (d*x^2 - 2*x)*e^(-d*x))/(b^2*d
^2*x^4*e^c + 2*a*b*d^2*x^2*e^c + a^2*d^2*e^c) + 1/2*integrate(-2*(2*a*d*x*e
^c - 3*b*x^2*e^c + a*e^c)*e^(d*x)/(b^3*d^2*x^6 + 3*a*b^2*d^2*x^4 + 3*a^2*b*
d^2*x^2 + a^3*d^2), x) + 1/2*integrate(2*(2*a*d*x + 3*b*x^2 - a)*e^(-d*x)/(
b^3*d^2*x^6*e^c + 3*a*b^2*d^2*x^4*e^c + 3*a^2*b*d^2*x^2*e^c + a^3*d^2*e^c),
 x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. $2(318) = 636$.
time = 0.43, size = 1162, normalized size = 2.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*b*d*x*cosh(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 -
 (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2
```

```
 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)
) - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh
(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x +
 c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b))
- (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(
d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x +
c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^2 + a^2*d^2)*c
osh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)
*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x
 - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - (((a*b*d^2*x^2 + a^2*d^2)*c
osh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)
*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x
- sqrt(-a*d^2/b)) + ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2
 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 +
 a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c +
sqrt(-a*d^2/b)) + (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2
+ a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 +
a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b*d^2*
x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 -
((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a
*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a*b^3*d*x^
2 + a^2*b^2*d)*cosh(d*x + c)^2 - (a*b^3*d*x^2 + a^2*b^2*d)*sinh(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cosh(c + dx)}{(a + bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**2+a)**2,x)
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(x^2*cosh(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\cosh(c+d\,x)}{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(c + d*x))/(a + b*x^2)^2,x)
```

```
[Out] int((x^2*cosh(c + d*x))/(a + b*x^2)^2, x)
```

3.68 $\int \frac{x \cosh(c+dx)}{(a+bx^2)^2}\, dx$

Optimal. Leaf size=239

$$-\frac{\cosh(c+dx)}{2b\,(a+bx^2)}-\frac{d\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4\sqrt{-a}\,b^{3/2}}+\frac{d\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4\sqrt{-a}\,b^{3/2}}-\frac{d\cos}{}$$

```
[Out] -1/2*cosh(d*x+c)/b/(b*x^2+a)+1/4*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(d*x-d*(
-a)^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/2)-1/4*d*cosh(c-d*(-a)^(1/2)/b^(1/2))*Sh
i(d*x+d*(-a)^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/2)-1/4*d*Chi(d*x+d*(-a)^(1/2)/b
^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/2)+1/4*d*Chi(-d*x+d*(-
a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5397, 5388, 3384, 3379, 3382}

$$-\frac{d\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4\sqrt{-a}\,b^{3/2}}+\frac{d\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}\,b^{3/2}}-\frac{d\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}\,b^{3/2}}-\frac{d\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4\sqrt{-a}\,b^{3/2}}-\frac{\cosh(c+dx)}{2b\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] -1/2*Cosh[c + d*x]/(b*(a + b*x^2)) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*Sqrt[-a]*b^(3/2)) + (d*CoshIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*Sqrt[-a]*b
^(3/2)) - (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[
b] - d*x])/(4*Sqrt[-a]*b^(3/2)) - (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhInt
egral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b^(3/2))
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx = -\frac{\cosh(c+dx)}{2b(a+bx^2)} + \frac{d\int \frac{\sinh(c+dx)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{\cosh(c+dx)}{2b(a+bx^2)} + \frac{d\int \left(\frac{\sqrt{-a}\,\sinh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{-a}\,\sinh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx}{2b}$$

$$= -\frac{\cosh(c+dx)}{2b(a+bx^2)} - \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{4\sqrt{-a}\,b} - \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{4\sqrt{-a}\,b}$$

$$= -\frac{\cosh(c+dx)}{2b(a+bx^2)} - \frac{\left(d\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{4\sqrt{-a}\,b} + \frac{\left(d\cosh\left(c+\right.\right.}{}$$

$$= -\frac{\cosh(c+dx)}{2b(a+bx^2)} - \frac{d\operatorname{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4\sqrt{-a}\,b^{3/2}} + \frac{d\operatorname{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right.}{4\sqrt{-}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.40, size = 239, normalized size = 1.00

$$\frac{i\left(d(a+bx^2)\operatorname{CosIntegral}\left(-\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)\sinh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)-d(a+bx^2)\operatorname{CosIntegral}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)\sinh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)+i\left(2\sqrt{a}\sqrt{b}\cosh(c+dx)+d(a+bx^2)\cosh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}-idx\right)+d(a+bx^2)\cosh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)\right)\right)}{4\sqrt{a}\,b^{3/2}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] ((I/4)*(d*(a + b*x^2)*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sinh[c -
(I*Sqrt[a]*d)/Sqrt[b]] - d*(a + b*x^2)*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*
d*x]*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]] + I*(2*Sqrt[a]*Sqrt[b]*Cosh[c + d*x] +
 d*(a + b*x^2)*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt
[b] - I*d*x] + d*(a + b*x^2)*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(S
qrt[a]*d)/Sqrt[b] + I*d*x])))/(Sqrt[a]*b^(3/2)*(a + b*x^2))
```

Maple [A]
time = 0.78, size = 291, normalized size = 1.22

method	result
risch	$-\frac{d^{2} {\mathrm e}^{-d x -c}}{4 b \left(b \,d^{2} x^{2}+a \,d^{2}\right)}+\frac{d \,{\mathrm e}^{-\frac{d \sqrt{-a b}+b c}{b}} \operatorname{expIntegral}\left(1, -\frac{d \sqrt{-a b}-b \left(d x +c \right)+b c}{b}\right)}{8 b \sqrt{-a b}}-\frac{d \,{\mathrm e}^{-\frac{-d \sqrt{-a b}+b c}{b}} \operatorname{expIntegral}\left(1, \frac{d \sqrt{-a b}+}{b}\right.}{8 b \sqrt{-a b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d^2*exp(-d*x-c)/b/(b*d^2*x^2+a*d^2)+1/8*d/b/(-a*b)^(1/2)*exp(-(d*(-a*b
)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)-1/8*d/b/(-a*b)^(1/2
)*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)-1/4*
d^2*exp(d*x+c)/b/(b*d^2*x^2+a*d^2)-1/8*d/b/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)
+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/8*d/b/(-a*b)^(1/2)*exp((-
d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b^2*d*x^4*e^c + 2*a*b*d*x^2*e^c + a^2*d
*e^c) + 1/2*integrate((3*b*x^2*e^c - a*e^c)*e^(d*x)/(b^3*d*x^6 + 3*a*b^2*d*
x^4 + 3*a^2*b*d*x^2 + a^3*d), x) - 1/2*integrate((3*b*x^2 - a)*e^(-d*x)/(b^
3*d*x^6*e^c + 3*a*b^2*d*x^4*e^c + 3*a^2*b*d*x^2*e^c + a^3*d*e^c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(181) = 362.
time = 0.52, size = 641, normalized size = 2.68

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*cosh(d*x + c) + (((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(
d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - ((b*x^2 + a)*cosh(d*x
 + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2
/b)))*cosh(c + sqrt(-a*d^2/b)) - (((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a
)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) - ((b*x^2 + a)*c
osh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt
(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + (((b*x^2 + a)*cosh(d*x + c)^2 - (b
*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + ((b*x^
2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*
x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (((b*x^2 + a)*cosh(d*x + c)
^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) +
 ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)
*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a*b^2*x^2 + a^2*b)
*cosh(d*x + c)^2 - (a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh\left(c+dx\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x*cosh(c + d*x)/(a + b*x**2)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*cosh(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\cosh(c+d\,x)}{\left(b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(c + d*x))/(a + b*x^2)^2,x)
```

```
[Out] int((x*cosh(c + d*x))/(a + b*x^2)^2, x)
```

3.69 $\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=476

$$-\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)}-\frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\cosh\left(c-\right.}{}$$

```
[Out] -1/4*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(d*x-d*(-a)^(1/2)/b^(1/2))/a/b-1/4*d
*cosh(c-d*(-a)^(1/2)/b^(1/2))*Shi(d*x+d*(-a)^(1/2)/b^(1/2))/a/b-1/4*d*Chi(d
*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a/b-1/4*d*Chi(-d*x+d*
(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a/b+1/4*Chi(d*x+d*(-a)^(1/
2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)-1/4*Chi(-d*x+d*
(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)+1/4*Shi
(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)-
1/4*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b
^(1/2)-1/4*cosh(d*x+c)/a/b^(1/2)/((-a)^(1/2)-x*b^(1/2))+1/4*cosh(d*x+c)/a/b
^(1/2)/((-a)^(1/2)+x*b^(1/2))
```

Rubi [A]
time = 0.60, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5389, 3378, 3384, 3379, 3382}

$$-\frac{d\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4ab}-\frac{d\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\operatorname{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4ab}-\frac{\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\operatorname{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\operatorname{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{d\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\operatorname{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4ab}-\frac{d\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4ab}-\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*x^2)^2,x]
```

```
[Out] -1/4*Cosh[c + d*x]/(a*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(4*a*
Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshInteg
ral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (Cosh[c - (Sqrt[-
a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt
[b]) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sq
rt[b]])/(4*a*b) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqr
t[-a]*d)/Sqrt[b]])/(4*a*b) + (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral
[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a*b) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*Sin
hIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (d*Cosh[c -
 (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a*b) +
(Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(
4*(-a)^(3/2)*Sqrt[b])
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \left(-\frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\,\sqrt{b}-bx\right)^2} - \frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\,\sqrt{b}+bx\right)^2} - \frac{b\cosh(c+dx)}{2a\left(-ab-b^2x^2\right)}\right) dx$$

$$= -\frac{b\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\,\sqrt{b}-bx\right)^2}\,dx}{4a} - \frac{b\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\,\sqrt{b}+bx\right)^2}\,dx}{4a} - \frac{b\int \frac{\cosh(c+dx)}{-ab-b^2x^2}\,dx}{2a}$$

$$= -\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{b\int \left(-\frac{\sqrt{-a}\cosh(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}\,x\right)} - \cdots\right.}{2a}$$

$$= -\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{4(-a)^{3/2}} + \frac{\int \frac{\cosh(\cdots}{\sqrt{-a}\cdots}}{4(-\cdots}$$

$$= -\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{d\,\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sinh\left(c\cdots\right.}{4ab}$$

$$= -\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-\cdots}}{\sqrt{\cdots}}\right.}{4(-a)^{3/2}\sqrt{b}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.56, size = 590, normalized size = 1.24

$$\frac{2\sqrt{a}\,bx\cosh(c+dx) - (a+bx^2)\,\mathrm{CosIntegral}\left(-\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)\left(-i\sqrt{b}\cosh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)+\sqrt{a}\,d\sinh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\right) - (a+bx^2)\,\mathrm{CosIntegral}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)\left(i\sqrt{b}\cosh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)+\sqrt{a}\,d\sinh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\right) - ia^{3/2}d\cosh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}-idx\right) - i\sqrt{a}\,bdx^2\cosh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}-idx\right) - a\sqrt{b}\sinh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}-idx\right) - b^{3/2}x^2\sinh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}-idx\right) + ia^{3/2}d\cosh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right) + i\sqrt{a}\,bdx^2\cosh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right) - a\sqrt{b}\sinh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right) - b^{3/2}x^2\sinh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)}{4a^{3/2}b\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x^2)^2,x]
```

```
[Out] (2*Sqrt[a]*b*x*Cosh[c + d*x] - (a + b*x^2)*CosIntegral[-((Sqrt[a]*d)/Sqrt[b
]) + I*d*x]*((-I)*Sqrt[b]*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[a]*d*Sinh[
c - (I*Sqrt[a]*d)/Sqrt[b]]) - (a + b*x^2)*CosIntegral[(Sqrt[a]*d)/Sqrt[b] +
 I*d*x]*(I*Sqrt[b]*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[a]*d*Sinh[c + (I*
Sqrt[a]*d)/Sqrt[b]]) - I*a^(3/2)*d*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinInteg
ral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - I*Sqrt[a]*b*d*x^2*Cosh[c - (I*Sqrt[a]*d)
/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - a*Sqrt[b]*Sinh[c - (I*
```

```
Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - b^(3/2)*x^2*
Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] +
I*a^(3/2)*d*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b]
 + I*d*x] + I*Sqrt[a]*b*d*x^2*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(
Sqrt[a]*d)/Sqrt[b] + I*d*x] - a*Sqrt[b]*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*Sin
Integral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - b^(3/2)*x^2*Sinh[c + (I*Sqrt[a]*d)/
Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/(4*a^(3/2)*b*(a + b*x^2)
)
```

Maple [A]
time = 0.73, size = 503, normalized size = 1.06

method	result
risch	$\frac{d^{2} \mathrm{e}^{-d x -c} x}{4 a \left(b \,d^{2} x^{2}+a \,d^{2}\right)}-\frac{d \,\mathrm{e}^{-\frac{d \sqrt{-a b}+b c}{b}} \mathrm{expIntegral}\left(1,-\frac{d \sqrt{-a b}-b \left(d x +c \right)+b c}{b}\right)}{8 b a}-\frac{d \,\mathrm{e}^{-\frac{-d \sqrt{-a b}+b c}{b}} \mathrm{expIntegral}\left(1,\frac{d \sqrt{-a b}+}{b}\right.}{8 b a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d^2*exp(-d*x-c)*x/a/(b*d^2*x^2+a*d^2)-1/8*d/b/a*exp(-(d*(-a*b)^(1/2)+b*
c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)-1/8*d/b/a*exp(-(-d*(-a*b)^(1/
2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)-1/8/(-a*b)^(1/2)/a*exp(-(
d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/8/(-a*b)^(
1/2)/a*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)
+1/4*d^2*exp(d*x+c)*x/a/(b*d^2*x^2+a*d^2)+1/8*d/b/a*exp((d*(-a*b)^(1/2)+b*c
)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/8*d/b/a*exp((-d*(-a*b)^(1/2)+
b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)-1/8/(-a*b)^(1/2)/a*exp((d*(
-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/8/(-a*b)^(1/2)
/a*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/(b*x^2 + a)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. 2(365) = 730.
time = 0.41, size = 1162, normalized size = 2.44

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/8*(4*a*b*d*x*cosh(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a^2*b^2*d*x^2 + a^3*b*d)*cosh(d*x + c)^2 - (a^2*b^2*d*x^2 + a^3*b*d)*sinh(d*x + c)^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c + dx)}{(a + bx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(b*x**2+a)**2,x)

[Out] Integral(cosh(c + d*x)/(a + b*x**2)**2, x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)}{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*x^2)^2,x)
```

```
[Out] int(cosh(c + d*x)/(a + b*x^2)^2, x)
```

3.70 $\int \frac{\cosh(c+dx)}{x(a+bx^2)^2}\,dx$

Optimal. Leaf size=435

$$\frac{\cosh(c+dx)}{2a\,(a+bx^2)}+\frac{\cosh(c)\mathrm{Chi}(dx)}{a^2}-\frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^2}-\frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}\right.}{2a^2}$$

```
[Out] Chi(d*x)*cosh(c)/a^2+1/2*cosh(d*x+c)/a/(b*x^2+a)-1/2*Chi(d*x+d*(-a)^(1/2)/b
^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/a^2-1/2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))
*cosh(c+d*(-a)^(1/2)/b^(1/2))/a^2+Shi(d*x)*sinh(c)/a^2-1/2*Shi(d*x+d*(-a)^(
1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a^2-1/2*Shi(d*x-d*(-a)^(1/2)/b^(
1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^2+1/4*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*S
hi(d*x-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)-1/4*d*cosh(c-d*(-a)^(1/2)/b
^(1/2))*Shi(d*x+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)-1/4*d*Chi(d*x+d*(-
a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)+1/4*d*Chi
(-d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)
```

Rubi [A]
time = 0.61, antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,
Rules used = {5401, 3384, 3379, 3382, 5397, 5388}

$$-\frac{\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^2}-\frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a^2}+\frac{\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^2}-\frac{\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a^2}+\frac{\cosh(c)\mathrm{Chi}(dx)}{a^2}+\frac{\sinh(c)\mathrm{Shi}(dx)}{a^2}-\frac{d\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{d\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{d\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{d\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\cosh(c+dx)}{2a\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x^2)^2),x]
```

```
[Out] Cosh[c + d*x]/(2*a*(a + b*x^2)) + (Cosh[c]*CoshIntegral[d*x])/a^2 - (Cosh[c
 + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2)
- (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])
/(2*a^2) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d
)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) + (Sinh[c]*Sin
hIntegral[d*x])/a^2 - (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[
-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (Sinh[c + (Sqrt[-a]*d)/Sqrt
[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) - (d*Cosh[c - (Sqrt[
-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqr
t[b]) - (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x])/(2*a^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^2}\,dx = \int \left(\frac{\cosh(c+dx)}{a^2x} - \frac{bx\cosh(c+dx)}{a\left(a+bx^2\right)^2} - \frac{bx\cosh(c+dx)}{a^2\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a^2} - \frac{b\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{a^2} - \frac{b\int \frac{x\cosh(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a}$$

$$= \frac{\cosh(c+dx)}{2a\left(a+bx^2\right)} - \frac{b\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx}{a^2} - \frac{d\int \frac{\sinh(c+dx)}{a+bx^2}}{2a}$$

$$= \frac{\cosh(c+dx)}{2a\left(a+bx^2\right)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} + \frac{\sqrt{b}\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2a^2} - \frac{\sqrt{b}\int}{}$$

$$= \frac{\cosh(c+dx)}{2a\left(a+bx^2\right)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} - \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{4(-a)^{3/2}} - \frac{d\int \frac{\sinh(c}{\sqrt{-a}}}{4(-a}$$

$$= \frac{\cosh(c+dx)}{2a\left(a+bx^2\right)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^2} - \frac{\cosh\left(c\right.}{}$$

$$= \frac{\cosh(c+dx)}{2a\left(a+bx^2\right)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^2} - \frac{\cosh\left(c\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.84, size = 501, normalized size = 1.15

$$\frac{\frac{a\cosh(c)\cosh(dx)}{a+bx^2} + \frac{a\sinh(c)\sinh(dx)}{a+bx^2} + 2(\cosh(c)\text{Chi}(dx) + \sinh(c)\text{Shi}(dx)) + \frac{\sqrt{a}\,d\left(-i\,\text{CosIntegral}\left(-\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)\sinh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right) + i\,\text{CosIntegral}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)\sinh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right) + \cosh\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}-idx\right) + \cosh\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)\right)}{2\sqrt{b}} + i\sinh(c)\left(\text{CosIntegral}\left(-\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)\sin\left(\frac{\sqrt{a}\,d}{\sqrt{b}}\right) - \text{CosIntegral}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)\sin\left(\frac{\sqrt{a}\,d}{\sqrt{b}}\right) + \cos\left(\frac{\sqrt{a}\,d}{\sqrt{b}}\right)\left(-\text{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}-idx\right) + \text{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)\right)\right) - \cosh(c)\left(\cos\left(\frac{\sqrt{a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(-\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right) + \cos\left(\frac{\sqrt{a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right) + \sin\left(\frac{\sqrt{a}\,d}{\sqrt{b}}\right)\left(\text{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}-idx\right) + \text{Si}\left(\frac{\sqrt{a}\,d}{\sqrt{b}}+idx\right)\right)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*Cosh[c]*Cosh[d*x])/(a + b*x^2) + (a*Sinh[c]*Sinh[d*x])/(a + b*x^2) + 2*
(Cosh[c]*CoshIntegral[d*x] + Sinh[c]*SinhIntegral[d*x]) + (Sqrt[a]*d*((-I)*
CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]
 + I*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b
]] + Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*
x] + Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*
x]))/(2*Sqrt[b]) + I*Sinh[c]*(CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*S
```

```
in[(Sqrt[a]*d)/Sqrt[b]] - CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqr
t[a]*d)/Sqrt[b]] + Cos[(Sqrt[a]*d)/Sqrt[b]]*(-SinIntegral[(Sqrt[a]*d)/Sqrt[
b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])) - Cosh[c]*(Cos[(Sq
rt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + Cos[(Sqrt[a
]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a]*d)/Sq
rt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/
Sqrt[b] + I*d*x])))/(2*a^2)
```

Maple [A]
time = 0.75, size = 546, normalized size = 1.26

method	result
risch	$\frac{e^{-dx-c} d^2}{4a\left((dx+c)^2 b-2b(dx+c)c+a\,d^2+b\,c^2\right)}-\frac{e^{-c}\,\text{expIntegral}(1,dx)}{2a^2}-\frac{e^{-\frac{d\sqrt{-ab}+bc}{b}}\,\text{expIntegral}\left(1,-\frac{d\sqrt{-ab}-b(dx+c)+bc}{b}\right)d}{8a\sqrt{-ab}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*exp(-d*x-c)*d^2/a/((d*x+c)^2*b-2*b*(d*x+c)*c+a*d^2+b*c^2)-1/2/a^2*exp(-
c)*Ei(1,d*x)-1/8/a/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b
)^(1/2)-b*(d*x+c)+b*c)/b)*d+1/8/a/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+b*c)/b
)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)*d+1/4/a^2*exp(-(d*(-a*b)^(1/2)+b*c
)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/4/a^2*exp(-(-d*(-a*b)^(1/2)+
b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)+1/4*exp(d*x+c)*d^2/a/((d*x+c
)^2*b-2*b*(d*x+c)*c+a*d^2+b*c^2)-1/2/a^2*exp(c)*Ei(1,-d*x)-1/8/a/(-a*b)^(1/
2)*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)*d+1
/8/a/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c
)+b*c)/b)*d+1/4/a^2*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d
*x+c)-b*c)/b)+1/4/a^2*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d
*x+c)+b*c)/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^2*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 992 vs. 2(337) = 674.
time = 0.49, size = 992, normalized size = 2.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*cosh(d*x + c) - ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*si
nh(d*x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)
*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + (2*(b*x^2 + a)*cosh(d*x + c)^2
- 2*(b*x^2 + a)*sinh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a
)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt
(-a*d^2/b)) + 4*((b*x^2 + a)*Ei(d*x) + (b*x^2 + a)*Ei(-d*x))*cosh(c) - ((2*
(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 + ((b*x^2 + a)*
cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqr
t(-a*d^2/b)) + (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)
^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^
2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - ((2*(b*x^2 + a
)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 - ((b*x^2 + a)*cosh(d*x +
 c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b
)) - (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 + ((b*x
^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-
d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + 4*((b*x^2 + a)*Ei(d*x) -
(b*x^2 + a)*Ei(-d*x))*sinh(c) + ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2
+ a)*sinh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x
+ c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - (2*(b*x^2 + a)*cosh(d*x
+ c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*
x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-
c + sqrt(-a*d^2/b)))/((a^2*b*x^2 + a^3)*cosh(d*x + c)^2 - (a^2*b*x^2 + a^3)
*sinh(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c+dx)}{x(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x**2+a)**2,x)
```

```
[Out] Integral(cosh(c + d*x)/(x*(a + b*x**2)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError >> type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)}{x\,{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x*(a + b*x^2)^2),x)
```

```
[Out] int(cosh(c + d*x)/(x*(a + b*x^2)^2), x)
```

3.71 $\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=500

$$-\frac{\cosh(c+dx)}{a^2x}+\frac{\sqrt{b}\,\cosh(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}\,x\right)}-\frac{\sqrt{b}\,\cosh(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}\,x\right)}-\frac{3\sqrt{b}\,\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}}$$

```
[Out] -cosh(d*x+c)/a^2/x+d*cosh(c)*Shi(d*x)/a^2+1/4*d*cosh(c+d*(-a)^(1/2)/b^(1/2)
)*Shi(d*x-d*(-a)^(1/2)/b^(1/2))/a^2+1/4*d*cosh(c-d*(-a)^(1/2)/b^(1/2))*Shi(
d*x+d*(-a)^(1/2)/b^(1/2))/a^2+d*Chi(d*x)*sinh(c)/a^2+1/4*d*Chi(d*x+d*(-a)^(
1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a^2+1/4*d*Chi(-d*x+d*(-a)^(1/2)/
b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^2+3/4*Chi(d*x+d*(-a)^(1/2)/b^(1/2))
*cosh(c-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(5/2)-3/4*Chi(-d*x+d*(-a)^(1/2)/
b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(5/2)+3/4*Shi(d*x+d*(-a)
^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(5/2)-3/4*Shi(d*x
-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(5/2)+1/4*
cosh(d*x+c)*b^(1/2)/a^2/((-a)^(1/2)-x*b^(1/2))-1/4*cosh(d*x+c)*b^(1/2)/a^2/
((-a)^(1/2)+x*b^(1/2))
```

Rubi [A]
time = 0.91, antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5401, 3378, 3384, 3379, 3382, 5389}

$$\frac{d\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4a^2}+\frac{d\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4a^2}-\frac{d\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4a^2}+\frac{d\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4a^2}+\frac{\sqrt{b}\,\cosh(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}\,x\right)}-\frac{\sqrt{b}\,\cosh(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}\,x\right)}+\frac{d\sinh(c)\mathrm{Chi}(dx)}{a^2}+\frac{d\cosh(c)\mathrm{Shi}(dx)}{a^2}-\frac{\cosh(c+dx)}{a^2x}-\frac{3\sqrt{b}\,\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}}+\frac{3\sqrt{b}\,\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4(-a)^{5/2}}+\frac{3\sqrt{b}\,\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}}+\frac{3\sqrt{b}\,\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4(-a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(Cosh[c + d*x]/(a^2*x)) + (Sqrt[b]*Cosh[c + d*x])/(4*a^2*(Sqrt[-a] - Sqrt[
b]*x)) - (Sqrt[b]*Cosh[c + d*x])/(4*a^2*(Sqrt[-a] + Sqrt[b]*x)) - (3*Sqrt[b
]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(4*(-a)^(5/2)) + (3*Sqrt[b]*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sq
rt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(5/2)) + (d*CoshIntegral[d*x]*Sinh[c])/a^
2 + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[
b]])/(4*a^2) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-
a]*d)/Sqrt[b]])/(4*a^2) + (d*Cosh[c]*SinhIntegral[d*x])/a^2 - (d*Cosh[c + (
Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a^2) + (3
*Sqrt[b]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x])/(4*(-a)^(5/2)) + (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqr
t[-a]*d)/Sqrt[b] + d*x])/(4*a^2) + (3*Sqrt[b]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]
]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(5/2))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)^2}\,dx = \int \left(\frac{\cosh(c+dx)}{a^2x^2} - \frac{b\cosh(c+dx)}{a\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^2\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a^2} - \frac{b\int \frac{\cosh(c+dx)}{a+bx^2}\,dx}{a^2} - \frac{b\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx}{a}$$

$$= -\frac{\cosh(c+dx)}{a^2x} - \frac{b\int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^2} - \frac{b\int\left(-\frac{b\cosh(\ldots}{4a\left(\sqrt{-a}\ldots\right.}\right.}{\ldots}$$

$$= -\frac{\cosh(c+dx)}{a^2x} + \frac{b\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{5/2}} + \frac{b\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{5/2}} + \frac{b^2\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,d\ldots}{4a^2}$$

$$= -\frac{\cosh(c+dx)}{a^2x} + \frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{d\,\mathrm{Chi}(dx)\sinh(c)}{a^2} + \ldots$$

$$= -\frac{\cosh(c+dx)}{a^2x} + \frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{\sqrt{b}\cosh\left(c+\frac{\sqrt{-\ldots}}{\sqrt{\ldots}}\right.}{2\ldots}$$

$$= -\frac{\cosh(c+dx)}{a^2x} + \frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{\sqrt{b}\cosh\left(c+\frac{\sqrt{-\ldots}}{\sqrt{\ldots}}\right.}{2\ldots}$$

$$= -\frac{\cosh(c+dx)}{a^2x} + \frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{3\sqrt{b}\cosh\left(c+\frac{\sqrt{-\ldots}}{\sqrt{\ldots}}\right.}{4\ldots}$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 0.70, size = 675, normalized size = 1.35

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] (-4*a^(3/2)*Cosh[c + d*x] - 6*Sqrt[a]*b*x^2*Cosh[c + d*x] + 4*a^(3/2)*d*x*C
oshIntegral[d*x]*Sinh[c] + 4*Sqrt[a]*b*d*x^3*CoshIntegral[d*x]*Sinh[c] + x*
```

```
(a + b*x^2)*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*((-3*I)*Sqrt[b]*Cos
h[c - (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[a]*d*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]) +
 x*(a + b*x^2)*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*((3*I)*Sqrt[b]*Cosh
[c + (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[a]*d*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]) +
4*a^(3/2)*d*x*Cosh[c]*SinhIntegral[d*x] + 4*Sqrt[a]*b*d*x^3*Cosh[c]*SinhInt
egral[d*x] + I*a^(3/2)*d*x*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqr
t[a]*d)/Sqrt[b] - I*d*x] + I*Sqrt[a]*b*d*x^3*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]
]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + 3*a*Sqrt[b]*x*Sinh[c - (I*Sqrt
[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + 3*b^(3/2)*x^3*Si
nh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - I*
a^(3/2)*d*x*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b]
 + I*d*x] - I*Sqrt[a]*b*d*x^3*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(
Sqrt[a]*d)/Sqrt[b] + I*d*x] + 3*a*Sqrt[b]*x*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]
*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + 3*b^(3/2)*x^3*Sinh[c + (I*Sqrt[
a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/(4*a^(5/2)*x*(a +
b*x^2))
```

Maple [A]
time = 0.78, size = 595, normalized size = 1.19

method	result
risch	$-\frac{3\,\mathrm{e}^{-dx-c} x\, d^{2} b}{4 a^{2} \left(b\, d^{2} x^{2}+a\, d^{2}\right)}-\frac{\mathrm{e}^{-dx-c} d^{2}}{2 a \left(b\, d^{2} x^{2}+a\, d^{2}\right) x}+\frac{d\,\mathrm{e}^{-c}\, \mathrm{expIntegral}(1,dx)}{2 a^{2}}+\frac{d\,\mathrm{e}^{-\frac{d\sqrt{-ab}+bc}{b}}\, \mathrm{expIntegral}\left(1,-\frac{d\sqrt{-ab}-b(dx+c)+b}{b}\right.}{8 a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*exp(-d*x-c)/a^2/(b*d^2*x^2+a*d^2)*x*d^2*b-1/2*exp(-d*x-c)/a/(b*d^2*x^2
+a*d^2)/x*d^2+1/2*d/a^2*exp(-c)*Ei(1,d*x)+1/8*d/a^2*exp(-(d*(-a*b)^(1/2)+b*
c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/8*d/a^2*exp(-(-d*(-a*b)^(1/
2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)+3/8/a^2/(-a*b)^(1/2)*exp(
-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)*b-3/8/a^2/
(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b
*c)/b)*b-3/4*exp(d*x+c)/a^2/(b*d^2*x^2+a*d^2)*x*d^2*b-1/2*exp(d*x+c)/a/(b*d
^2*x^2+a*d^2)/x*d^2-1/2*d/a^2*exp(c)*Ei(1,-d*x)-1/8*d/a^2*exp((d*(-a*b)^(1/
2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)-1/8*d/a^2*exp((-d*(-a*b)^
(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)+3/8/a^2/(-a*b)^(1/2)*
exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)*b-3/8/a^
2/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)
-b*c)/b)*b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^2*x^2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1310 vs. $2(389) = 778$.
time = 0.48, size = 1310, normalized size = 2.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(3*a*b*d*x^2 + 2*a^2*d)*cosh(d*x + c) - (((a*b*d^2*x^3 + a^2*d^2*x)
*cosh(d*x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 + 3*((b^2*x^3
+ a*b*x)*cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b
))*Ei(d*x - sqrt(-a*d^2/b)) - ((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x + c)^2 -
(a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 - 3*((b^2*x^3 + a*b*x)*cosh(d*x +
 c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-
a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - 4*((a*b*d^2*x^3 + a^2*d^2*x)*Ei(d*x)
- (a*b*d^2*x^3 + a^2*d^2*x)*Ei(-d*x))*cosh(c) - (((a*b*d^2*x^3 + a^2*d^2*x)
*cosh(d*x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 - 3*((b^2*x^3
+ a*b*x)*cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b
))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x + c)^2 -
(a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 + 3*((b^2*x^3 + a*b*x)*cosh(d*x +
 c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-
a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - (((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x
 + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 + 3*((b^2*x^3 + a*b*x)*
cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x
 - sqrt(-a*d^2/b)) + ((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x + c)^2 - (a*b*d^2*
x^3 + a^2*d^2*x)*sinh(d*x + c)^2 - 3*((b^2*x^3 + a*b*x)*cosh(d*x + c)^2 - (
b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b))
)*sinh(c + sqrt(-a*d^2/b)) - 4*((a*b*d^2*x^3 + a^2*d^2*x)*Ei(d*x) + (a*b*d^
2*x^3 + a^2*d^2*x)*Ei(-d*x))*sinh(c) + (((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x
 + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 - 3*((b^2*x^3 + a*b*x)*
cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x
 + sqrt(-a*d^2/b)) + ((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x + c)^2 - (a*b*d^2*
x^3 + a^2*d^2*x)*sinh(d*x + c)^2 + 3*((b^2*x^3 + a*b*x)*cosh(d*x + c)^2 - (
b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b))
)*sinh(-c + sqrt(-a*d^2/b)))/((a^3*b*d*x^3 + a^4*d*x)*cosh(d*x + c)^2 - (a^
3*b*d*x^3 + a^4*d*x)*sinh(d*x + c)^2)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Integral(cosh(c + d*x)/(x**2*(a + b*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)}{x^2\,(b\,x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] int(cosh(c + d*x)/(x^2*(a + b*x^2)^2), x)
```

3.72 $\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=476

$$-\frac{x^2\cosh(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{\cosh(c+dx)}{4b^2\left(a+bx^2\right)}+\frac{d^2\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16b^3}+\frac{d^2\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\cdots\right.}{16b^3}$$

[Out] -1/4*x^2*cosh(d*x+c)/b/(b*x^2+a)^2-1/4*cosh(d*x+c)/b^2/(b*x^2+a)+1/16*d^2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/b^3+1/16*d^2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/b^3-1/8*d*x*sinh(d*x+c)/b^2/(b*x^2+a)+1/16*d^2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/b^3+1/16*d^2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/b^3+3/16*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(d*x-d*(-a)^(1/2)/b^(1/2))/b^(5/2)/(-a)^(1/2)-3/16*d*cosh(c-d*(-a)^(1/2)/b^(1/2))*Shi(d*x+d*(-a)^(1/2)/b^(1/2))/b^(5/2)/(-a)^(1/2)-3/16*d*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/b^(5/2)/(-a)^(1/2)+3/16*d*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/b^(5/2)/(-a)^(1/2)

Rubi [A]
time = 0.75, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {5399, 5397, 5388, 3384, 3379, 3382, 5398, 5401}

$$-\frac{3d\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16\sqrt{-a}\,b^{5/2}}+\frac{3d\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}\,b^{5/2}}-\frac{3d\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}\,b^{5/2}}-\frac{3d\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16\sqrt{-a}\,b^{5/2}}+\frac{d^2\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16b^3}+\frac{d^2\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16b^3}-\frac{d^2\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16b^3}+\frac{d^2\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16b^3}-\frac{dx\sinh(c+dx)}{8b^2\left(a+bx^2\right)}-\frac{\cosh(c+dx)}{4b^2\left(a+bx^2\right)}-\frac{x^2\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^2)^3,x]

[Out] -1/4*(x^2*Cosh[c + d*x])/(b*(a + b*x^2)^2) - Cosh[c + d*x]/(4*b^2*(a + b*x^2)) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*b^3) + (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*b^3) - (3*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*Sqrt[-a]*b^(5/2)) + (3*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*Sqrt[-a]*b^(5/2)) - (d*x*Sinh[c + d*x])/(8*b^2*(a + b*x^2)) - (3*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/2)) - (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*b^3) - (3*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*Sqrt[-a]*b^(5/2)) + (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*b^3)

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5398

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Sinh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^2)^3}\,dx = -\frac{x^2\cosh(c+dx)}{4b(a+bx^2)^2} + \frac{\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx}{2b} + \frac{d\int \frac{x^2\sinh(c+dx)}{(a+bx^2)^2}\,dx}{4b}$$

$$= -\frac{x^2\cosh(c+dx)}{4b(a+bx^2)^2} - \frac{\cosh(c+dx)}{4b^2(a+bx^2)} - \frac{dx\sinh(c+dx)}{8b^2(a+bx^2)} + \frac{d\int \frac{\sinh(c+dx)}{a+bx^2}\,dx}{8b^2} + \frac{d\int \frac{\sinh(c+}{a+b}}{4b^2}$$

$$= -\frac{x^2\cosh(c+dx)}{4b(a+bx^2)^2} - \frac{\cosh(c+dx)}{4b^2(a+bx^2)} - \frac{dx\sinh(c+dx)}{8b^2(a+bx^2)} + \frac{d\int \left(\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{}}{2a}\right.}{8b^2}$$

$$= -\frac{x^2\cosh(c+dx)}{4b(a+bx^2)^2} - \frac{\cosh(c+dx)}{4b^2(a+bx^2)} - \frac{dx\sinh(c+dx)}{8b^2(a+bx^2)} - \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{16\sqrt{-a}\,b^2} - \frac{d\int -}{1}$$

$$= -\frac{x^2\cosh(c+dx)}{4b(a+bx^2)^2} - \frac{\cosh(c+dx)}{4b^2(a+bx^2)} - \frac{dx\sinh(c+dx)}{8b^2(a+bx^2)} - \frac{\left(d\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\right)\int}{16\sqrt{-a}\,b}$$

$$= -\frac{x^2\cosh(c+dx)}{4b(a+bx^2)^2} - \frac{\cosh(c+dx)}{4b^2(a+bx^2)} + \frac{d^2\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16b^3} + \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.42, size = 648, normalized size = 1.36

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] ((-2*Cosh[d*x]*(2*(a + 2*b*x^2)*Cosh[c] + d*x*(a + b*x^2)*Sinh[c]))/(a + b*
x^2)^2 - (2*(d*x*(a + b*x^2)*Cosh[c] + 2*(a + 2*b*x^2)*Sinh[c])*Sinh[d*x])/
(a + b*x^2)^2 + ((3*I)*d*Sinh[c]*(Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((S
qrt[a]*d)/Sqrt[b]) + I*d*x] - Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]
```

```
*d)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sq
rt[b] - I*d*x] - SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/(Sqrt[a]*Sqrt[
b]) - (I*d^2*Sinh[c]*(CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sin[(Sqrt
[a]*d)/Sqrt[b]] - CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqrt[a]*d)/
Sqrt[b]] + Cos[(Sqrt[a]*d)/Sqrt[b]]*(-SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d
*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/b + (3*d*Cosh[c]*(CosInte
gral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] + CosIntegral
[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] - Cos[(Sqrt[a]*d)/Sq
rt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/
Sqrt[b] + I*d*x])))/(Sqrt[a]*Sqrt[b]) + (d^2*Cosh[c]*(Cos[(Sqrt[a]*d)/Sqrt[
b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + Cos[(Sqrt[a]*d)/Sqrt[b]]*
CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a]*d)/Sqrt[b]]*(SinInt
egral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*
x])))/b)/(16*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(374) = 748$.
time = 1.10, size = 820, normalized size = 1.72

method	result
risch	$-\frac{d^4 e^{-dx-c} x^2}{4b(x^4b^2d^4+2ab\,d^4x^2+a^2d^4)} - \frac{d^2 e^{-\frac{-d\sqrt{-ab}+bc}{b}} \operatorname{expIntegral}\left(1, \frac{d\sqrt{-ab}+b(dx+c)-bc}{b}\right)}{32b^3} - \frac{d^2 e^{-\frac{d\sqrt{-ab}+bc}{b}} \operatorname{expIntegral}(\ldots}{32b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d^4*exp(-d*x-c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2-1/32*d^2/b^3
*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)-1/32*
d^2/b^3*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b
)-3/32*d/b^2/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2
)+b*(d*x+c)-b*c)/b)+3/32*d/b^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei
(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/16*d^5*exp(-d*x-c)/b/(b^2*d^4*x^4+2
*a*b*d^4*x^2+a^2*d^4)*x^3+1/16*d^5*exp(-d*x-c)*a/b^2/(b^2*d^4*x^4+2*a*b*d^4
*x^2+a^2*d^4)*x-1/8*d^4*exp(-d*x-c)*a/b^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^
4)-1/4*d^4*exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2-1/32*d^2/b^
3*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)-1/32*d
^2/b^3*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)
-3/32*d/b^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b
*(d*x+c)+b*c)/b)+3/32*d/b^2/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,
-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)-1/16*d^5*exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b
*d^4*x^2+a^2*d^4)*x^3-1/16*d^5*exp(d*x+c)*a/b^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+
a^2*d^4)*x-1/8*d^4*exp(d*x+c)*a/b^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*((d^2*x^3*e^(2*c) + 3*d*x^2*e^(2*c) + 12*x*e^(2*c))*e^(d*x) - (d^2*x^3
- 3*d*x^2 + 12*x)*e^(-d*x))/(b^3*d^3*x^6*e^c + 3*a*b^2*d^3*x^4*e^c + 3*a^2*
b*d^3*x^2*e^c + a^3*d^3*e^c) - 1/2*integrate(6*(3*a*d*x*e^c + (a*d^2*e^c -
10*b*e^c)*x^2 + 2*a*e^c)*e^(d*x)/(b^4*d^3*x^8 + 4*a*b^3*d^3*x^6 + 6*a^2*b^2
*d^3*x^4 + 4*a^3*b*d^3*x^2 + a^4*d^3), x) + 1/2*integrate(-6*(3*a*d*x - (a*
d^2 - 10*b)*x^2 - 2*a)*e^(-d*x)/(b^4*d^3*x^8*e^c + 4*a*b^3*d^3*x^6*e^c + 6*
a^2*b^2*d^3*x^4*e^c + 4*a^3*b*d^3*x^2*e^c + a^4*d^3*e^c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1620 vs. 2(374) = 748.
time = 0.44, size = 1620, normalized size = 3.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(8*(2*a*b^2*x^2 + a^2*b)*cosh(d*x + c) - (((a*b^2*d^2*x^4 + 2*a^2*b*d
^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*
d^2)*sinh(d*x + c)^2 - 3*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 -
 (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x -
sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c
)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + 3*((b^3
*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*
b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqr
t(-a*d^2/b)) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^
2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + 3*((b^3*x
^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^
4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d
^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - 3*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh
(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/
b))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + 4*(a*b^2*d*x^3 +
 a^2*b*d*x)*sinh(d*x + c) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*c
osh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^
2 - 3*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2
*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - (
(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^
```

```
4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + 3*((b^3*x^4 + 2*a*b^2*x^2
+ a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)
*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (((a
*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4
+ 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + 3*((b^3*x^4 + 2*a*b^2*x^2 +
a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*s
qrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2
 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*s
inh(d*x + c)^2 - 3*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*
x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(
-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^
3)*cosh(d*x + c)^2 - (a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3)*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\cosh(c+d\,x)}{\left(b\,x^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*cosh(c + d*x))/(a + b*x^2)^3,x)
```

```
[Out] int((x^3*cosh(c + d*x))/(a + b*x^2)^3, x)
```

3.73 $\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=746

$$-\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)}-\frac{x\cosh(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-\right.}{16(-a)^{3/2}b^{3/2}}$$

```
[Out] -1/4*x*cosh(d*x+c)/b/(b*x^2+a)^2+1/16*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-
d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)-1/16*Chi(-d*x+d*(-a)^(1/2)/b^(1/2)
)*cosh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)-1/16*d*cosh(c+d*(-a)^(1/2
)/b^(1/2))*Shi(d*x-d*(-a)^(1/2)/b^(1/2))/a/b^2-1/16*d*cosh(c-d*(-a)^(1/2)/b
^(1/2))*Shi(d*x+d*(-a)^(1/2)/b^(1/2))/a/b^2-1/8*d*sinh(d*x+c)/b^2/(b*x^2+a)
-1/16*d*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a/b^2+1/
16*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^
(3/2)-1/16*d*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a/
b^2-1/16*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3
/2)/b^(3/2)-1/16*d^2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1
/2))/b^(5/2)/(-a)^(1/2)+1/16*d^2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-
a)^(1/2)/b^(1/2))/b^(5/2)/(-a)^(1/2)-1/16*d^2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))
*sinh(c-d*(-a)^(1/2)/b^(1/2))/b^(5/2)/(-a)^(1/2)+1/16*d^2*Shi(d*x-d*(-a)^(1
/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/b^(5/2)/(-a)^(1/2)-1/16*cosh(d*x+
c)/a/b^(3/2)/((-a)^(1/2)-x*b^(1/2))+1/16*cosh(d*x+c)/a/b^(3/2)/((-a)^(1/2)+
x*b^(1/2))
```

Rubi [A]
time = 0.82, antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5399, 5389, 3378, 3384, 3379, 3382, 5396}

$$\frac{d^2\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}\,b^{5/2}}-\frac{d^2\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16\sqrt{-a}\,b^{5/2}}-\frac{\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}}+\frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}-\frac{d^2\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}\,b^{5/2}}-\frac{d^2\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16\sqrt{-a}\,b^{5/2}}+\frac{\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}}+\frac{\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}-\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)}-\frac{d\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16ab^2}-\frac{d\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16ab^2}+\frac{d\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16ab^2}-\frac{d\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16ab^2}-\frac{d\sinh(c+dx)}{8b^2\left(a+bx^2\right)}-\frac{x\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] -1/16*Cosh[c + d*x]/(a*b^(3/2)*(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(16*
a*b^(3/2)*(Sqrt[-a] + Sqrt[b]*x)) - (x*Cosh[c + d*x])/(4*b*(a + b*x^2)^2) -
 (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(16*(-a)^(3/2)*b^(3/2)) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[
(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/2)) + (Cosh[c - (Sqrt[-a]*d)
/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/2))
 - (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] +
d*x])/(16*Sqrt[-a]*b^(5/2)) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*S
```

```
inh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a*b^2) - (d*CoshIntegral[(Sqrt[-a]*d)/Sq
rt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*a*b^2) - (d*Sinh[c + d*x])
/(8*b^2*(a + b*x^2)) + (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt
[-a]*d)/Sqrt[b] - d*x])/(16*a*b^2) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIn
tegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) - (d^2*Sinh[c +
 (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-
a]*b^(5/2)) - (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/S
qrt[b] + d*x])/(16*a*b^2) + (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(S
qrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/2)) - (d^2*Sinh[c - (Sqrt[-a
]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*Sqrt[-a]*b^(5/2
))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5396

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_
)], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)))
```

```
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cosh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rubi steps

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^2)^3}\,dx = -\frac{x\cosh(c+dx)}{4b(a+bx^2)^2} + \frac{\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx}{4b} + \frac{d\int \frac{x\sinh(c+dx)}{(a+bx^2)^2}\,dx}{4b}$$

$$= -\frac{x\cosh(c+dx)}{4b(a+bx^2)^2} - \frac{d\sinh(c+dx)}{8b^2(a+bx^2)} + \frac{\int\left(-\frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)}\right.}{4b}$$

$$= -\frac{x\cosh(c+dx)}{4b(a+bx^2)^2} - \frac{d\sinh(c+dx)}{8b^2(a+bx^2)} - \frac{\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a} - \frac{\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}}{16a}$$

$$= -\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{x\cosh(c+dx)}{4b(a+bx^2)^2} - \frac{d\sinh}{8b^2(}$$

$$= -\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{x\cosh(c+dx)}{4b(a+bx^2)^2} - \frac{d\sinh}{8b^2(}$$

$$= -\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{x\cosh(c+dx)}{4b(a+bx^2)^2} + \frac{d^2\cos}{}$$

$$= -\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{x\cosh(c+dx)}{4b(a+bx^2)^2} - \frac{\cosh\left(\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.90, size = 932, normalized size = 1.25

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] ((-2*a^(3/2)*b*x*Cosh[c]*Cosh[d*x])/(a + b*x^2)^2 + (2*Sqrt[a]*b^2*x^3*Cosh
[c]*Cosh[d*x])/(a + b*x^2)^2 - (2*a^(5/2)*d*Cosh[d*x]*Sinh[c])/(a + b*x^2)^
2 - (2*a^(3/2)*b*d*x^2*Cosh[d*x]*Sinh[c])/(a + b*x^2)^2 + (I*CosIntegral[-(
```

```
(Sqrt[a]*d)/Sqrt[b]) + I*d*x]*((b + a*d^2)*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]
+ I*Sqrt[a]*Sqrt[b]*d*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]))/Sqrt[b] - (I*CosInt
egral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*((b + a*d^2)*Cosh[c + (I*Sqrt[a]*d)/Sqrt
[b]] - I*Sqrt[a]*Sqrt[b]*d*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]))/Sqrt[b] - (2*a
^(5/2)*d*Cosh[c]*Sinh[d*x])/(a + b*x^2)^2 - (2*a^(3/2)*b*d*x^2*Cosh[c]*Sinh
[d*x])/(a + b*x^2)^2 - (2*a^(3/2)*b*x*Sinh[c]*Sinh[d*x])/(a + b*x^2)^2 + (2
*Sqrt[a]*b^2*x^3*Sinh[c]*Sinh[d*x])/(a + b*x^2)^2 - I*Sqrt[a]*d*Cos[(Sqrt[a
]*d)/Sqrt[b]]*Cosh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + I*Sqrt[b]*
Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] +
 (I*a*d^2*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b]
- I*d*x])/Sqrt[b] - Sqrt[b]*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(S
qrt[a]*d)/Sqrt[b] - I*d*x] - (a*d^2*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinInt
egral[(Sqrt[a]*d)/Sqrt[b] - I*d*x])/Sqrt[b] - Sqrt[a]*d*Sin[(Sqrt[a]*d)/Sqr
t[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + I*Sqrt[a]*d*Cos[(S
qrt[a]*d)/Sqrt[b]]*Cosh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - I*Sqr
t[b]*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d
*x] - (I*a*d^2*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqr
t[b] + I*d*x])/Sqrt[b] - Sqrt[b]*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegr
al[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - (a*d^2*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*S
inIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/Sqrt[b] - Sqrt[a]*d*Sin[(Sqrt[a]*d
)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/(16*a^(3/2)*b^
2)
```

Maple [A]
time = 0.93, size = 1064, normalized size = 1.43

method	result
risch	$\frac{d^5 \mathrm{e}^{-dx-c} x^2}{16b(x^4b^2d^4+2ab\,d^4x^2+a^2d^4)}+\frac{d^4 \mathrm{e}^{-dx-c} x^3}{16a(x^4b^2d^4+2ab\,d^4x^2+a^2d^4)}+\frac{d^5 \mathrm{e}^{-dx-c} a}{16b^2(x^4b^2d^4+2ab\,d^4x^2+a^2d^4)}-\frac{d^4 \mathrm{e}^{-dx-c} x}{16b(x^4b^2d^4+2ab\,d^4x^2+a^2d^4)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*d^5*exp(-d*x-c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2+1/16*d^4*exp
(-d*x-c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3+1/16*d^5*exp(-d*x-c)*a/b
^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-1/16*d^4*exp(-d*x-c)/b/(b^2*d^4*x^4+
2*a*b*d^4*x^2+a^2*d^4)*x+1/32*d^2/b^2/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+b*
c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)-1/32*d^2/b^2/(-a*b)^(1/2)*exp(
-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)-1/32*d/b^2
/a*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)-1/3
2*d/b^2/a*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)
/b)+1/32/b/a/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2
)+b*(d*x+c)-b*c)/b)-1/32/b/a/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1
,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)-1/16*d^5*exp(d*x+c)/b/(b^2*d^4*x^4+2*a*
```

```
b*d^4*x^2+a^2*d^4)*x^2+1/16*d^4*exp(d*x+c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2
*d^4)*x^3-1/16*d^5*exp(d*x+c)*a/b^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-1/1
6*d^4*exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x-1/32/b/a/(-a*b)^(1
/2)*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)-1/32
*d^2/b^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d
*x+c)+b*c)/b)+1/32*d^2/b^2/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-
(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)+1/32*d/b^2/a*exp((d*(-a*b)^(1/2)+b*c)/b)*
Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/32*d/b^2/a*exp((-d*(-a*b)^(1/2)+b*
c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)+1/32/b/a/(-a*b)^(1/2)*exp((-d
*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*((d*x^2*e^(2*c) + 4*x*e^(2*c))*e^(d*x) - (d*x^2 - 4*x)*e^(-d*x))/(b^3*d
^2*x^6*e^c + 3*a*b^2*d^2*x^4*e^c + 3*a^2*b*d^2*x^2*e^c + a^3*d^2*e^c) + 1/2
*integrate(-2*(3*a*d*x*e^c - 10*b*x^2*e^c + 2*a*e^c)*e^(d*x)/(b^4*d^2*x^8 +
 4*a*b^3*d^2*x^6 + 6*a^2*b^2*d^2*x^4 + 4*a^3*b*d^2*x^2 + a^4*d^2), x) + 1/2
*integrate(2*(3*a*d*x + 10*b*x^2 - 2*a)*e^(-d*x)/(b^4*d^2*x^8*e^c + 4*a*b^3
*d^2*x^6*e^c + 6*a^2*b^2*d^2*x^4*e^c + 4*a^3*b*d^2*x^2*e^c + a^4*d^2*e^c),
x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2047 vs. 2(575) = 1150.
time = 0.38, size = 2047, normalized size = 2.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(4*(a*b^2*d*x^3 - a^2*b*d*x)*cosh(d*x + c) - (((a*b^2*d^2*x^4 + 2*a^2*
b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a
^3*d^2)*sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^
2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 +
a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x
- sqrt(-a*d^2/b)) - ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x +
 c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3
*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x
+ c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x
^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sq
```

```
rt(-a*d^2/b)) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)
^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^
2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c
)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)
*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b^2*d^2*x^
4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d
^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2
*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b
^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b
))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - 4*(a^2*b*d^2*x^2
+ a^3*d^2)*sinh(d*x + c) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*co
sh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2
 + ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*c
osh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 +
a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + ((a
*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4
+ 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 + b^3
)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*
b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sq
rt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (((a*b^
2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2
*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 + b^3)*x
^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2
*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(
-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a
^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(
d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b
^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^
2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^
2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a^2*b^4*d*x^4 + 2*a^3*b^3*d*x^2 + a^4*b
^2*d)*cosh(d*x + c)^2 - (a^2*b^4*d*x^4 + 2*a^3*b^3*d*x^2 + a^4*b^2*d)*sinh(
d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \cosh(c + d\,x)}{\left(b\,x^2 + a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(c + d*x))/(a + b*x^2)^3,x)
```

```
[Out] int((x^2*cosh(c + d*x))/(a + b*x^2)^3, x)
```

3.74 $\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=512

$$-\frac{\cosh(c+dx)}{4b\,(a+bx^2)^2}-\frac{d^2\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16ab^2}-\frac{d^2\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{16ab^2}+-\frac{d\ \cdots}{}$$

```
[Out] -1/4*cosh(d*x+c)/b/(b*x^2+a)^2-1/16*d^2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(
c-d*(-a)^(1/2)/b^(1/2))/a/b^2-1/16*d^2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(
c+d*(-a)^(1/2)/b^(1/2))/a/b^2-1/16*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(d*x-d
*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)+1/16*d*cosh(c-d*(-a)^(1/2)/b^(1/2))
*Shi(d*x+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)+1/16*d*Chi(d*x+d*(-a)^(1/
2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)-1/16*d^2*Shi(d*
x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a/b^2-1/16*d*Chi(-d*x+
d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)-1/16*
d^2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a/b^2-1/16*d
*sinh(d*x+c)/a/b^(3/2)/((-a)^(1/2)-x*b^(1/2))+1/16*d*sinh(d*x+c)/a/b^(3/2)/
((-a)^(1/2)+x*b^(1/2))
```

Rubi [A]
time = 0.60, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5397, 5388, 3378, 3384, 3379, 3382}

$$\frac{d\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}-\frac{d\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}}+\frac{d\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}}+\frac{d\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}-\frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)}-\frac{d^2\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16ab^2}-\frac{d^2\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16ab^2}+\frac{d^2\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16ab^2}-\frac{d^2\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16ab^2}-\frac{\cosh(c+dx)}{4b\,(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] -1/4*Cosh[c + d*x]/(b*(a + b*x^2)^2) - (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*
CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a*b^2) - (d^2*Cosh[c - (Sqrt[
-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a*b^2) + (d*C
oshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16
*(-a)^(3/2)*b^(3/2)) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c +
 (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(3/2)*b^(3/2)) - (d*Sinh[c + d*x])/(16*a*b
^(3/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Sinh[c + d*x])/(16*a*b^(3/2)*(Sqrt[-a]
+ Sqrt[b]*x)) + (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)
/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) + (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[
b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a*b^2) + (d*Cosh[c - (Sqr
t[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*
b^(3/2)) - (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sq
rt[b] + d*x])/(16*a*b^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx = -\frac{\cosh(c+dx)}{4b(a+bx^2)^2} + \frac{d\int \frac{\sinh(c+dx)}{(a+bx^2)^2}\,dx}{4b}$$

$$= -\frac{\cosh(c+dx)}{4b(a+bx^2)^2} + \frac{d\int \left(-\frac{b\sinh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\sinh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{b\sinh(c+dx)}{2a(-ab-b^2x^2)}\right) dx}{4b}$$

$$= -\frac{\cosh(c+dx)}{4b(a+bx^2)^2} - \frac{d\int \frac{\sinh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a} - \frac{d\int \frac{\sinh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16a} - \frac{d\int \frac{\sinh(c+dx)}{-ab-b^2x^2}}{8a}$$

$$= -\frac{\cosh(c+dx)}{4b(a+bx^2)^2} - \frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{d\int \left(-\frac{}{2a}\right.}{}$$

$$= -\frac{\cosh(c+dx)}{4b(a+bx^2)^2} - \frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)} + \frac{d\int \frac{\sinh(}{\sqrt{-a}}}{16(-}$$

$$= -\frac{\cosh(c+dx)}{4b(a+bx^2)^2} - \frac{d^2\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d^2\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right.}{16}$$

$$= -\frac{\cosh(c+dx)}{4b(a+bx^2)^2} - \frac{d^2\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d^2\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right.}{16}$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 1.35, size = 637, normalized size = 1.24

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] ((2*Cosh[d*x]*(-2*a*Cosh[c] + d*x*(a + b*x^2)*Sinh[c]))/(a + b*x^2)^2 + (2*
(d*x*(a + b*x^2)*Cosh[c] - 2*a*Sinh[c])*Sinh[d*x])/(a + b*x^2)^2 + (I*d*Sin
h[c]*(Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]
- Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + Sin[(
```

```
Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - SinIntegral
[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/(Sqrt[a]*Sqrt[b]) + (I*d^2*Sinh[c]*(CosInt
egral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] - CosIntegra
l[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] + Cos[(Sqrt[a]*d)/S
qrt[b]]*(-SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d
)/Sqrt[b] + I*d*x])))/b + (d*Cosh[c]*(CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) +
I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] + CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*
Sin[(Sqrt[a]*d)/Sqrt[b]] - Cos[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d
)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/(Sqrt[a]*S
qrt[b]) - (d^2*Cosh[c]*(Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/
Sqrt[b]) + I*d*x] + Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b
] + I*d*x] + Sin[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*
d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/b)/(16*a*b)
```

Maple [A]
time = 0.84, size = 743, normalized size = 1.45

method	result
risch	$-\frac{d^{5} \mathrm{e}^{-d x -c} x^{3}}{16 a \left(x^{4} b^{2} d^{4}+2 a b \,d^{4} x^{2}+a^{2} d^{4}\right)}-\frac{d^{5} \mathrm{e}^{-d x -c} x}{16 b \left(x^{4} b^{2} d^{4}+2 a b \,d^{4} x^{2}+a^{2} d^{4}\right)}-\frac{d^{4} \mathrm{e}^{-d x -c}}{8 b \left(x^{4} b^{2} d^{4}+2 a b \,d^{4} x^{2}+a^{2} d^{4}\right)}+\frac{d^{2} \mathrm{e}^{-\frac{-d \sqrt{-a b}+b c}{b}} \mathrm{expInteg}}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*d^5*exp(-d*x-c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3-1/16*d^5*ex
p(-d*x-c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x-1/8*d^4*exp(-d*x-c)/b/(b^
2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+1/32*d^2/b^2/a*exp(-(-d*(-a*b)^(1/2)+b*c)/
b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)+1/32*d^2/b^2/a*exp(-(d*(-a*b)^(1/
2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)-1/32*d/b/a/(-a*b)^(1/2)*
exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)+1/32*d
/b/a/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x
+c)+b*c)/b)+1/16*d^5*exp(d*x+c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3+1
/16*d^5*exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x-1/8*d^4*exp(d*x+
c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+1/32*d^2/b^2/a*exp((-d*(-a*b)^(1/2
)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)+1/32*d^2/b^2/a*exp((d*(-a
*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/32*d/b/a/(-a*b)^
(1/2)*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)-
1/32*d/b/a/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*
(d*x+c)+b*c)/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b^3*d*x^6*e^c + 3*a*b^2*d*x^4*e^c + 3*a
^2*b*d*x^2*e^c + a^3*d*e^c) + 1/2*integrate((5*b*x^2*e^c - a*e^c)*e^(d*x)/(
b^4*d*x^8 + 4*a*b^3*d*x^6 + 6*a^2*b^2*d*x^4 + 4*a^3*b*d*x^2 + a^4*d), x) -
1/2*integrate((5*b*x^2 - a)*e^(-d*x)/(b^4*d*x^8*e^c + 4*a*b^3*d*x^6*e^c + 6
*a^2*b^2*d*x^4*e^c + 4*a^3*b*d*x^2*e^c + a^4*d*e^c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1607 vs. $2(399) = 798$.
time = 0.58, size = 1607, normalized size = 3.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(8*a^2*b*cosh(d*x + c) + (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2
)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x +
c)^2 + ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^
2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) +
((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x
^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2*x^2 +
 a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*
sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) + (((a*
b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 +
 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2*x^2 + a^2
*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt
(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 +
a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh
(d*x + c)^2 + ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 +
 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^
2/b)))*cosh(-c + sqrt(-a*d^2/b)) - 4*(a*b^2*d*x^3 + a^2*b*d*x)*sinh(d*x + c
) + (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*
d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((b^3*x^4 + 2*a*b^2*
x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c
)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - ((a*b^2*d^2*x^4 + 2*a^2*b*d
^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*
d^2)*sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (
b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + s
qrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x
^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)
*sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*
x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-
```

```
a*d^2/b)) - ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 -
(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((b^3*x^4 + 2
*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(
d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^
2/b)))/((a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)*cosh(d*x + c)^2 - (a^2*b^4*
x^4 + 2*a^3*b^3*x^2 + a^4*b^2)*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x**2+a)**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
[Out] integrate(x*cosh(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\cosh(c+d\,x)}{{\left(b\,x^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(c + d*x))/(a + b*x^2)^3,x)
[Out] int((x*cosh(c + d*x))/(a + b*x^2)^3, x)
```

3.75 $\int \frac{\cosh(c+dx)}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=856

$$-\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)^2}-\frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)^2}+\frac{3\,\mathrm{c}}{16a^2\sqrt{b}}$$

```
[Out] -1/16*d^2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(
3/2)/b^(3/2)+1/16*d^2*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^
(1/2))/(-a)^(3/2)/b^(3/2)-3/16*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(d*x-d*(-a
)^(1/2)/b^(1/2))/a^2/b-3/16*d*cosh(c-d*(-a)^(1/2)/b^(1/2))*Shi(d*x+d*(-a)^(
1/2)/b^(1/2))/a^2/b-3/16*d*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2
)/b^(1/2))/a^2/b-1/16*d^2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)
/b^(1/2))/(-a)^(3/2)/b^(3/2)-3/16*d*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c+d
*(-a)^(1/2)/b^(1/2))/a^2/b+1/16*d^2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*
(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)-3/16*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*c
osh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+3/16*Chi(-d*x+d*(-a)^(1/2)/b
^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)-3/16*Shi(d*x+d*(-a)
^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+3/16*Shi(d*
x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)-1/1
6*cosh(d*x+c)/(-a)^(3/2)/b^(1/2)/((-a)^(1/2)-x*b^(1/2))^2+1/16*d*sinh(d*x+c
)/(-a)^(3/2)/b/((-a)^(1/2)-x*b^(1/2))-3/16*cosh(d*x+c)/a^2/b^(1/2)/((-a)^(1
/2)-x*b^(1/2))+1/16*cosh(d*x+c)/(-a)^(3/2)/b^(1/2)/((-a)^(1/2)+x*b^(1/2))^2
+1/16*d*sinh(d*x+c)/(-a)^(3/2)/b/((-a)^(1/2)+x*b^(1/2))+3/16*cosh(d*x+c)/a^
2/b^(1/2)/((-a)^(1/2)+x*b^(1/2))
```

Rubi [A]
time = 0.88, antiderivative size = 856, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5389, 3378, 3384, 3379, 3382}

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*x^2)^3,x]
```

```
[Out] -1/16*Cosh[c + d*x]/((-a)^(3/2)*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)^2) - (3*Cosh
[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(16*(-a)
^(3/2)*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)^2) + (3*Cosh[c + d*x])/(16*a^2*Sqrt[b
]*(Sqrt[-a] + Sqrt[b]*x)) + (3*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[
(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (d^2*Cosh[c + (Sqrt[
-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^
```

```
(3/2)) - (3*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b
] + d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*Cos
hIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/2)) - (3*d*CoshI
ntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^2
*b) - (3*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/S
qrt[b]])/(16*a^2*b) + (d*Sinh[c + d*x])/(16*(-a)^(3/2)*b*(Sqrt[-a] - Sqrt[b
]*x)) + (d*Sinh[c + d*x])/(16*(-a)^(3/2)*b*(Sqrt[-a] + Sqrt[b]*x)) + (3*d*C
osh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16
*a^2*b) - (3*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[
b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*Si
nhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) - (3*d*Cosh
[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^
2*b) - (3*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b]
+ d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhI
ntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/2))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
```

```
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{\left(a+bx^2\right)^3}\,dx = \int \left(-\frac{b^{3/2}\cosh(c+dx)}{8(-a)^{3/2}\left(\sqrt{-a}\,\sqrt{b}-bx\right)^3} - \frac{3b\cosh(c+dx)}{16a^2\left(\sqrt{-a}\,\sqrt{b}-bx\right)^2} - \frac{b^{3/2}\cosh(c+d}{8(-a)^{3/2}\left(\sqrt{-a}\,\sqrt{b}\right.}\right.$$

$$= -\frac{(3b)\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\,\sqrt{b}-bx\right)^2}\,dx}{16a^2} - \frac{(3b)\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\,\sqrt{b}+bx\right)^2}\,dx}{16a^2} - \frac{(3b)\int \frac{\cosh(c+dx)}{-ab-b^2x^2}\,dx}{8a^2} - \frac{b^{3/2}}{}$$

$$= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)^2} - \frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\cosh(c+}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-}\right.}$$

$$= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)^2} - \frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\cosh(c+}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-}\right.}$$

$$= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)^2} - \frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\cosh(c+}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-}\right.}$$

$$= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)^2} - \frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\cosh(c+}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-}\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.78, size = 933, normalized size = 1.09

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x^2)^3,x]
```

```
[Out] ((10*a*b^(3/2)*x*Cosh[c]*Cosh[d*x])/(a + b*x^2)^2 + (6*b^(5/2)*x^3*Cosh[c]*
Cosh[d*x])/(a + b*x^2)^2 + (2*a^2*Sqrt[b]*d*Cosh[d*x]*Sinh[c])/(a + b*x^2)^
2 + (2*a*b^(3/2)*d*x^2*Cosh[d*x]*Sinh[c])/(a + b*x^2)^2 + (CosIntegral[-((S
```

```
qrt[a]*d)/Sqrt[b]) + I*d*x]*(I*(3*b - a*d^2)*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]
] - 3*Sqrt[a]*Sqrt[b]*d*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]))/Sqrt[a] + (I*CosI
ntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*((-3*b + a*d^2)*Cosh[c + (I*Sqrt[a]*d)
/Sqrt[b]] + (3*I)*Sqrt[a]*Sqrt[b]*d*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]))/Sqrt[
a] + (2*a^2*Sqrt[b]*d*Cosh[c]*Sinh[d*x])/(a + b*x^2)^2 + (2*a*b^(3/2)*d*x^2
*Cosh[c]*Sinh[d*x])/(a + b*x^2)^2 + (10*a*b^(3/2)*x*Sinh[c]*Sinh[d*x])/(a +
 b*x^2)^2 + (6*b^(5/2)*x^3*Sinh[c]*Sinh[d*x])/(a + b*x^2)^2 - (3*I)*Sqrt[b]
*d*Cos[(Sqrt[a]*d)/Sqrt[b]]*Cosh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x
] + ((3*I)*b*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[
b] - I*d*x])/Sqrt[a] - I*Sqrt[a]*d^2*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIn
tegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - (3*b*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]
*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x])/Sqrt[a] + Sqrt[a]*d^2*Cos[(Sqrt[
a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - 3*Sqrt[b]
*d*Sin[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x
] + (3*I)*Sqrt[b]*d*Cos[(Sqrt[a]*d)/Sqrt[b]]*Cosh[c]*SinIntegral[(Sqrt[a]*d
)/Sqrt[b] + I*d*x] - ((3*I)*b*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[
(Sqrt[a]*d)/Sqrt[b] + I*d*x])/Sqrt[a] + I*Sqrt[a]*d^2*Cosh[c]*Sin[(Sqrt[a]*
d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - (3*b*Cos[(Sqrt[a]*d)
/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/Sqrt[a] + Sqrt[
a]*d^2*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I
*d*x] - 3*Sqrt[b]*d*Sin[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d
)/Sqrt[b] + I*d*x])/(16*a^2*b^(3/2))
```

Maple [A]
time = 0.76, size = 1064, normalized size = 1.24

method	result
risch	$-\frac{d^5 e^{-dx-c} x^2}{16a(x^4b^2d^4+2ab\,d^4x^2+a^2d^4)}+\frac{3d^4 e^{-dx-c} b\,x^3}{16a^2(x^4b^2d^4+2ab\,d^4x^2+a^2d^4)}-\frac{d^5 e^{-dx-c}}{16b(x^4b^2d^4+2ab\,d^4x^2+a^2d^4)}+\frac{5d^4 e^{-dx-c} x}{16a(x^4b^2d^4+2ab\,d^4x^2+a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*d^5*exp(-d*x-c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2+3/16*d^4*ex
p(-d*x-c)/a^2*b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3-1/16*d^5*exp(-d*x-c
)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+5/16*d^4*exp(-d*x-c)/a/(b^2*d^4*x^4
+2*a*b*d^4*x^2+a^2*d^4)*x-1/32*d^2/b/a/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+b
*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)+1/32*d^2/b/a/(-a*b)^(1/2)*exp
(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)-3/32*d/b/
a^2*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)-3/
32*d/b/a^2*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c
)/b)+3/32/a^2/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/
2)+b*(d*x+c)-b*c)/b)-3/32/a^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(
1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/16*d^5*exp(d*x+c)/a/(b^2*d^4*x^4+2*a
```

```
*b*d^4*x^2+a^2*d^4)*x^2+3/16*d^4*exp(d*x+c)/a^2*b/(b^2*d^4*x^4+2*a*b*d^4*x^
2+a^2*d^4)*x^3+1/16*d^5*exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+5/
16*d^4*exp(d*x+c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x+1/32*d^2/b/a/(-a*
b)^(1/2)*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)
-1/32*d^2/b/a/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/
2)+b*(d*x+c)-b*c)/b)+3/32*d/b/a^2*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b
)^(1/2)-b*(d*x+c)+b*c)/b)+3/32*d/b/a^2*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(
d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)-3/32/a^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+
b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+3/32/a^2/(-a*b)^(1/2)*exp((-
d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/(b*x^2 + a)^3, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2116 vs. 2(655) = 1310.
time = 0.39, size = 2116, normalized size = 2.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(4*(3*a*b^2*d*x^3 + 5*a^2*b*d*x)*cosh(d*x + c) - ((3*(a*b^2*d^2*x^4 +
2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2
*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a
^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2
 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqr
t(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - (3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2
 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)
*sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b
*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 -
 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei
(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - ((3*(a*b^2*d^2*x^4 + 2*
a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x
^2 + a^3*d^2)*sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2
*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 -
 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(
-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - (3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 +
```

```
 a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*s
inh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d
^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3
*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-
d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + 4*(a^2*b*d^2*x^2 + a^3*d
^2)*sinh(d*x + c) - ((3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*
x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 -
((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2
)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b
*d^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/
b)) + (3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a
*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((a^3*d^2 + (a*
b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)
^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)
*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c +
sqrt(-a*d^2/b)) + ((3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x
+ c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((
a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*
cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d
^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)
) + (3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b
^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^
2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2
 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x
^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + s
qrt(-a*d^2/b)))/((a^3*b^3*d*x^4 + 2*a^4*b^2*d*x^2 + a^5*b*d)*cosh(d*x + c)^
2 - (a^3*b^3*d*x^4 + 2*a^4*b^2*d*x^2 + a^5*b*d)*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)}{\left(b\,x^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*x^2)^3,x)
```

```
[Out] int(cosh(c + d*x)/(a + b*x^2)^3, x)
```

3.76 $\int \frac{\cosh(c+dx)}{x(a+bx^2)^3}\,dx$

Optimal. Leaf size=730

$$\frac{\cosh(c+dx)}{4a(a+bx^2)^2}+\frac{\cosh(c+dx)}{2a^2(a+bx^2)}+\frac{\cosh(c)\text{Chi}(dx)}{a^3}-\frac{\cosh\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^3}+\frac{d^2\cosh\left(c+\sqrt{-\right.}{\ldots}$$

```
[Out] Chi(d*x)*cosh(c)/a^3+1/4*cosh(d*x+c)/a/(b*x^2+a)^2+1/2*cosh(d*x+c)/a^2/(b*x
^2+a)-1/2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/a^3+1/
16*d^2*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/a^2/b-1/2
*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/a^3+1/16*d^2*C
hi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/a^2/b+Shi(d*x)*s
inh(c)/a^3-1/2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a
^3+1/16*d^2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a^2/
b-1/2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^3+1/16*d
^2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^2/b-5/16*d*
cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(d*x-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/
2)+5/16*d*cosh(c-d*(-a)^(1/2)/b^(1/2))*Shi(d*x+d*(-a)^(1/2)/b^(1/2))/(-a)^(
5/2)/b^(1/2)+5/16*d*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/
2))/(-a)^(5/2)/b^(1/2)-5/16*d*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^
(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+1/16*d*sinh(d*x+c)/a^2/b^(1/2)/((-a)^(1/2
)-x*b^(1/2))-1/16*d*sinh(d*x+c)/a^2/b^(1/2)/((-a)^(1/2)+x*b^(1/2))
```

Rubi [A]

time = 1.26, antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5401, 3384, 3379, 3382, 5397, 5388, 3378}

$$-\frac{\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^3}-\frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a^3}+\frac{\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^3}-\frac{\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a^3}+\frac{\cosh(c)\text{Chi}(dx)}{a^3}+\frac{\sinh(c)\text{Shi}(dx)}{a^3}+\frac{d^2\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16a^2b}+\frac{d^2\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16a^2b}-\frac{d^2\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16a^2b}+\frac{d^2\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16a^2b}+\frac{\cosh(c+dx)}{2a^2(a+bx^2)}+\frac{d\sinh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)}-\frac{d\sinh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)}+\frac{5d\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}-\frac{5d\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}}+\frac{5d\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}}+\frac{5d\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}+\frac{\cosh(c+dx)}{4a(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x^2)^3),x]
```

```
[Out] Cosh[c + d*x]/(4*a*(a + b*x^2)^2) + Cosh[c + d*x]/(2*a^2*(a + b*x^2)) + (Co
sh[c]*CoshIntegral[d*x])/a^3 - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral
[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^3) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]
*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (Cosh[c - (Sqrt[-a]
*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^3) + (d^2*Cosh[
c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2
*b) + (5*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/S
qrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) - (5*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) + (d*Sinh[c +
```

```
 d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) - (d*Sinh[c + d*x])/(16*a^2*
Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) + (Sinh[c]*SinhIntegral[d*x])/a^3 + (5*d*Co
sh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*
(-a)^(5/2)*Sqrt[b]) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a
]*d)/Sqrt[b] - d*x])/(2*a^3) - (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhInte
gral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) + (5*d*Cosh[c - (Sqrt[-a]*d)/S
qrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b]) -
 (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(2*a^3) + (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqr
t[b] + d*x])/(16*a^2*b)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
```

```
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x(a+bx^2)^3}\,dx = \int \left(\frac{\cosh(c+dx)}{a^3x} - \frac{bx\cosh(c+dx)}{a(a+bx^2)^3} - \frac{bx\cosh(c+dx)}{a^2(a+bx^2)^2} - \frac{bx\cosh(c+dx)}{a^3(a+bx^2)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a^3} - \frac{b\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{a^3} - \frac{b\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx}{a^2} - \frac{b\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx}{a}$$

$$= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} - \frac{b\int\left(-\frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}-\sqrt{b}x)} + \frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}+\sqrt{b}x)}\right)d}{a^3}$$

$$= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} + \frac{\sinh(c)\text{Shi}(dx)}{a^3} + \frac{\sqrt{b}\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}}}{2a^3}$$

$$= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} + \frac{\sinh(c)\text{Shi}(dx)}{a^3} + \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}}{4(-a)^{5/2}}$$

$$= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} -\right.}{2a^3}$$

$$= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} -\right.}{2a^3}$$

$$= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} -\right.}{2a^3}$$

$$= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}} -\right.}{2a^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.71, size = 1046, normalized size = 1.43

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^2)^3),x]
```

```
[Out] ((12*a^2*Cosh[c + d*x])/(a + b*x^2)^2 + (8*a*b*x^2*Cosh[c + d*x])/(a + b*x^
2)^2 + 16*Cosh[c]*CoshIntegral[d*x] - 8*Cos[(Sqrt[a]*d)/Sqrt[b]]*Cosh[c]*Co
sIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + (a*d^2*Cos[(Sqrt[a]*d)/Sqrt[b]]*Co
sh[c]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/b - (5*Sqrt[a]*d*Cosh[c]*Co
sIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]])/Sqrt[b] +
((5*I)*Sqrt[a]*d*Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] +
 I*d*x]*Sinh[c])/Sqrt[b] - (8*I)*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*S
in[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c] + (I*a*d^2*CosIntegral[(Sqrt[a]*d)/Sqrt[b]
+ I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c])/b - (CosIntegral[-((Sqrt[a]*d)/S
qrt[b]) + I*d*x]*((8*b - a*d^2)*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]] + (5*I)*Sqr
t[a]*Sqrt[b]*d*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]))/b - (2*a^2*d*x*Sinh[c + d*
x])/(a + b*x^2)^2 - (2*a*b*d*x^3*Sinh[c + d*x])/(a + b*x^2)^2 + (16*a^2*Sin
h[c]*SinhIntegral[d*x])/(a + b*x^2)^2 + (32*a*b*x^2*Sinh[c]*SinhIntegral[d*
x])/(a + b*x^2)^2 + (16*b^2*x^4*Sinh[c]*SinhIntegral[d*x])/(a + b*x^2)^2 +
(5*Sqrt[a]*d*Cos[(Sqrt[a]*d)/Sqrt[b]]*Cosh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[
b] - I*d*x])/Sqrt[b] - 8*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt
[a]*d)/Sqrt[b] - I*d*x] + (a*d^2*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegr
al[(Sqrt[a]*d)/Sqrt[b] - I*d*x])/b - (8*I)*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]
*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + (I*a*d^2*Cos[(Sqrt[a]*d)/Sqrt[b
]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x])/b - ((5*I)*Sqrt[a]*d*S
in[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x])/S
qrt[b] + (5*Sqrt[a]*d*Cos[(Sqrt[a]*d)/Sqrt[b]]*Cosh[c]*SinIntegral[(Sqrt[a]
*d)/Sqrt[b] + I*d*x])/Sqrt[b] - 8*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinInteg
ral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + (a*d^2*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*
SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/b + (8*I)*Cos[(Sqrt[a]*d)/Sqrt[b]
]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - (I*a*d^2*Cos[(Sqrt[a]*
d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/b + ((5*I)*Sq
rt[a]*d*Sin[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] +
I*d*x])/Sqrt[b])/(16*a^3)
```

Maple [A]

time = 0.78, size = 1090, normalized size = 1.49

method	result
risch	$\frac{e^{-dx-c} d^{2} \left((dx+c)^{3} b-3(dx+c)^{2} bc+(dx+c) a\, d^{2}+3(dx+c) b\, c^{2}-ac\, d^{2}-b\, c^{3}+4(dx+c)^{2} b-8b(dx+c)c+6a\, d^{2}+4b\, c^{2}\right)}{16a^{2} \left((dx+c)^{4} b^{2}-4(dx+c)^{3} c\, b^{2}+2(dx+c)^{2} ab\, d^{2}+6(dx+c)^{2} c^{2} b^{2}-4ab(dx+c)c\, d^{2}-4b^{2}(dx+c)c^{3}+a^{2} d^{4}+2ab\, c^{2} d^{2}+b^{2} c^{4}\right)} - \frac{e^{-c} \mathrm{expl}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*exp(-d*x-c)*d^2*((d*x+c)^3*b-3*(d*x+c)^2*b*c+(d*x+c)*a*d^2+3*(d*x+c)*b
*c^2-a*c*d^2-b*c^3+4*(d*x+c)^2*b-8*b*(d*x+c)*c+6*a*d^2+4*b*c^2)/a^2/((d*x+c
)^4*b^2-4*(d*x+c)^3*c*b^2+2*(d*x+c)^2*a*b*d^2+6*(d*x+c)^2*c^2*b^2-4*a*b*(d*
x+c)*c*d^2-4*b^2*(d*x+c)*c^3+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4)-1/2/a^3*exp(-c)
*Ei(1,d*x)-1/32/b/a^2*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*
(d*x+c)-b*c)/b)*d^2-1/32/b/a^2*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)
^(1/2)-b*(d*x+c)+b*c)/b)*d^2+5/32/a^2/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+b*
c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)*d-5/32/a^2/(-a*b)^(1/2)*exp(-(
d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)*d+1/4/a^3*ex
p(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)+1/4/a^3*
exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)-1/16*e
xp(d*x+c)*d^2*((d*x+c)^3*b-3*(d*x+c)^2*b*c+(d*x+c)*a*d^2+3*(d*x+c)*b*c^2-a*
c*d^2-b*c^3-4*(d*x+c)^2*b+8*b*(d*x+c)*c-6*a*d^2-4*b*c^2)/a^2/((d*x+c)^4*b^2
-4*(d*x+c)^3*c*b^2+2*(d*x+c)^2*a*b*d^2+6*(d*x+c)^2*c^2*b^2-4*a*b*(d*x+c)*c*
d^2-4*b^2*(d*x+c)*c^3+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4)-1/2/a^3*exp(c)*Ei(1,-d
*x)-1/32/b/a^2*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b
*c)/b)*d^2-1/32/b/a^2*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*
(d*x+c)-b*c)/b)*d^2+5/32/a^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,
(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)*d-5/32/a^2/(-a*b)^(1/2)*exp((-d*(-a*b)^(1
/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)*d+1/4/a^3*exp((d*(-a*b)
^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/4/a^3*exp((-d*(-a*b
)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2076 vs. 2(575) = 1150.
time = 0.54, size = 2076, normalized size = 2.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(8*(2*a*b^2*x^2 + 3*a^2*b)*cosh(d*x + c) + (((a^3*d^2 + (a*b^2*d^2 - 8
*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d
^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(
d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4
```

```
+ 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^
2/b)) + ((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*
b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b +
2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x + c)^2 - 5*((b^3*x^4 + 2*a*b^2*x^2 +
a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*s
qrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) + 16*((b
^3*x^4 + 2*a*b^2*x^2 + a^2*b)*Ei(d*x) + (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*Ei(
-d*x))*cosh(c) + (((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*
d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 -
8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x + c)^2 - 5*((b^3*x^4 + 2*a*
b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x
 + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a^3*d^2 + (a*b^2*d^2
- 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^
3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*si
nh(d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x
^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-
a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - 4*(a*b^2*d*x^3 + a^2*b*d*x)*sinh(d*x
 + c) + (((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a
*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b +
 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^2 +
 a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*
sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - ((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*
x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (
a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x +
c)^2 - 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*
b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b))
)*sinh(c + sqrt(-a*d^2/b)) + 16*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*Ei(d*x) -
(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*Ei(-d*x))*sinh(c) - (((a^3*d^2 + (a*b^2*d^2
 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a
^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*s
inh(d*x + c)^2 - 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*
x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-
a*d^2/b)) - ((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 -
8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*
b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^
2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^
2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((
a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)*cosh(d*x + c)^2 - (a^3*b^3*x^4 + 2*a^4
*b^2*x^2 + a^5*b)*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)}{x\,{\left(b\,x^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x*(a + b*x^2)^3),x)
```

```
[Out] int(cosh(c + d*x)/(x*(a + b*x^2)^3), x)
```

3.77 $\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^3}\,dx$

Optimal. Leaf size=874

$$-\frac{\cosh(c+dx)}{a^3x}-\frac{\sqrt{b}\,\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)^2}+\frac{7\sqrt{b}\,\cosh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\sqrt{b}\,\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)^2}-\frac{7}{16a}$$

```
[Out] -cosh(d*x+c)/a^3/x+d*cosh(c)*Shi(d*x)/a^3+7/16*d*cosh(c+d*(-a)^(1/2)/b^(1/2
))*Shi(d*x-d*(-a)^(1/2)/b^(1/2))/a^3+7/16*d*cosh(c-d*(-a)^(1/2)/b^(1/2))*Sh
i(d*x+d*(-a)^(1/2)/b^(1/2))/a^3+d*Chi(d*x)*sinh(c)/a^3+7/16*d*Chi(d*x+d*(-a
)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a^3+7/16*d*Chi(-d*x+d*(-a)^(1
/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^3-1/16*d^2*Chi(d*x+d*(-a)^(1/2)
/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+1/16*d^2*Chi(-d*x
+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)-1/16
*d^2*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/
b^(1/2)+1/16*d^2*Shi(d*x-d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))
/(-a)^(5/2)/b^(1/2)-15/16*Chi(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)
/b^(1/2))*b^(1/2)/(-a)^(7/2)+15/16*Chi(-d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c+d*
(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)-15/16*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*
sinh(c-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)+15/16*Shi(d*x-d*(-a)^(1/2)/
b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)-1/16*cosh(d*x+c)*b
^(1/2)/(-a)^(5/2)/((-a)^(1/2)-x*b^(1/2))^2+1/16*d*sinh(d*x+c)/(-a)^(5/2)/((
-a)^(1/2)-x*b^(1/2))+7/16*cosh(d*x+c)*b^(1/2)/a^3/((-a)^(1/2)-x*b^(1/2))+1/
16*cosh(d*x+c)*b^(1/2)/(-a)^(5/2)/((-a)^(1/2)+x*b^(1/2))^2+1/16*d*sinh(d*x+
c)/(-a)^(5/2)/((-a)^(1/2)+x*b^(1/2))-7/16*cosh(d*x+c)*b^(1/2)/a^3/((-a)^(1/
2)+x*b^(1/2))
```

Rubi [A]

time = 2.12, antiderivative size = 874, normalized size of antiderivative = 1.00, number of steps used = 60, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5401, 3378, 3384, 3379, 3382, 5389}

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(Cosh[c + d*x]/(a^3*x)) - (Sqrt[b]*Cosh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a]
 - Sqrt[b]*x)^2) + (7*Sqrt[b]*Cosh[c + d*x])/(16*a^3*(Sqrt[-a] - Sqrt[b]*x)
) + (Sqrt[b]*Cosh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)^2) - (7*S
qrt[b]*Cosh[c + d*x])/(16*a^3*(Sqrt[-a] + Sqrt[b]*x)) + (15*Sqrt[b]*Cosh[c
+ (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^
```

```
(7/2)) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (15*Sqrt[b]*Cosh[c - (Sqrt[-a]*d)/Sqr
t[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) - (d^2*Cosh
[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-
a)^(5/2)*Sqrt[b]) + (d*CoshIntegral[d*x]*Sinh[c])/a^3 + (7*d*CoshIntegral[(
Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^3) + (7*d*
CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(1
6*a^3) + (d*Sinh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Sinh
[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)) + (d*Cosh[c]*SinhIntegral
[d*x])/a^3 - (7*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/
Sqrt[b] - d*x])/(16*a^3) - (15*Sqrt[b]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhI
ntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(7/2)) - (d^2*Sinh[c + (Sqrt[
-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sq
rt[b]) + (7*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt
[b] + d*x])/(16*a^3) - (15*Sqrt[b]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhInteg
ral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) - (d^2*Sinh[c - (Sqrt[-a]*
d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b
])
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx = \int \left(\frac{\cosh(c+dx)}{a^3x^2} - \frac{b\cosh(c+dx)}{a\left(a+bx^2\right)^3} - \frac{b\cosh(c+dx)}{a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)}\right)\,dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a^3} - \frac{b\int \frac{\cosh(c+dx)}{a+bx^2}\,dx}{a^3} - \frac{b\int \frac{\cosh(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a^2} - \frac{b\int \frac{\cosh(c+dx)}{\left(a+bx^2\right)^3}\,dx}{a}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{b\int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)\,dx}{a^3} - \frac{b\int \left(-\frac{b\cosh(\cdots}{4a\left(\sqrt{-a}\cdots\right.}\right.}{\cdots}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{b\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2(-a)^{7/2}} - \frac{b\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2(-a)^{7/2}} + \frac{(3b^2)\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)\cdots}}{16a^3}\cdots$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)^2} + \frac{7\sqrt{b}\cosh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{b}\,\cdots}{16(-a)^{5/2}\cdots}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)^2} + \frac{7\sqrt{b}\cosh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{b}\,\cdots}{16(-a)^{5/2}\cdots}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)^2} + \frac{7\sqrt{b}\cosh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{b}\,\cdots}{16(-a)^{5/2}\cdots}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)^2} + \frac{7\sqrt{b}\cosh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{b}\,\cdots}{16(-a)^{5/2}\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.94, size = 1009, normalized size = 1.15

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x^2)^3),x]
```

```
[Out] ((-16*a^(5/2)*Cosh[c]*Cosh[d*x])/(x*(a + b*x^2)^2) - (50*a^(3/2)*b*x*Cosh[c
]*Cosh[d*x])/(a + b*x^2)^2 - (30*Sqrt[a]*b^2*x^3*Cosh[c]*Cosh[d*x])/(a + b*
```

```
x^2)^2 - (2*a^(5/2)*d*Cosh[d*x]*Sinh[c])/(a + b*x^2)^2 - (2*a^(3/2)*b*d*x^2
*Cosh[d*x]*Sinh[c])/(a + b*x^2)^2 + 16*Sqrt[a]*d*CoshIntegral[d*x]*Sinh[c]
+ (CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*((-I)*(15*b - a*d^2)*Cosh[c
- (I*Sqrt[a]*d)/Sqrt[b]] + 7*Sqrt[a]*Sqrt[b]*d*Sinh[c - (I*Sqrt[a]*d)/Sqrt[
b]]))/Sqrt[b] + (CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*(I*(15*b - a*d^2)
*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]] + 7*Sqrt[a]*Sqrt[b]*d*Sinh[c + (I*Sqrt[a]*
d)/Sqrt[b]]))/Sqrt[b] - (2*a^(5/2)*d*Cosh[c]*Sinh[d*x])/(a + b*x^2)^2 - (2*
a^(3/2)*b*d*x^2*Cosh[c]*Sinh[d*x])/(a + b*x^2)^2 - (16*a^(5/2)*Sinh[c]*Sinh
[d*x])/(x*(a + b*x^2)^2) - (50*a^(3/2)*b*x*Sinh[c]*Sinh[d*x])/(a + b*x^2)^2
 - (30*Sqrt[a]*b^2*x^3*Sinh[c]*Sinh[d*x])/(a + b*x^2)^2 + 16*Sqrt[a]*d*Cosh
[c]*SinhIntegral[d*x] + (7*I)*Sqrt[a]*d*Cos[(Sqrt[a]*d)/Sqrt[b]]*Cosh[c]*Si
nIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - (15*I)*Sqrt[b]*Cosh[c]*Sin[(Sqrt[a
]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + (I*a*d^2*Cosh[c]*S
in[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x])/Sqrt[b] +
 15*Sqrt[b]*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b
] - I*d*x] - (a*d^2*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d
)/Sqrt[b] - I*d*x])/Sqrt[b] + 7*Sqrt[a]*d*Sin[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*
SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - (7*I)*Sqrt[a]*d*Cos[(Sqrt[a]*d)/
Sqrt[b]]*Cosh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + (15*I)*Sqrt[b]*
Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] -
 (I*a*d^2*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b]
+ I*d*x])/Sqrt[b] + 15*Sqrt[b]*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral
[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - (a*d^2*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*Sin
Integral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/Sqrt[b] + 7*Sqrt[a]*d*Sin[(Sqrt[a]*d
)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/(16*a^(7/2))
```

Maple [A]
time = 0.79, size = 1178, normalized size = 1.35

method	result
risch	$\frac{e^{-dx-c}x^2d^5b}{16a^2(x^4b^2d^4+2ab\,d^4x^2+a^2d^4)} - \frac{15\,e^{-dx-c}x^3d^4b^2}{16a^3(x^4b^2d^4+2ab\,d^4x^2+a^2d^4)} + \frac{e^{-dx-c}d^5}{16a(x^4b^2d^4+2ab\,d^4x^2+a^2d^4)} - \frac{25\,e^{-dx-c}x\,d^4b}{16a^2(x^4b^2d^4+2ab\,d^4x^2+a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*exp(-d*x-c)/a^2*x^2*d^5/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-15/16*ex
p(-d*x-c)/a^3*x^3*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2+1/16*exp(-d*x
-c)/a*d^5/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-25/16*exp(-d*x-c)/a^2*x*d^4/(
b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-1/2*exp(-d*x-c)/a/x*d^4/(b^2*d^4*x^4+2
*a*b*d^4*x^2+a^2*d^4)+1/2*d/a^3*exp(-c)*Ei(1,d*x)-1/32/a^2*d^2/(-a*b)^(1/2)
*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/32/
a^2*d^2/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(
d*x+c)-b*c)/b)+7/32*d/a^3*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2
```

```
)-b*(d*x+c)+b*c)/b)+7/32*d/a^3*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)
^(1/2)+b*(d*x+c)-b*c)/b)+15/32/a^3/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+b*c)/b
)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)*b-15/32/a^3/(-a*b)^(1/2)*exp(-(-d
*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)*b-1/16*exp(d*x
+c)/a^2*x^2*d^5/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-15/16*exp(d*x+c)/a^3*
x^3*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2-1/16*exp(d*x+c)/a*d^5/(b^2*
d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-25/16*exp(d*x+c)/a^2*x*d^4/(b^2*d^4*x^4+2*a*
b*d^4*x^2+a^2*d^4)*b-1/2*exp(d*x+c)/a/x*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*
d^4)-1/2*d/a^3*exp(c)*Ei(1,-d*x)-1/32/a^2*d^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1
/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)+1/32/a^2*d^2/(-a*b)^(1/2
)*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)-7/32
*d/a^3*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)-7
/32*d/a^3*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)
/b)+15/32/a^3/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)
-b*(d*x+c)+b*c)/b)*b-15/32/a^3/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei
(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)*b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x^2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2346 vs. $2(673) = 1346$.
time = 0.52, size = 2346, normalized size = 2.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(4*(15*a*b^2*d*x^4 + 25*a^2*b*d*x^2 + 8*a^3*d)*cosh(d*x + c) - ((7*(a
*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2 - 7*(a*b^2*d^2*
x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 - (((a*b^2*d^2 - 15*b^3)
*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*cosh(d*x + c)
^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 -
15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - (7
*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2 - 7*(a*b^2*d
^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 + (((a*b^2*d^2 - 15*b
^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*cosh(d*x +
 c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2
```

```
 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))
*cosh(c + sqrt(-a*d^2/b)) - 16*((a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*
x)*Ei(d*x) - (a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*Ei(-d*x))*cosh(c
) - ((7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2 - 7*(
a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 + (((a*b^2*d^2
 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*cos
h(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (
a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^
2/b)) - (7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2 -
7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 - (((a*b^2*
d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*
cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3
+ (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-
a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + 4*(a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d
*x + c) - ((7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2
 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 - (((a*b
^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*
x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x
^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt
(-a*d^2/b)) + (7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c
)^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 + (((
a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*
b)*x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2
)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x +
sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - 16*((a*b^2*d^2*x^5 + 2*a^2*b*d^
2*x^3 + a^3*d^2*x)*Ei(d*x) + (a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*
Ei(-d*x))*sinh(c) + ((7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(
d*x + c)^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^
2 + (((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 -
15*a^2*b)*x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 1
5*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(
d*x + sqrt(-a*d^2/b)) + (7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*co
sh(d*x + c)^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x +
c)^2 - (((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2
 - 15*a^2*b)*x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2
- 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*
Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a^4*b^2*d*x^5 + 2*a
^5*b*d*x^3 + a^6*d*x)*cosh(d*x + c)^2 - (a^4*b^2*d*x^5 + 2*a^5*b*d*x^3 + a^
6*d*x)*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**2/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)}{x^2\,{\left(b\,x^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x^2*(a + b*x^2)^3),x)
```

```
[Out] int(cosh(c + d*x)/(x^2*(a + b*x^2)^3), x)
```

3.78 $\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=791

$$-\frac{\cosh(c+dx)}{2a^3x^2}-\frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2}-\frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)}-\frac{3b\cosh(c)\text{Chi}(dx)}{a^4}+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a^3}+\frac{3b\cosh\left(c+\frac{\sqrt{-\cdots}}{\sqrt{\cdots}}\right.}{\cdots}$$

```
[Out] -3*b*Chi(d*x)*cosh(c)/a^4+1/2*d^2*Chi(d*x)*cosh(c)/a^3-1/2*cosh(d*x+c)/a^3/
x^2-1/4*b*cosh(d*x+c)/a^2/(b*x^2+a)^2-b*cosh(d*x+c)/a^3/(b*x^2+a)+3/2*b*Chi
(d*x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/a^4-1/16*d^2*Chi(d*
x+d*(-a)^(1/2)/b^(1/2))*cosh(c-d*(-a)^(1/2)/b^(1/2))/a^3+3/2*b*Chi(-d*x+d*(
-a)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/a^4-1/16*d^2*Chi(-d*x+d*(-a
)^(1/2)/b^(1/2))*cosh(c+d*(-a)^(1/2)/b^(1/2))/a^3-3*b*Shi(d*x)*sinh(c)/a^4+
1/2*d^2*Shi(d*x)*sinh(c)/a^3-1/2*d*sinh(d*x+c)/a^3/x+3/2*b*Shi(d*x+d*(-a)^(
1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a^4-1/16*d^2*Shi(d*x+d*(-a)^(1/2
)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))/a^3+3/2*b*Shi(d*x-d*(-a)^(1/2)/b^(1
/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^4-1/16*d^2*Shi(d*x-d*(-a)^(1/2)/b^(1/2)
)*sinh(c+d*(-a)^(1/2)/b^(1/2))/a^3-9/16*d*cosh(c+d*(-a)^(1/2)/b^(1/2))*Shi(
d*x-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)+9/16*d*cosh(c-d*(-a)^(1/2)/b^(
1/2))*Shi(d*x+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)+9/16*d*Chi(d*x+d*(-a
)^(1/2)/b^(1/2))*sinh(c-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)-9/16*d*Chi
(-d*x+d*(-a)^(1/2)/b^(1/2))*sinh(c+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)
-1/16*d*sinh(d*x+c)*b^(1/2)/a^3/((-a)^(1/2)-x*b^(1/2))+1/16*d*sinh(d*x+c)*b
^(1/2)/a^3/((-a)^(1/2)+x*b^(1/2))
```

Rubi [A]
time = 1.41, antiderivative size = 791, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5401, 3378, 3384, 3379, 3382, 5397, 5388}

$$-\frac{3b\cosh(c)\text{Chi}(dx)}{a^4}+\frac{3b\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^4}+\frac{3b\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a^4}-\frac{3b\sinh(c)\text{Shi}(dx)}{a^4}-\frac{3b\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^4}+\frac{3b\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a^4}-\frac{d^2\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16a^3}-\frac{d^2\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16a^3}+\frac{d^2\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16a^3}-\frac{d^2\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16a^3}-\frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)}-\frac{\sqrt{b}\,d\sinh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\sqrt{b}\,d\sinh(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}\,x\right)}+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a^3}+\frac{d^2\sinh(c)\text{Shi}(dx)}{2a^3}-\frac{\cosh(c+dx)}{2a^3x^2}-\frac{d\sinh(c+dx)}{2a^3x}-\frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2}+\frac{9\sqrt{b}\,d\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16(-a)^{7/2}}-\frac{9\sqrt{b}\,d\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}}+\frac{9\sqrt{b}\,d\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}}+\frac{9\sqrt{b}\,d\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16(-a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -1/2*Cosh[c + d*x]/(a^3*x^2) - (b*Cosh[c + d*x])/(4*a^2*(a + b*x^2)^2) - (b
*Cosh[c + d*x])/(a^3*(a + b*x^2)) - (3*b*Cosh[c]*CoshIntegral[d*x])/a^4 + (
d^2*Cosh[c]*CoshIntegral[d*x])/(2*a^3) + (3*b*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]
]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^4) - (d^2*Cosh[c + (Sqrt[-
a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) + (3*b*Co
sh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a
^4) - (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b]
```

```
 + d*x])/(16*a^3) + (9*Sqrt[b]*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*S
inh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2)) - (9*Sqrt[b]*d*CoshIntegral[
(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2))
 - (d*Sinh[c + d*x])/(2*a^3*x) - (Sqrt[b]*d*Sinh[c + d*x])/(16*a^3*(Sqrt[-a
] - Sqrt[b]*x)) + (Sqrt[b]*d*Sinh[c + d*x])/(16*a^3*(Sqrt[-a] + Sqrt[b]*x))
 - (3*b*Sinh[c]*SinhIntegral[d*x])/a^4 + (d^2*Sinh[c]*SinhIntegral[d*x])/(2
*a^3) + (9*Sqrt[b]*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*
d)/Sqrt[b] - d*x])/(16*(-a)^(7/2)) - (3*b*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*Si
nhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^4) + (d^2*Sinh[c + (Sqrt[-a]*d
)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) + (9*Sqrt[b]*
d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(16*(-a)^(7/2)) + (3*b*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x])/(2*a^4) - (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhInte
gral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx = \int \left(\frac{\cosh(c+dx)}{a^3x^3} - \frac{3b\cosh(c+dx)}{a^4x} + \frac{b^2x\cosh(c+dx)}{a^2\left(a+bx^2\right)^3} + \frac{2b^2x\cosh(c+dx)}{a^3\left(a+bx^2\right)^2} + \frac{3b^2x\,\mathrm{c}}{a^4}\right.$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^3}\,dx}{a^3} - \frac{(3b)\int \frac{\cosh(c+dx)}{x}\,dx}{a^4} + \frac{(3b^2)\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{a^4} + \frac{(2b^2)\int \frac{x\cosh(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a^3} +$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} + \frac{(3b^2)\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \right.}{a^4}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cosh(c)\mathrm{Chi}(dx)}{a^4} - \frac{d\sinh(c+d}{2a^3x}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cosh(c)\mathrm{Chi}(dx)}{a^4} - \frac{d\sinh(c+d}{2a^3x}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cosh(c)\mathrm{Chi}(dx)}{a^4} + \frac{d^2\cosh(c)\mathrm{C}}{2a^3}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cosh(c)\mathrm{Chi}(dx)}{a^4} + \frac{d^2\cosh(c)\mathrm{C}}{2a^3}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cosh(c)\mathrm{Chi}(dx)}{a^4} + \frac{d^2\cosh(c)\mathrm{C}}{2a^3}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cosh(c)\mathrm{Chi}(dx)}{a^4} + \frac{d^2\cosh(c)\mathrm{C}}{2a^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.63, size = 998, normalized size = 1.26

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -1/16*((2*a*Cosh[d*x]*(2*(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4)*Cosh[c] + d*x*(4*a
^2 + 7*a*b*x^2 + 3*b^2*x^4)*Sinh[c]))/(x^2*(a + b*x^2)^2) + (2*a*(d*x*(4*a^
2 + 7*a*b*x^2 + 3*b^2*x^4)*Cosh[c] + 2*(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4)*Sinh
[c])*Sinh[d*x])/(x^2*(a + b*x^2)^2) + 8*(6*b - a*d^2)*(Cosh[c]*CoshIntegral
[d*x] + Sinh[c]*SinhIntegral[d*x]) - (9*I)*Sqrt[a]*Sqrt[b]*d*Sinh[c]*(Cos[(
Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - Cos[(Sqrt
[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a]*d)/
Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - SinIntegral[(Sqrt[a]*d
)/Sqrt[b] + I*d*x])) + (24*I)*b*Sinh[c]*(CosIntegral[-((Sqrt[a]*d)/Sqrt[b])
 + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] - CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*
x]*Sin[(Sqrt[a]*d)/Sqrt[b]] + Cos[(Sqrt[a]*d)/Sqrt[b]]*(-SinIntegral[(Sqrt[
a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])) - I*a*d
^2*Sinh[c]*(CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sin[(Sqrt[a]*d)/Sqr
t[b]] - CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] +
 Cos[(Sqrt[a]*d)/Sqrt[b]]*(-SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinI
ntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])) - 9*Sqrt[a]*Sqrt[b]*d*Cosh[c]*(CosIn
tegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] + CosIntegr
al[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] - Cos[(Sqrt[a]*d)/
Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d
)/Sqrt[b] + I*d*x])) - 24*b*Cosh[c]*(Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-
((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt
[a]*d)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)
/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])) + a*d^2*Cosh
[c]*(Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] +
 Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + Sin[(S
qrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[
(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/a^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1293 vs. $2(629) = 1258$.
time = 0.82, size = 1294, normalized size = 1.64

method	result	size
risch	Expression too large to display	1294

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/16*d^5*exp(d*x+c)/a^3*x^3/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2-7/16*d
^5*exp(d*x+c)/a^2*x/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-3/4*d^4*exp(d*x+c
)/a^3*x^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2-9/32*d/a^3/(-a*b)^(1/2)*e
xp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+b*c)/b)*b+9/32*d/
```

```
a^3/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+
c)-b*c)/b)*b+7/16*d^5*exp(-d*x-c)/a^2*x/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)
*b-3/4*d^4*exp(-d*x-c)/a^3*x^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2+9/32
*d/a^3/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d
*x+c)+b*c)/b)*b-9/32*d/a^3/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,
(d*(-a*b)^(1/2)+b*(d*x+c)-b*c)/b)*b+3/16*d^5*exp(-d*x-c)/a^3*x^3/(b^2*d^4*x
^4+2*a*b*d^4*x^2+a^2*d^4)*b^2-1/4*d^2/a^3*exp(c)*Ei(1,-d*x)+3/2/a^4*exp(c)*
Ei(1,-d*x)*b+1/32*d^2/a^3*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)-
b*(d*x+c)+b*c)/b)+1/32*d^2/a^3*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)
^(1/2)+b*(d*x+c)-b*c)/b)-3/4/a^4*exp((d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)
^(1/2)-b*(d*x+c)+b*c)/b)*b-3/4/a^4*exp((-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-
a*b)^(1/2)+b*(d*x+c)-b*c)/b)*b-9/8*d^4*exp(-d*x-c)/a^2/(b^2*d^4*x^4+2*a*b*d
^4*x^2+a^2*d^4)*b-1/4*d^4*exp(-d*x-c)/a/x^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*
d^4)+1/4*d^5*exp(-d*x-c)/a/x/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-1/4*d^5*ex
p(d*x+c)/a/x/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-9/8*d^4*exp(d*x+c)/a^2/(b^
2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-1/4*d^4*exp(d*x+c)/a/x^2/(b^2*d^4*x^4+2*
a*b*d^4*x^2+a^2*d^4)-1/4*d^2/a^3*exp(-c)*Ei(1,d*x)+3/2/a^4*exp(-c)*Ei(1,d*x
)*b+1/32*d^2/a^3*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+
c)+b*c)/b)+1/32*d^2/a^3*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^(1/2)+
b*(d*x+c)-b*c)/b)-3/4/a^4*exp(-(d*(-a*b)^(1/2)+b*c)/b)*Ei(1,-(d*(-a*b)^(1/2
)-b*(d*x+c)+b*c)/b)*b-3/4/a^4*exp(-(-d*(-a*b)^(1/2)+b*c)/b)*Ei(1,(d*(-a*b)^
(1/2)+b*(d*x+c)-b*c)/b)*b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x^3), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2363 vs. $2(630) = 1260$.
time = 0.49, size = 2363, normalized size = 2.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(8*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)*cosh(d*x + c) + ((((a*b^2*d^2
- 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*co
sh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 +
(a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 + 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2
```

```
*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c)
^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + (((a*b^2*d^2 - 24*b^3)*x^6 +
 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 -
 ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a
^2*b)*x^2)*sinh(d*x + c)^2 - 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d*
x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2
/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - 8*(((a*b^2*d^2 -
 6*b^3)*x^6 + 2*(a^2*b*d^2 - 6*a*b^2)*x^4 + (a^3*d^2 - 6*a^2*b)*x^2)*Ei(d*x
) + ((a*b^2*d^2 - 6*b^3)*x^6 + 2*(a^2*b*d^2 - 6*a*b^2)*x^4 + (a^3*d^2 - 6*a
^2*b)*x^2)*Ei(-d*x))*cosh(c) + ((((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 -
 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 - ((a*b^2*d^2 -
24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*sinh
(d*x + c)^2 - 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d*x + c)^2 - (b^3
*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + s
qrt(-a*d^2/b)) + (((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4
 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2
*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 + 9
*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*
x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b))
)*cosh(-c + sqrt(-a*d^2/b)) + 4*(3*a*b^2*d*x^5 + 7*a^2*b*d*x^3 + 4*a^3*d*x)
*sinh(d*x + c) + ((((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4
 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 +
2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 +
9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2
*x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b))
 - (((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 2
4*a^2*b)*x^2)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 -
24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 - 9*((b^3*x^6 + 2
*a*b^2*x^4 + a^2*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^
2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqr
t(-a*d^2/b)) - 8*(((a*b^2*d^2 - 6*b^3)*x^6 + 2*(a^2*b*d^2 - 6*a*b^2)*x^4 +
(a^3*d^2 - 6*a^2*b)*x^2)*Ei(d*x) - ((a*b^2*d^2 - 6*b^3)*x^6 + 2*(a^2*b*d^2
- 6*a*b^2)*x^4 + (a^3*d^2 - 6*a^2*b)*x^2)*Ei(-d*x))*sinh(c) - ((((a*b^2*d^2
 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*c
osh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 +
 (a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 - 9*((b^3*x^6 + 2*a*b^2*x^4 + a^
2*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c
)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - (((a*b^2*d^2 - 24*b^3)*x^6
+ 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2
- ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*
a^2*b)*x^2)*sinh(d*x + c)^2 + 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d
*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^
2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a^4*b^2*x^6 +
 2*a^5*b*x^4 + a^6*x^2)*cosh(d*x + c)^2 - (a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*
x^2)*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**3/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)}{x^3\,{\left(b\,x^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x^3*(a + b*x^2)^3),x)
```

```
[Out] int(cosh(c + d*x)/(x^3*(a + b*x^2)^3), x)
```

3.79 $\int x^3(a+bx^3)\cosh(c+dx)\,dx$

Optimal. Leaf size=154

$$-\frac{6a\cosh(c+dx)}{d^4}-\frac{720bx\cosh(c+dx)}{d^6}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{120bx^3\cosh(c+dx)}{d^4}-\frac{6bx^5\cosh(c+dx)}{d^2}+\frac{720}{}$$

```
[Out] -6*a*cosh(d*x+c)/d^4-720*b*x*cosh(d*x+c)/d^6-3*a*x^2*cosh(d*x+c)/d^2-120*b*
x^3*cosh(d*x+c)/d^4-6*b*x^5*cosh(d*x+c)/d^2+720*b*sinh(d*x+c)/d^7+6*a*x*sin
h(d*x+c)/d^3+360*b*x^2*sinh(d*x+c)/d^5+a*x^3*sinh(d*x+c)/d+30*b*x^4*sinh(d*
x+c)/d^3+b*x^6*sinh(d*x+c)/d
```

Rubi [A]
time = 0.21, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5395, 3377, 2718, 2717}

$$-\frac{6a\cosh(c+dx)}{d^4}+\frac{6ax\sinh(c+dx)}{d^3}-\frac{3ax^2\cosh(c+dx)}{d^2}+\frac{ax^3\sinh(c+dx)}{d}+\frac{720b\sinh(c+dx)}{d^7}-\frac{720bx\cosh(c+dx)}{d^6}+\frac{360bx^2\sinh(c+dx)}{d^5}-\frac{120bx^3\cosh(c+dx)}{d^4}+\frac{30bx^4\sinh(c+dx)}{d^3}-\frac{6bx^5\cosh(c+dx)}{d^2}+\frac{bx^6\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-6*a*Cosh[c + d*x])/d^4 - (720*b*x*Cosh[c + d*x])/d^6 - (3*a*x^2*Cosh[c +
d*x])/d^2 - (120*b*x^3*Cosh[c + d*x])/d^4 - (6*b*x^5*Cosh[c + d*x])/d^2 + (
720*b*Sinh[c + d*x])/d^7 + (6*a*x*Sinh[c + d*x])/d^3 + (360*b*x^2*Sinh[c +
d*x])/d^5 + (a*x^3*Sinh[c + d*x])/d + (30*b*x^4*Sinh[c + d*x])/d^3 + (b*x^6
*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
```

```
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+bx^3\right)\cosh(c+dx)\,dx &= \int \left(ax^3\cosh(c+dx)+bx^6\cosh(c+dx)\right)\,dx \\
&= a\int x^3\cosh(c+dx)\,dx+b\int x^6\cosh(c+dx)\,dx \\
&= \frac{ax^3\sinh(c+dx)}{d}+\frac{bx^6\sinh(c+dx)}{d}-\frac{(3a)\int x^2\sinh(c+dx)\,dx}{d}-\frac{(6b)\int
\end{aligned}$$

$$\begin{aligned}
&= -\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{6bx^5\cosh(c+dx)}{d^2}+\frac{ax^3\sinh(c+dx)}{d}+\frac{bx^6\sinh(c
\end{aligned}$$

$$\begin{aligned}
&= -\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{6bx^5\cosh(c+dx)}{d^2}+\frac{6ax\sinh(c+dx)}{d^3}+\frac{ax^3\sinh(c
\end{aligned}$$

$$\begin{aligned}
&= -\frac{6a\cosh(c+dx)}{d^4}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{120bx^3\cosh(c+dx)}{d^4}-\frac{6bx^5\cosh
\end{aligned}$$

$$\begin{aligned}
&= -\frac{6a\cosh(c+dx)}{d^4}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{120bx^3\cosh(c+dx)}{d^4}-\frac{6bx^5\cosh
\end{aligned}$$

$$\begin{aligned}
&= -\frac{6a\cosh(c+dx)}{d^4}-\frac{720bx\cosh(c+dx)}{d^6}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{120bx^3\cos
\end{aligned}$$

$$\begin{aligned}
&= -\frac{6a\cosh(c+dx)}{d^4}-\frac{720bx\cosh(c+dx)}{d^6}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{120bx^3\cos
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 100, normalized size = 0.65

$$\frac{-3d(ad^2(2+d^2x^2)+2bx(120+20d^2x^2+d^4x^4))\cosh(c+dx)+(ad^4x(6+d^2x^2)+b(720+360d^2x^2+30d^4x^4+d^6x^6))\sinh(c+dx)}{d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-3*d*(a*d^2*(2 + d^2*x^2) + 2*b*x*(120 + 20*d^2*x^2 + d^4*x^4))*Cosh[c + d
*x] + (a*d^4*x*(6 + d^2*x^2) + b*(720 + 360*d^2*x^2 + 30*d^4*x^4 + d^6*x^6)
)*Sinh[c + d*x])/d^7
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(154) = 308$.
time = 0.67, size = 551, normalized size = 3.58 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(b/d^3*c^6*sinh(d*x+c)-6*b/d^3*c^5*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+
15*b/d^3*c^4*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-20
*b/d^3*c^3*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*
x+c)-6*cosh(d*x+c))+15*b/d^3*c^2*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*
x+c)+12*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))-6*b/d^
3*c*((d*x+c)^5*sinh(d*x+c)-5*(d*x+c)^4*cosh(d*x+c)+20*(d*x+c)^3*sinh(d*x+c)
-60*(d*x+c)^2*cosh(d*x+c)+120*(d*x+c)*sinh(d*x+c)-120*cosh(d*x+c))+b/d^3*((
d*x+c)^6*sinh(d*x+c)-6*(d*x+c)^5*cosh(d*x+c)+30*(d*x+c)^4*sinh(d*x+c)-120*(
d*x+c)^3*cosh(d*x+c)+360*(d*x+c)^2*sinh(d*x+c)-720*(d*x+c)*cosh(d*x+c)+720*
sinh(d*x+c))-a*c^3*sinh(d*x+c)+3*a*c^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-3*
a*c*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+a*((d*x+c)^
3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c)))
```

Maxima [A]
time = 0.28, size = 268, normalized size = 1.74

$$-\frac{1}{56}\,d\left(\frac{7\,(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2e^c-24\,dxe^c+24\,e^c)ae^{(dx)}}{d^5}+\frac{7\,(d^4x^4+4\,d^3x^3+12\,d^2x^2+24\,dx+24)ae^{(-dx-c)}}{d^5}+\frac{4\,(d^7x^7e^c-7\,d^6x^6e^c+42\,d^5x^5e^c-210\,d^4x^4e^c+840\,d^3x^3e^c-2520\,d^2x^2e^c+5040\,dxe^c-5040\,e^c)be^{(dx)}}{d^8}+\frac{4\,(d^7x^7+7\,d^6x^6+42\,d^5x^5+210\,d^4x^4+840\,d^3x^3+2520\,d^2x^2+5040\,dx+5040)be^{(-dx-c)}}{d^8}\right)+\frac{1}{28}\,(4\,bx^7+7\,ax^4)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/56*d*(7*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*
e^c)*a*e^(d*x)/d^5 + 7*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a*e
^(-d*x - c)/d^5 + 4*(d^7*x^7*e^c - 7*d^6*x^6*e^c + 42*d^5*x^5*e^c - 210*d^4
*x^4*e^c + 840*d^3*x^3*e^c - 2520*d^2*x^2*e^c + 5040*d*x*e^c - 5040*e^c)*b*
e^(d*x)/d^8 + 4*(d^7*x^7 + 7*d^6*x^6 + 42*d^5*x^5 + 210*d^4*x^4 + 840*d^3*x
^3 + 2520*d^2*x^2 + 5040*d*x + 5040)*b*e^(-d*x - c)/d^8) + 1/28*(4*b*x^7 +
7*a*x^4)*cosh(d*x + c)
```

Fricas [A]
time = 0.42, size = 105, normalized size = 0.68

$$-\frac{3\,(2\,bd^5x^5+ad^5x^2+40\,bd^3x^3+2\,ad^3+240\,bdx)\cosh\left(dx+c\right)-(bd^6x^6+ad^6x^3+30\,bd^4x^4+6\,ad^4x+360\,bd^2x^2+720\,b)\sinh\left(dx+c\right)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(3*(2*b*d^5*x^5 + a*d^5*x^2 + 40*b*d^3*x^3 + 2*a*d^3 + 240*b*d*x)*cosh(d*x
 + c) - (b*d^6*x^6 + a*d^6*x^3 + 30*b*d^4*x^4 + 6*a*d^4*x + 360*b*d^2*x^2 +
 720*b)*sinh(d*x + c))/d^7
```

Sympy [A]
time = 0.68, size = 185, normalized size = 1.20

$$\begin{cases} \frac{ax^3\sinh\left(c+dx\right)}{d}-\frac{3ax^2\cosh\left(c+dx\right)}{d^2}+\frac{6ax\sinh\left(c+dx\right)}{d^3}-\frac{6a\cosh\left(c+dx\right)}{d^4}+\frac{bx^6\sinh\left(c+dx\right)}{d}-\frac{6bx^5\cosh\left(c+dx\right)}{d^2}+\frac{30bx^4\sinh\left(c+dx\right)}{d^3}-\frac{120bx^3\cosh\left(c+dx\right)}{d^4}+\frac{360bx^2\sinh\left(c+dx\right)}{d^5}-\frac{720bx\cosh\left(c+dx\right)}{d^6}+\frac{720b\sinh\left(c+dx\right)}{d^7} & \text{for}\: d\neq 0 \\ \left(\frac{ax^4}{4}+\frac{bx^7}{7}\right)\cosh\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x**3*sinh(c + d*x)/d - 3*a*x**2*cosh(c + d*x)/d**2 + 6*a*x*sin
h(c + d*x)/d**3 - 6*a*cosh(c + d*x)/d**4 + b*x**6*sinh(c + d*x)/d - 6*b*x**
5*cosh(c + d*x)/d**2 + 30*b*x**4*sinh(c + d*x)/d**3 - 120*b*x**3*cosh(c + d
*x)/d**4 + 360*b*x**2*sinh(c + d*x)/d**5 - 720*b*x*cosh(c + d*x)/d**6 + 720
*b*sinh(c + d*x)/d**7, Ne(d, 0)), ((a*x**4/4 + b*x**7/7)*cosh(c), True))
```

Giac [A]
time = 0.41, size = 192, normalized size = 1.25

$$\frac{(bd^6x^6 - 6\,bd^5x^5 + ad^6x^3 + 30\,bd^4x^4 - 3\,ad^5x^2 - 120\,bd^3x^3 + 6\,ad^4x + 360\,bd^2x^2 - 6\,ad^3 - 720\,bdx + 720\,b)e^{(dx+c)}}{2\,d^7} - \frac{(bd^6x^6 + 6\,bd^5x^5 + ad^6x^3 + 30\,bd^4x^4 + 3\,ad^5x^2 + 120\,bd^3x^3 + 6\,ad^4x + 360\,bd^2x^2 + 6\,ad^3 + 720\,bdx + 720\,b)e^{(-dx-c)}}{2\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^6*x^6 - 6*b*d^5*x^5 + a*d^6*x^3 + 30*b*d^4*x^4 - 3*a*d^5*x^2 - 120
*b*d^3*x^3 + 6*a*d^4*x + 360*b*d^2*x^2 - 6*a*d^3 - 720*b*d*x + 720*b)*e^(d*
x + c)/d^7 - 1/2*(b*d^6*x^6 + 6*b*d^5*x^5 + a*d^6*x^3 + 30*b*d^4*x^4 + 3*a*
d^5*x^2 + 120*b*d^3*x^3 + 6*a*d^4*x + 360*b*d^2*x^2 + 6*a*d^3 + 720*b*d*x +
 720*b)*e^(-d*x - c)/d^7
```

Mupad [B]
time = 1.05, size = 152, normalized size = 0.99

$$\frac{30\,b\,x^4\,\sinh(c+d\,x)+6\,a\,x\,\sinh(c+d\,x)}{d^3}-\frac{3\,a\,x^2\,\cosh(c+d\,x)+6\,b\,x^5\,\cosh(c+d\,x)}{d^2}+\frac{a\,x^3\,\sinh(c+d\,x)+b\,x^6\,\sinh(c+d\,x)}{d}-\frac{6\,a\,\cosh(c+d\,x)+120\,b\,x^3\,\cosh(c+d\,x)}{d^4}+\frac{720\,b\,\sinh(c+d\,x)}{d^7}-\frac{720\,b\,x\,\cosh(c+d\,x)}{d^6}+\frac{360\,b\,x^2\,\sinh(c+d\,x)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(c + d*x)*(a + b*x^3),x)
```

```
[Out] (30*b*x^4*sinh(c + d*x) + 6*a*x*sinh(c + d*x))/d^3 - (3*a*x^2*cosh(c + d*x)
 + 6*b*x^5*cosh(c + d*x))/d^2 + (a*x^3*sinh(c + d*x) + b*x^6*sinh(c + d*x))
/d - (6*a*cosh(c + d*x) + 120*b*x^3*cosh(c + d*x))/d^4 + (720*b*sinh(c + d*
x))/d^7 - (720*b*x*cosh(c + d*x))/d^6 + (360*b*x^2*sinh(c + d*x))/d^5
```

3.80 $\int x^2(a+bx^3)\cosh(c+dx)\,dx$

Optimal. Leaf size=124

$$-\frac{120b\cosh(c+dx)}{d^6}-\frac{2ax\cosh(c+dx)}{d^2}-\frac{60bx^2\cosh(c+dx)}{d^4}-\frac{5bx^4\cosh(c+dx)}{d^2}+\frac{2a\sinh(c+dx)}{d^3}+\frac{120bx\,\mathrm{s}}{}$$

```
[Out] -120*b*cosh(d*x+c)/d^6-2*a*x*cosh(d*x+c)/d^2-60*b*x^2*cosh(d*x+c)/d^4-5*b*x
^4*cosh(d*x+c)/d^2+2*a*sinh(d*x+c)/d^3+120*b*x*sinh(d*x+c)/d^5+a*x^2*sinh(d
*x+c)/d+20*b*x^3*sinh(d*x+c)/d^3+b*x^5*sinh(d*x+c)/d
```

Rubi [A]
time = 0.16, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5395, 3377, 2717, 2718}

$$\frac{2a\sinh(c+dx)}{d^3}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{ax^2\sinh(c+dx)}{d}-\frac{120b\cosh(c+dx)}{d^6}+\frac{120bx\sinh(c+dx)}{d^5}-\frac{60bx^2\cosh(c+dx)}{d^4}+\frac{20bx^3\sinh(c+dx)}{d^3}-\frac{5bx^4\cosh(c+dx)}{d^2}+\frac{bx^5\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-120*b*Cosh[c + d*x])/d^6 - (2*a*x*Cosh[c + d*x])/d^2 - (60*b*x^2*Cosh[c +
 d*x])/d^4 - (5*b*x^4*Cosh[c + d*x])/d^2 + (2*a*Sinh[c + d*x])/d^3 + (120*b
*x*Sinh[c + d*x])/d^5 + (a*x^2*Sinh[c + d*x])/d + (20*b*x^3*Sinh[c + d*x])/
d^3 + (b*x^5*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
```

```
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2(a+bx^3)\cosh(c+dx)\,dx = \int \left(ax^2\cosh(c+dx)+bx^5\cosh(c+dx)\right)\,dx$$
$$= a\int x^2\cosh(c+dx)\,dx + b\int x^5\cosh(c+dx)\,dx$$
$$= \frac{ax^2\sinh(c+dx)}{d} + \frac{bx^5\sinh(c+dx)}{d} - \frac{(2a)\int x\sinh(c+dx)\,dx}{d} - \frac{(5b)\int x$$
$$= -\frac{2ax\cosh(c+dx)}{d^2} - \frac{5bx^4\cosh(c+dx)}{d^2} + \frac{ax^2\sinh(c+dx)}{d} + \frac{bx^5\sinh(c+$$
$$= -\frac{2ax\cosh(c+dx)}{d^2} - \frac{5bx^4\cosh(c+dx)}{d^2} + \frac{2a\sinh(c+dx)}{d^3} + \frac{ax^2\sinh(c+$$
$$= -\frac{2ax\cosh(c+dx)}{d^2} - \frac{60bx^2\cosh(c+dx)}{d^4} - \frac{5bx^4\cosh(c+dx)}{d^2} + \frac{2a\sinh(c$$
$$= -\frac{2ax\cosh(c+dx)}{d^2} - \frac{60bx^2\cosh(c+dx)}{d^4} - \frac{5bx^4\cosh(c+dx)}{d^2} + \frac{2a\sinh(c$$
$$= -\frac{120b\cosh(c+dx)}{d^6} - \frac{2ax\cosh(c+dx)}{d^2} - \frac{60bx^2\cosh(c+dx)}{d^4} - \frac{5bx^4\cosh$$

Mathematica [A]
time = 0.11, size = 84, normalized size = 0.68

$$\frac{-((2ad^4x+5b(24+12d^2x^2+d^4x^4))\cosh(c+dx))+d(ad^2(2+d^2x^2)+bx(120+20d^2x^2+d^4x^4))\sinh(c+dx)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)*Cosh[c + d*x],x]
[Out] (-((2*a*d^4*x + 5*b*(24 + 12*d^2*x^2 + d^4*x^4))*Cosh[c + d*x]) + d*(a*d^2*
(2 + d^2*x^2) + b*x*(120 + 20*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(124) = 248$.
time = 0.66, size = 389, normalized size = 3.14

method	result
risch	$\frac{(b\,x^5d^5-5b\,x^4d^4+a\,d^5x^2+20b\,d^3x^3-2a\,d^4x-60b\,d^2x^2+2a\,d^3+120bdx-120b)\,e^{dx+c}}{2d^6} - \frac{(b\,x^5d^5+5b\,x^4d^4+a\,d^5x^2+20b\,d^3$
meijerg	$-\frac{32b\cosh(c)\sqrt{\pi}\left(-\frac{15}{4\sqrt{\pi}}+\frac{\left(\frac{15}{8}d^4x^4+\frac{45}{2}d^2x^2+45\right)\cosh(dx)}{12\sqrt{\pi}}-\frac{xd\left(\frac{3}{8}d^4x^4+\frac{15}{2}d^2x^2+45\right)\sinh(dx)}{12\sqrt{\pi}}\right)}{d^6}+\frac{32ib\sinh(c)\sqrt{\pi}$

derivativedivides	$\frac{-\frac{b\,c^5 \sinh(dx+c)}{d^3}+\frac{5b\,c^4((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^3}-\frac{10b\,c^3\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^3}+\frac{10b}{}}{}$
default	$\frac{-\frac{b\,c^5 \sinh(dx+c)}{d^3}+\frac{5b\,c^4((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^3}-\frac{10b\,c^3\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^3}+\frac{10b}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(-b/d^3*c^5*sinh(d*x+c)+5*b/d^3*c^4*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))
-10*b/d^3*c^3*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+1
0*b/d^3*c^2*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d
*x+c)-6*cosh(d*x+c))-5*b/d^3*c*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+
c)+12*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))+b/d^3*((
d*x+c)^5*sinh(d*x+c)-5*(d*x+c)^4*cosh(d*x+c)+20*(d*x+c)^3*sinh(d*x+c)-60*(d
*x+c)^2*cosh(d*x+c)+120*(d*x+c)*sinh(d*x+c)-120*cosh(d*x+c))+a*c^2*sinh(d*x
+c)-2*a*c*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+a*((d*x+c)^2*sinh(d*x+c)-2*(d*x
+c)*cosh(d*x+c)+2*sinh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(124) = 248$.
time = 0.28, size = 267, normalized size = 2.15

$$\frac{(bx^3+a)^2\cosh(dx+c)}{6b}-\frac{\left(\frac{a^2e^{(dx+c)}}{d}+\frac{a^2e^{(-dx-c)}}{d}+\frac{2\,(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c)abe^{(dx)}}{d^4}+\frac{2\,(d^3x^3+3\,d^2x^2+6\,dx+6)abe^{(-dx-c)}}{d^4}+\frac{(d^6x^6e^c-6\,d^5x^5e^c+30\,d^4x^4e^c-120\,d^3x^3e^c+360\,d^2x^2e^c-720\,dxe^c+720\,e^c)b^2e^{(dx)}}{d^7}+\frac{(d^6x^6+6\,d^5x^5+30\,d^4x^4+120\,d^3x^3+360\,d^2x^2+720\,dx+720)b^2e^{(-dx-c)}}{d^7}\right)d}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^3 + a)^2*cosh(d*x + c)/b - 1/12*(a^2*e^(d*x + c)/d + a^2*e^(-d*x -
 c)/d + 2*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*b*e^(d*x)/d^4
 + 2*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*b*e^(-d*x - c)/d^4 + (d^6*x^6*e^c
- 6*d^5*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*
d*x*e^c + 720*e^c)*b^2*e^(d*x)/d^7 + (d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 12
0*d^3*x^3 + 360*d^2*x^2 + 720*d*x + 720)*b^2*e^(-d*x - c)/d^7)*d/b
```

Fricas [A]
time = 0.37, size = 87, normalized size = 0.70

$$-\frac{(5\,bd^4x^4+2\,ad^4x+60\,bd^2x^2+120\,b)\cosh(dx+c)-(bd^5x^5+ad^5x^2+20\,bd^3x^3+2\,ad^3+120\,bdx)\sinh(dx+c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((5*b*d^4*x^4 + 2*a*d^4*x + 60*b*d^2*x^2 + 120*b)*cosh(d*x + c) - (b*d^5*x
^5 + a*d^5*x^2 + 20*b*d^3*x^3 + 2*a*d^3 + 120*b*d*x)*sinh(d*x + c))/d^6
```

Sympy [A]
time = 0.45, size = 151, normalized size = 1.22

$$\begin{cases} \frac{ax^2\sinh(c+dx)}{d} - \frac{2ax\cosh(c+dx)}{d^2} + \frac{2a\sinh(c+dx)}{d^3} + \frac{bx^5\sinh(c+dx)}{d} - \frac{5bx^4\cosh(c+dx)}{d^2} + \frac{20bx^3\sinh(c+dx)}{d^3} - \frac{60bx^2\cosh(c+dx)}{d^4} + \frac{120bx\sinh(c+dx)}{d^5} - \frac{120b\cosh(c+dx)}{d^6} & \text{for } d \neq 0 \\ \left(\frac{ax^3}{3} + \frac{bx^6}{6}\right)\cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x**2*sinh(c + d*x)/d - 2*a*x*cosh(c + d*x)/d**2 + 2*a*sinh(c +
 d*x)/d**3 + b*x**5*sinh(c + d*x)/d - 5*b*x**4*cosh(c + d*x)/d**2 + 20*b*x*
*3*sinh(c + d*x)/d**3 - 60*b*x**2*cosh(c + d*x)/d**4 + 120*b*x*sinh(c + d*x
)/d**5 - 120*b*cosh(c + d*x)/d**6, Ne(d, 0)), ((a*x**3/3 + b*x**6/6)*cosh(c
), True))
```

Giac [A]
time = 0.41, size = 156, normalized size = 1.26

$$\frac{(bd^5x^5 - 5\,bd^4x^4 + ad^5x^2 + 20\,bd^3x^3 - 2\,ad^4x - 60\,bd^2x^2 + 2\,ad^3 + 120\,bdx - 120\,b)e^{(dx+c)}}{2\,d^6} - \frac{(bd^5x^5 + 5\,bd^4x^4 + ad^5x^2 + 20\,bd^3x^3 + 2\,ad^4x + 60\,bd^2x^2 + 2\,ad^3 + 120\,bdx + 120\,b)e^{(-dx-c)}}{2\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^5*x^5 - 5*b*d^4*x^4 + a*d^5*x^2 + 20*b*d^3*x^3 - 2*a*d^4*x - 60*b*
d^2*x^2 + 2*a*d^3 + 120*b*d*x - 120*b)*e^(d*x + c)/d^6 - 1/2*(b*d^5*x^5 + 5
*b*d^4*x^4 + a*d^5*x^2 + 20*b*d^3*x^3 + 2*a*d^4*x + 60*b*d^2*x^2 + 2*a*d^3
+ 120*b*d*x + 120*b)*e^(-d*x - c)/d^6
```

Mupad [B]
time = 0.96, size = 122, normalized size = 0.98

$$\frac{a\,x^2\sinh(c+d\,x) + b\,x^5\sinh(c+d\,x)}{d} - \frac{2\,a\,x\cosh(c+d\,x) + 5\,b\,x^4\cosh(c+d\,x)}{d^2} + \frac{2\,a\sinh(c+d\,x) + 20\,b\,x^3\sinh(c+d\,x)}{d^3} - \frac{120\,b\cosh(c+d\,x)}{d^6} + \frac{120\,b\,x\sinh(c+d\,x)}{d^5} - \frac{60\,b\,x^2\cosh(c+d\,x)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(c + d*x)*(a + b*x^3),x)
```

```
[Out] (a*x^2*sinh(c + d*x) + b*x^5*sinh(c + d*x))/d - (2*a*x*cosh(c + d*x) + 5*b*
x^4*cosh(c + d*x))/d^2 + (2*a*sinh(c + d*x) + 20*b*x^3*sinh(c + d*x))/d^3 -
 (120*b*cosh(c + d*x))/d^6 + (120*b*x*sinh(c + d*x))/d^5 - (60*b*x^2*cosh(c
 + d*x))/d^4
```

3.81 $\int x(a+bx^3)\cosh(c+dx)\,dx$

Optimal. Leaf size=94

$$-\frac{a\cosh(c+dx)}{d^2}-\frac{24bx\cosh(c+dx)}{d^4}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{24b\sinh(c+dx)}{d^5}+\frac{ax\sinh(c+dx)}{d}+\frac{12bx^2\sinh(c}{d^3}$$

```
[Out] -a*cosh(d*x+c)/d^2-24*b*x*cosh(d*x+c)/d^4-4*b*x^3*cosh(d*x+c)/d^2+24*b*sinh
(d*x+c)/d^5+a*x*sinh(d*x+c)/d+12*b*x^2*sinh(d*x+c)/d^3+b*x^4*sinh(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5395, 3377, 2718, 2717}

$$-\frac{a\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}+\frac{24b\sinh(c+dx)}{d^5}-\frac{24bx\cosh(c+dx)}{d^4}+\frac{12bx^2\sinh(c+dx)}{d^3}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{bx^4\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] -((a*Cosh[c + d*x])/d^2) - (24*b*x*Cosh[c + d*x])/d^4 - (4*b*x^3*Cosh[c + d
*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (a*x*Sinh[c + d*x])/d + (12*b*x^2*Sin
h[c + d*x])/d^3 + (b*x^4*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x(a+bx^3)\cosh(c+dx)\,dx &= \int (ax\cosh(c+dx)+bx^4\cosh(c+dx))\,dx \\
&= a\int x\cosh(c+dx)\,dx+b\int x^4\cosh(c+dx)\,dx \\
&= \frac{ax\sinh(c+dx)}{d}+\frac{bx^4\sinh(c+dx)}{d}-\frac{a\int\sinh(c+dx)\,dx}{d}-\frac{(4b)\int x^3\sinh(\cdots}{d} \\
&= -\frac{a\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}+\frac{bx^4\sinh(c+dx\cdots}{d} \\
&= -\frac{a\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}+\frac{12bx^2\sinh(c+\cdots}{d^3} \\
&= -\frac{a\cosh(c+dx)}{d^2}-\frac{24bx\cosh(c+dx)}{d^4}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+d\cdots}{d} \\
&= -\frac{a\cosh(c+dx)}{d^2}-\frac{24bx\cosh(c+dx)}{d^4}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{24b\sinh(c+\cdots}{d^5}
\end{aligned}
$$

Mathematica [**A**]
time = 0.09, size = 66, normalized size = 0.70

$$\frac{-d(ad^2+4bx(6+d^2x^2))\cosh(c+dx)+(ad^4x+b(24+12d^2x^2+d^4x^4))\sinh(c+dx)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-(d*(a*d^2 + 4*b*x*(6 + d^2*x^2))*Cosh[c + d*x]) + (a*d^4*x + b*(24 + 12*d
^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(94) = 188$.
time = 0.66, size = 257, normalized size = 2.73

method	result
risch	$\frac{(b x^4 d^4-4b d^3 x^3+a d^4 x+12b d^2 x^2-a d^3-24bdx+24b)\mathrm{e}^{dx+c}}{2d^5}-\frac{(b x^4 d^4+4b d^3 x^3+a d^4 x+12b d^2 x^2+a d^3+24bdx+24b)\mathrm{e}^{-\cdots}}{2d^5}$
meijerg	$-\frac{16ib\cosh(c)\sqrt{\pi}\left(-\frac{ixd\left(\frac{5d^2x^2}{2}+15\right)\cosh(dx)}{10\sqrt{\pi}}+\frac{i\left(\frac{5}{8}d^4x^4+\frac{15}{2}d^2x^2+15\right)\sinh(dx)}{10\sqrt{\pi}}\right)}{d^5}-\frac{16b\sinh(c)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d^{\cdots}\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{-\frac{4b c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^3}+\frac{6b c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^3}-\frac{4bc\left((dx+c)^3\sinh(dx+c)-\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{4b c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^3}+\frac{6b c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^3}-\frac{4bc\left((dx+c)^3\sinh(dx+c)-\cdots\right.}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(-4*b/d^3*c^3*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+6*b/d^3*c^2*((d*x+c)^
2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-4*b/d^3*c*((d*x+c)^3*sin
h(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))+b/d^3
*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12*(d*x+c)^2*sinh(d*x+c)-24
*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))+a*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+b/
d^3*c^4*sinh(d*x+c)-c*a*sinh(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(94) = 188$.
time = 0.26, size = 196, normalized size = 2.09

$$-\frac{1}{20}\,d\left(\frac{5\,(d^2x^2e^c-2\,dxe^c+2\,e^c)ae^{(dx)}}{d^3}+\frac{5\,(d^2x^2+2\,dx+2)ae^{(-dx-c)}}{d^3}+\frac{2\,(d^5x^5e^c-5\,d^4x^4e^c+20\,d^3x^3e^c-60\,d^2x^2e^c+120\,dxe^c-120\,e^c)be^{(dx)}}{d^6}+\frac{2\,(d^5x^5+5\,d^4x^4+20\,d^3x^3+60\,d^2x^2+120\,dx+120)be^{(-dx-c)}}{d^6}\right)+\frac{1}{10}\,(2\,bx^5+5\,ax^2)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/20*d*(5*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*e^(d*x)/d^3 + 5*(d^2*x^2 + 2
*d*x + 2)*a*e^(-d*x - c)/d^3 + 2*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*
e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*b*e^(d*x)/d^6 + 2*(d^5*x^5 +
5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*b*e^(-d*x - c)/d^6) +
1/10*(2*b*x^5 + 5*a*x^2)*cosh(d*x + c)
```

Fricas [A]
time = 0.39, size = 68, normalized size = 0.72

$$-\frac{(4\,bd^3x^3+ad^3+24\,bdx)\cosh\left(dx+c\right)-(bd^4x^4+ad^4x+12\,bd^2x^2+24\,b)\sinh\left(dx+c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((4*b*d^3*x^3 + a*d^3 + 24*b*d*x)*cosh(d*x + c) - (b*d^4*x^4 + a*d^4*x + 1
2*b*d^2*x^2 + 24*b)*sinh(d*x + c))/d^5
```

Sympy [A]
time = 0.30, size = 116, normalized size = 1.23

$$\begin{cases} \frac{ax\sinh\left(c+dx\right)}{d}-\frac{a\cosh\left(c+dx\right)}{d^2}+\frac{bx^4\sinh\left(c+dx\right)}{d}-\frac{4bx^3\cosh\left(c+dx\right)}{d^2}+\frac{12bx^2\sinh\left(c+dx\right)}{d^3}-\frac{24bx\cosh\left(c+dx\right)}{d^4}+\frac{24b\sinh\left(c+dx\right)}{d^5} & \text{for}\ d\neq 0\\ \left(\frac{ax^2}{2}+\frac{bx^5}{5}\right)\cosh\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x*sinh(c + d*x)/d - a*cosh(c + d*x)/d**2 + b*x**4*sinh(c + d*x
)/d - 4*b*x**3*cosh(c + d*x)/d**2 + 12*b*x**2*sinh(c + d*x)/d**3 - 24*b*x*c
osh(c + d*x)/d**4 + 24*b*sinh(c + d*x)/d**5, Ne(d, 0)), ((a*x**2/2 + b*x**5
/5)*cosh(c), True))
```

Giac [A]
time = 0.40, size = 119, normalized size = 1.27

$$\frac{(bd^4x^4 - 4\,bd^3x^3 + ad^4x + 12\,bd^2x^2 - ad^3 - 24\,bdx + 24\,b)e^{(dx+c)}}{2\,d^5} - \frac{(bd^4x^4 + 4\,bd^3x^3 + ad^4x + 12\,bd^2x^2 + ad^3 + 24\,bdx + 24\,b)e^{(-dx-c)}}{2\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^4*x^4 - 4*b*d^3*x^3 + a*d^4*x + 12*b*d^2*x^2 - a*d^3 - 24*b*d*x +
24*b)*e^(d*x + c)/d^5 - 1/2*(b*d^4*x^4 + 4*b*d^3*x^3 + a*d^4*x + 12*b*d^2*x
^2 + a*d^3 + 24*b*d*x + 24*b)*e^(-d*x - c)/d^5
```

Mupad [B]
time = 0.92, size = 92, normalized size = 0.98

$$\frac{b\,x^4\,\mathrm{sinh}\left(c+d\,x\right)+a\,x\,\mathrm{sinh}\left(c+d\,x\right)}{d}-\frac{a\,\mathrm{cosh}\left(c+d\,x\right)+4\,b\,x^3\,\mathrm{cosh}\left(c+d\,x\right)}{d^2}+\frac{24\,b\,\mathrm{sinh}\left(c+d\,x\right)}{d^5}-\frac{24\,b\,x\,\mathrm{cosh}\left(c+d\,x\right)}{d^4}+\frac{12\,b\,x^2\,\mathrm{sinh}\left(c+d\,x\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(c + d*x)*(a + b*x^3),x)
```

```
[Out] (b*x^4*sinh(c + d*x) + a*x*sinh(c + d*x))/d - (a*cosh(c + d*x) + 4*b*x^3*co
sh(c + d*x))/d^2 + (24*b*sinh(c + d*x))/d^5 - (24*b*x*cosh(c + d*x))/d^4 +
(12*b*x^2*sinh(c + d*x))/d^3
```

3.82 $\int (a+bx^3)\cosh(c+dx)\,dx$

Optimal. Leaf size=66

$$-\frac{6b\cosh(c+dx)}{d^4}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{a\sinh(c+dx)}{d}+\frac{6bx\sinh(c+dx)}{d^3}+\frac{bx^3\sinh(c+dx)}{d}$$

```
[Out] -6*b*cosh(d*x+c)/d^4-3*b*x^2*cosh(d*x+c)/d^2+a*sinh(d*x+c)/d+6*b*x*sinh(d*x
+c)/d^3+b*x^3*sinh(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5385, 2717, 3377, 2718}

$$\frac{a\sinh(c+dx)}{d}-\frac{6b\cosh(c+dx)}{d^4}+\frac{6bx\sinh(c+dx)}{d^3}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{bx^3\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-6*b*Cosh[c + d*x])/d^4 - (3*b*x^2*Cosh[c + d*x])/d^2 + (a*Sinh[c + d*x])/
d + (6*b*x*Sinh[c + d*x])/d^3 + (b*x^3*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5385

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c,
d, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int (a+bx^3)\cosh(c+dx)\,dx &= \int \left(a\cosh(c+dx)+bx^3\cosh(c+dx)\right)\,dx \\
&= a\int \cosh(c+dx)\,dx + b\int x^3\cosh(c+dx)\,dx \\
&= \frac{a\sinh(c+dx)}{d} + \frac{bx^3\sinh(c+dx)}{d} - \frac{(3b)\int x^2\sinh(c+dx)\,dx}{d} \\
&= -\frac{3bx^2\cosh(c+dx)}{d^2} + \frac{a\sinh(c+dx)}{d} + \frac{bx^3\sinh(c+dx)}{d} + \frac{(6b)\int x\cosh(c+
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3bx^2\cosh(c+dx)}{d^2} + \frac{a\sinh(c+dx)}{d} + \frac{6bx\sinh(c+dx)}{d^3} + \frac{bx^3\sinh(c+dx)}{d} \\
&= -\frac{6b\cosh(c+dx)}{d^4} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{a\sinh(c+dx)}{d} + \frac{6bx\sinh(c+dx)}{d^3}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 0.74

$$
\frac{-3b(2+d^2x^2)\cosh(c+dx)+d(ad^2+bx(6+d^2x^2))\sinh(c+dx)}{d^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-3*b*(2 + d^2*x^2)*Cosh[c + d*x] + d*(a*d^2 + b*x*(6 + d^2*x^2))*Sinh[c +
d*x])/d^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(66) = 132$.
time = 0.66, size = 158, normalized size = 2.39

method	result
risch	$\frac{(b\,d^3x^3-3b\,d^2x^2+a\,d^3+6bdx-6b)\mathrm{e}^{dx+c}}{2d^4}-\frac{(b\,d^3x^3+3b\,d^2x^2+a\,d^3+6bdx+6b)\mathrm{e}^{-dx-c}}{2d^4}$
meijerg	$\frac{8b\cosh(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(\frac{3d^2x^2}{2}+3\right)\cosh(dx)}{4\sqrt{\pi}}+\frac{dx\left(\frac{d^2x^2}{2}+3\right)\sinh(dx)}{4\sqrt{\pi}}\right)}{d^4}-\frac{8ib\sinh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{5d^2x^2}{2}+15\right)\cosh(dx)}{20\sqrt{\pi}}\right.}{d^4}$
derivativedivides	$\frac{-\frac{b\,c^3\sinh(dx+c)}{d^3}+\frac{3b\,c^2((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^3}-\frac{3bc\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^3}+\frac{b\left((dx+c\right.}{}}{d}$
default	$\frac{-\frac{b\,c^3\sinh(dx+c)}{d^3}+\frac{3b\,c^2((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^3}-\frac{3bc\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^3}+\frac{b\left((dx+c\right.}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b/d^3*c^3*sinh(d*x+c)+3*b/d^3*c^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-3
*b/d^3*c*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+b/d^3*
((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh
(d*x+c))+a*sinh(d*x+c))
```

Maxima [A]
time = 0.27, size = 104, normalized size = 1.58

$$\frac{ae^{(dx+c)}}{2\,d} - \frac{ae^{(-dx-c)}}{2\,d} + \frac{(d^3x^3e^c - 3\,d^2x^2e^c + 6\,dxe^c - 6\,e^c)be^{(dx)}}{2\,d^4} - \frac{(d^3x^3 + 3\,d^2x^2 + 6\,dx + 6)be^{(-dx-c)}}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a*e^(d*x + c)/d - 1/2*a*e^(-d*x - c)/d + 1/2*(d^3*x^3*e^c - 3*d^2*x^2*e
^c + 6*d*x*e^c - 6*e^c)*b*e^(d*x)/d^4 - 1/2*(d^3*x^3 + 3*d^2*x^2 + 6*d*x +
6)*b*e^(-d*x - c)/d^4
```

Fricas [A]
time = 0.42, size = 53, normalized size = 0.80

$$-\frac{3\,(bd^2x^2 + 2\,b)\cosh\left(dx + c\right) - (bd^3x^3 + ad^3 + 6\,bdx)\sinh\left(dx + c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(3*(b*d^2*x^2 + 2*b)*cosh(d*x + c) - (b*d^3*x^3 + a*d^3 + 6*b*d*x)*sinh(d*
x + c))/d^4
```

Sympy [A]
time = 0.19, size = 82, normalized size = 1.24

$$\begin{cases} \frac{a\sinh\left(c+dx\right)}{d} + \frac{bx^3\sinh\left(c+dx\right)}{d} - \frac{3bx^2\cosh\left(c+dx\right)}{d^2} + \frac{6bx\sinh\left(c+dx\right)}{d^3} - \frac{6b\cosh\left(c+dx\right)}{d^4} & \text{for}\ d \neq 0 \\ \left(ax + \frac{bx^4}{4}\right)\cosh\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)/d + b*x**3*sinh(c + d*x)/d - 3*b*x**2*cosh(c + d
*x)/d**2 + 6*b*x*sinh(c + d*x)/d**3 - 6*b*cosh(c + d*x)/d**4, Ne(d, 0)), ((
a*x + b*x**4/4)*cosh(c), True))
```

Giac [A]
time = 0.41, size = 88, normalized size = 1.33

$$\frac{(bd^3x^3 - 3\,bd^2x^2 + ad^3 + 6\,bdx - 6\,b)e^{(dx+c)}}{2\,d^4} - \frac{(bd^3x^3 + 3\,bd^2x^2 + ad^3 + 6\,bdx + 6\,b)e^{(-dx-c)}}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^3*x^3 - 3*b*d^2*x^2 + a*d^3 + 6*b*d*x - 6*b)*e^(d*x + c)/d^4 - 1/2
*(b*d^3*x^3 + 3*b*d^2*x^2 + a*d^3 + 6*b*d*x + 6*b)*e^(-d*x - c)/d^4
```

Mupad [B]
time = 0.10, size = 65, normalized size = 0.98

$$\frac{a\sinh(c+d\,x)+b\,x^3\sinh(c+d\,x)}{d}-\frac{6\,b\cosh(c+d\,x)}{d^4}+\frac{6\,b\,x\sinh(c+d\,x)}{d^3}-\frac{3\,b\,x^2\cosh(c+d\,x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b*x^3),x)
```

```
[Out] (a*sinh(c + d*x) + b*x^3*sinh(c + d*x))/d - (6*b*cosh(c + d*x))/d^4 + (6*b*
x*sinh(c + d*x))/d^3 - (3*b*x^2*cosh(c + d*x))/d^2
```

3.83 $\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx$

Optimal. Leaf size=56

$$-\frac{2bx\cosh(c+dx)}{d^2}+a\cosh(c)\text{Chi}(dx)+\frac{2b\sinh(c+dx)}{d^3}+\frac{bx^2\sinh(c+dx)}{d}+a\sinh(c)\text{Shi}(dx)$$

```
[Out] a*Chi(d*x)*cosh(c)-2*b*x*cosh(d*x+c)/d^2+a*Shi(d*x)*sinh(c)+2*b*sinh(d*x+c)
/d^3+b*x^2*sinh(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5395, 3384, 3379, 3382, 3377, 2717}

$$a\cosh(c)\text{Chi}(dx)+a\sinh(c)\text{Shi}(dx)+\frac{2b\sinh(c+dx)}{d^3}-\frac{2bx\cosh(c+dx)}{d^2}+\frac{bx^2\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*Cosh[c + d*x])/x,x]
```

```
[Out] (-2*b*x*Cosh[c + d*x])/d^2 + a*Cosh[c]*CoshIntegral[d*x] + (2*b*Sinh[c + d*
x])/d^3 + (b*x^2*Sinh[c + d*x])/d + a*Sinh[c]*SinhIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx &= \int \left(\frac{a\cosh(c+dx)}{x}+bx^2\cosh(c+dx)\right)dx\\ &= a\int \frac{\cosh(c+dx)}{x}\,dx+b\int x^2\cosh(c+dx)\,dx\\ &= \frac{bx^2\sinh(c+dx)}{d}-\frac{(2b)\int x\sinh(c+dx)\,dx}{d}+(a\cosh(c))\int \frac{\cosh(dx)}{x}\,dx+(\\ &= -\frac{2bx\cosh(c+dx)}{d^2}+a\cosh(c)\text{Chi}(dx)+\frac{bx^2\sinh(c+dx)}{d}+a\sinh(c)\text{Shi}(dx\\ &= -\frac{2bx\cosh(c+dx)}{d^2}+a\cosh(c)\text{Chi}(dx)+\frac{2b\sinh(c+dx)}{d^3}+\frac{bx^2\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 49, normalized size = 0.88

$$a\cosh(c)\text{Chi}(dx)+\frac{b(-2dx\cosh(c+dx)+(2+d^2x^2)\sinh(c+dx))}{d^3}+a\sinh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*Cosh[c + d*x])/x,x]
```

```
[Out] a*Cosh[c]*CoshIntegral[d*x] + (b*(-2*d*x*Cosh[c + d*x] + (2 + d^2*x^2)*Sinh
[c + d*x]))/d^3 + a*Sinh[c]*SinhIntegral[d*x]
```

Maple [A]
time = 0.90, size = 113, normalized size = 2.02

method	result

risch	$\frac{b\,\mathrm{e}^{dx+c}x^{2}}{2d}-\frac{b\,\mathrm{e}^{dx+c}x}{d^{2}}-\frac{a\,\mathrm{e}^{c}\,\mathrm{expIntegral}(1,-dx)}{2}-\frac{a\,\mathrm{e}^{-c}\,\mathrm{expIntegral}(1,dx)}{2}-\frac{b\,\mathrm{e}^{-dx-c}x^{2}}{2d}-\frac{b\,\mathrm{e}^{-dx-c}x}{d^{2}}+\frac{b\,\mathrm{e}^{dx+c}}{d^{3}}-\frac{b\,\mathrm{e}^{-}}{d}$
meijerg	$\frac{4ib\cosh(c)\sqrt{\pi}\left(\frac{ixd\cosh(dx)}{2\sqrt{\pi}}-\frac{i\left(\frac{3d^{2}x^{2}}{2}+3\right)\sinh(dx)}{6\sqrt{\pi}}\right)}{d^{3}}+\frac{4b\sinh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(\frac{d^{2}x^{2}}{2}+1\right)\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^{3}}+\frac{a\,\mathrm{co}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*cosh(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b/d*exp(d*x+c)*x^2-b/d^2*exp(d*x+c)*x-1/2*a*exp(c)*Ei(1,-d*x)-1/2*a*exp
(-c)*Ei(1,d*x)-1/2*b/d*exp(-d*x-c)*x^2-b/d^2*exp(-d*x-c)*x+b/d^3*exp(d*x+c)
-b/d^3*exp(-d*x-c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(56) = 112$.
time = 0.31, size = 140, normalized size = 2.50

$$-\frac{1}{6}\left(b\left(\frac{(d^{3}x^{3}e^{c}-3\,d^{2}x^{2}e^{c}+6\,dxe^{c}-6\,e^{c})e^{(dx)}}{d^{4}}+\frac{(d^{3}x^{3}+3\,d^{2}x^{2}+6\,dx+6)e^{(-dx-c)}}{d^{4}}\right)+\frac{2\,a\cosh\left(dx+c\right)\log\left(x^{3}\right)}{d}-\frac{3\,(\mathrm{Ei}(-dx)\,e^{(-c)}+\mathrm{Ei}(dx)\,e^{c})a}{d}\right)d+\frac{1}{3}\left(bx^{3}+a\log\left(x^{3}\right)\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/6*(b*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4 + (d
^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4) + 2*a*cosh(d*x + c)*log(x
^3)/d - 3*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*a/d)*d + 1/3*(b*x^3 + a*log(x^3))
*cosh(d*x + c)
```

Fricas [A]
time = 0.35, size = 86, normalized size = 1.54

$$-\frac{4\,bdx\cosh\left(dx+c\right)-\left(ad^{3}\mathrm{Ei}(dx)+ad^{3}\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-2\left(bd^{2}x^{2}+2\,b\right)\sinh\left(dx+c\right)-\left(ad^{3}\mathrm{Ei}(dx)-ad^{3}\mathrm{Ei}(-dx)\right)\sinh\left(c\right)}{2\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b*d*x*cosh(d*x + c) - (a*d^3*Ei(d*x) + a*d^3*Ei(-d*x))*cosh(c) - 2*
(b*d^2*x^2 + 2*b)*sinh(d*x + c) - (a*d^3*Ei(d*x) - a*d^3*Ei(-d*x))*sinh(c))
/d^3
```

Sympy [A]
time = 2.05, size = 66, normalized size = 1.18

$$a\sinh\left(c\right)\mathrm{Shi}\left(dx\right)+a\cosh\left(c\right)\mathrm{Chi}\left(dx\right)+b\left(\begin{cases}\frac{x^{2}\sinh\left(c+dx\right)}{d}-\frac{2x\cosh\left(c+dx\right)}{d^{2}}+\frac{2\sinh\left(c+dx\right)}{d^{3}} & \text{for}\ d\neq 0\\ \frac{x^{3}\cosh\left(c\right)}{3} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*cosh(d*x+c)/x,x)
```

```
[Out] a*sinh(c)*Shi(d*x) + a*cosh(c)*Chi(d*x) + b*Piecewise((x**2*sinh(c + d*x)/d
 - 2*x*cosh(c + d*x)/d**2 + 2*sinh(c + d*x)/d**3, Ne(d, 0)), (x**3*cosh(c)/
3, True))
```

Giac [A]
time = 0.41, size = 109, normalized size = 1.95

$$\frac{bd^2x^2e^{(dx+c)} - bd^2x^2e^{(-dx-c)} + ad^3\text{Ei}(-dx)\,e^{(-c)} + ad^3\text{Ei}(dx)\,e^c - 2\,bdxe^{(dx+c)} - 2\,bdxe^{(-dx-c)} + 2\,be^{(dx+c)} - 2\,be^{(-dx-c)}}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(b*d^2*x^2*e^(d*x + c) - b*d^2*x^2*e^(-d*x - c) + a*d^3*Ei(-d*x)*e^(-c)
 + a*d^3*Ei(d*x)*e^c - 2*b*d*x*e^(d*x + c) - 2*b*d*x*e^(-d*x - c) + 2*b*e^(
d*x + c) - 2*b*e^(-d*x - c))/d^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$a\,\mathrm{coshint}(d\,x)\,\cosh(c) + a\,\mathrm{sinhint}(d\,x)\,\sinh(c) + \frac{b\,(2\sinh(c+d\,x) + d^2\,x^2\sinh(c+d\,x) - 2\,d\,x\cosh(c+d\,x))}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^3))/x,x)
```

```
[Out] a*coshint(d*x)*cosh(c) + a*sinhint(d*x)*sinh(c) + (b*(2*sinh(c + d*x) + d^2
*x^2*sinh(c + d*x) - 2*d*x*cosh(c + d*x)))/d^3
```

3.84 $\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx$

Optimal. Leaf size=55

$$-\frac{b\cosh(c+dx)}{d^2}-\frac{a\cosh(c+dx)}{x}+ad\text{Chi}(dx)\sinh(c)+\frac{bx\sinh(c+dx)}{d}+ad\cosh(c)\text{Shi}(dx)$$

```
[Out] -b*cosh(d*x+c)/d^2-a*cosh(d*x+c)/x+a*d*cosh(c)*Shi(d*x)+a*d*Chi(d*x)*sinh(c
)+b*x*sinh(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5395, 3378, 3384, 3379, 3382, 3377, 2718}

$$ad\sinh(c)\text{Chi}(dx)+ad\cosh(c)\text{Shi}(dx)-\frac{a\cosh(c+dx)}{x}-\frac{b\cosh(c+dx)}{d^2}+\frac{bx\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((b*Cosh[c + d*x])/d^2) - (a*Cosh[c + d*x])/x + a*d*CoshIntegral[d*x]*Sinh
[c] + (b*x*Sinh[c + d*x])/d + a*d*Cosh[c]*SinhIntegral[d*x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx &= \int \left(\frac{a\cosh(c+dx)}{x^2}+bx\cosh(c+dx)\right)dx \\
&= a\int \frac{\cosh(c+dx)}{x^2}\,dx+b\int x\cosh(c+dx)\,dx \\
&= -\frac{a\cosh(c+dx)}{x}+\frac{bx\sinh(c+dx)}{d}-\frac{b\int \sinh(c+dx)\,dx}{d}+(ad)\int \frac{\sinh(c+
\end{aligned}$$

$$= -\frac{b\cosh(c+dx)}{d^2}-\frac{a\cosh(c+dx)}{x}+\frac{bx\sinh(c+dx)}{d}+(ad\cosh(c))\int \frac{\sinh(}{x}$$

$$= -\frac{b\cosh(c+dx)}{d^2}-\frac{a\cosh(c+dx)}{x}+ad\text{Chi}(dx)\sinh(c)+\frac{bx\sinh(c+dx)}{d}+$$

Mathematica [A]
time = 0.10, size = 55, normalized size = 1.00

$$-\frac{b\cosh(c+dx)}{d^2}-\frac{a\cosh(c+dx)}{x}+ad\text{Chi}(dx)\sinh(c)+\frac{bx\sinh(c+dx)}{d}+ad\cosh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((b*Cosh[c + d*x])/d^2) - (a*Cosh[c + d*x])/x + a*d*CoshIntegral[d*x]*Sinh
[c] + (b*x*Sinh[c + d*x])/d + a*d*Cosh[c]*SinhIntegral[d*x]
```

Maple [A]
time = 0.92, size = 110, normalized size = 2.00

method	result
risch	$-\frac{a\,e^{-dx-c}}{2x}+\frac{da\,e^{-c}\,\mathrm{expIntegral}(1,dx)}{2}-\frac{b\,e^{-dx-c}x}{2d}-\frac{b\,e^{-dx-c}}{2d^2}-\frac{a\,e^{dx+c}}{2x}-\frac{da\,e^{c}\,\mathrm{expIntegral}(1,-dx)}{2}+\frac{b\,e^{dx+c}x}{2d}-b\,e$
meijerg	$-\frac{2b\cosh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{b\sinh(c)(\cosh(dx)dx-\sinh(dx))}{d^2}+\frac{ia\cosh(c)\sqrt{\pi}\,d\left(\frac{4i\cosh(dx)}{dx\sqrt{\pi}}-\frac{4i\,\mathrm{hy}}{}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*cosh(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*exp(-d*x-c)/x+1/2*d*a*exp(-c)*Ei(1,d*x)-1/2*b/d*exp(-d*x-c)*x-1/2*b/
d^2*exp(-d*x-c)-1/2*a/x*exp(d*x+c)-1/2*d*a*exp(c)*Ei(1,-d*x)+1/2*b/d*exp(d*
x+c)*x-1/2*b/d^2*exp(d*x+c)
```

Maxima [A]
time = 0.31, size = 102, normalized size = 1.85

$$-\frac{1}{4}\left(2\,a\mathrm{Ei}(-dx)\,e^{(-c)}-2\,a\mathrm{Ei}(dx)\,e^{c}+\frac{(d^2x^2e^c-2\,dxe^c+2\,e^c)be^{(dx)}}{d^3}+\frac{(d^2x^2+2\,dx+2)be^{(-dx-c)}}{d^3}\right)d+\frac{1}{2}\left(bx^2-\frac{2\,a}{x}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a*Ei(-d*x)*e^(-c) - 2*a*Ei(d*x)*e^c + (d^2*x^2*e^c - 2*d*x*e^c + 2*
e^c)*b*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*b*e^(-d*x - c)/d^3)*d + 1/2*(b*x
^2 - 2*a/x)*cosh(d*x + c)
```

Fricas [A]
time = 0.41, size = 90, normalized size = 1.64

$$\frac{2\,bdx^2\sinh\left(dx+c\right)-2\left(ad^2+bx\right)\cosh\left(dx+c\right)+\left(ad^3x\mathrm{Ei}(dx)-ad^3x\mathrm{Ei}(-dx)\right)\cosh\left(c\right)+\left(ad^3x\mathrm{Ei}(dx)+ad^3x\mathrm{Ei}(-dx)\right)\sinh\left(c\right)}{2\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x^2*sinh(d*x + c) - 2*(a*d^2 + b*x)*cosh(d*x + c) + (a*d^3*x*Ei(
d*x) - a*d^3*x*Ei(-d*x))*cosh(c) + (a*d^3*x*Ei(d*x) + a*d^3*x*Ei(-d*x))*sin
h(c))/(d^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+bx^3)\cosh\left(c+dx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*cosh(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**3)*cosh(c + d*x)/x**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(55) = 110.
time = 0.42, size = 111, normalized size = 2.02

$$-\frac{ad^3x\text{Ei}(-dx)\,e^{(-c)} - ad^3x\text{Ei}(dx)\,e^c - bdx^2e^{(dx+c)} + bdx^2e^{(-dx-c)} + ad^2e^{(dx+c)} + ad^2e^{(-dx-c)} + bxe^{(dx+c)} + bxe^{(-dx-c)}}{2\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*d^3*x*Ei(-d*x)*e^(-c) - a*d^3*x*Ei(d*x)*e^c - b*d*x^2*e^(d*x + c) +
 b*d*x^2*e^(-d*x - c) + a*d^2*e^(d*x + c) + a*d^2*e^(-d*x - c) + b*x*e^(d*x
 + c) + b*x*e^(-d*x - c))/(d^2*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^3))/x^2,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^3))/x^2, x)
```

3.85 $\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}\,dx$

Optimal. Leaf size=69

$$-\frac{a\cosh(c+dx)}{2x^2}+\frac{1}{2}ad^2\cosh(c)\text{Chi}(dx)+\frac{b\sinh(c+dx)}{d}-\frac{ad\sinh(c+dx)}{2x}+\frac{1}{2}ad^2\sinh(c)\text{Shi}(dx)$$

```
[Out] 1/2*a*d^2*Chi(d*x)*cosh(c)-1/2*a*cosh(d*x+c)/x^2+1/2*a*d^2*Shi(d*x)*sinh(c)
+b*sinh(d*x+c)/d-1/2*a*d*sinh(d*x+c)/x
```

Rubi [A]
time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5395, 2717, 3378, 3384, 3379, 3382}

$$\frac{1}{2}ad^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}ad^2\sinh(c)\text{Shi}(dx)-\frac{a\cosh(c+dx)}{2x^2}-\frac{ad\sinh(c+dx)}{2x}+\frac{b\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*Cosh[c + d*x])/x^3,x]
```

```
[Out] -1/2*(a*Cosh[c + d*x])/x^2 + (a*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (b*Sinh[
c + d*x])/d - (a*d*Sinh[c + d*x])/(2*x) + (a*d^2*Sinh[c]*SinhIntegral[d*x])
/2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}\,dx &= \int \left(b\cosh(c+dx)+\frac{a\cosh(c+dx)}{x^3}\right)dx \\
&= a\int \frac{\cosh(c+dx)}{x^3}\,dx+b\int \cosh(c+dx)\,dx \\
&= -\frac{a\cosh(c+dx)}{2x^2}+\frac{b\sinh(c+dx)}{d}+\frac{1}{2}(ad)\int \frac{\sinh(c+dx)}{x^2}\,dx \\
&= -\frac{a\cosh(c+dx)}{2x^2}+\frac{b\sinh(c+dx)}{d}-\frac{ad\sinh(c+dx)}{2x}+\frac{1}{2}(ad^2)\int \frac{\cosh(c+d
\\
&= -\frac{a\cosh(c+dx)}{2x^2}+\frac{b\sinh(c+dx)}{d}-\frac{ad\sinh(c+dx)}{2x}+\frac{1}{2}(ad^2\cosh(c))\int \frac{\cos
\\
&= -\frac{a\cosh(c+dx)}{2x^2}+\frac{1}{2}ad^2\cosh(c)\mathrm{Chi}(dx)+\frac{b\sinh(c+dx)}{d}-\frac{ad\sinh(c+dx)}{2x}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 86, normalized size = 1.25

$$\frac{b\cosh(dx)\sinh(c)}{d}-\frac{a\cosh(dx)(\cosh(c)+dx\sinh(c))}{2x^2}+\frac{b\cosh(c)\sinh(dx)}{d}-\frac{a(dx\cosh(c)+\sinh(c))\sinh(dx)}{2x^2}+\frac{1}{2}ad^2(\cosh(c)\mathrm{Chi}(dx)+\sinh(c)\mathrm{Shi}(dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*Cosh[c + d*x])/x^3,x]
```

```
[Out] (b*Cosh[d*x]*Sinh[c])/d - (a*Cosh[d*x]*(Cosh[c] + d*x*Sinh[c]))/(2*x^2) + (
b*Cosh[c]*Sinh[d*x])/d - (a*(d*x*Cosh[c] + Sinh[c])*Sinh[d*x])/(2*x^2) + (a
*d^2*(Cosh[c]*CoshIntegral[d*x] + Sinh[c]*SinhIntegral[d*x]))/2
```

Maple [A]
time = 0.93, size = 114, normalized size = 1.65

method	result
risch	$\frac{da\,\mathrm{e}^{-dx-c}}{4x}-\frac{a\,\mathrm{e}^{-dx-c}}{4x^{2}}-\frac{d^{2}a\,\mathrm{e}^{-c}\,\mathrm{expIntegral}(1,dx)}{4}-\frac{b\,\mathrm{e}^{-dx-c}}{2d}-\frac{a\,\mathrm{e}^{dx+c}}{4x^{2}}-\frac{da\,\mathrm{e}^{dx+c}}{4x}-\frac{d^{2}a\,\mathrm{e}^{c}\,\mathrm{expIntegral}(1,-dx)}{4}+ b\,\mathrm{e}$
meijerg	$\frac{b\cosh(c)\sinh(dx)}{d}-\frac{b\sinh(c)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(dx)}{\sqrt{\pi}}\right)}{d}-\frac{a\cosh(c)\sqrt{\pi}\,d^{2}\left(-\frac{4\left(\frac{9d^{2}x^{2}}{2}+3\right)}{3\sqrt{\pi}\,d^{2}x^{2}}+\frac{4\cosh(dx)}{\sqrt{\pi}\,d^{2}x^{2}}+\frac{4\sinh(dx)}{\sqrt{\pi}\,dx}-\frac{4(\mathrm{hyperb}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*cosh(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d*a*exp(-d*x-c)/x-1/4*a*exp(-d*x-c)/x^2-1/4*d^2*a*exp(-c)*Ei(1,d*x)-1/2
/d*b*exp(-d*x-c)-1/4*a/x^2*exp(d*x+c)-1/4*d*a/x*exp(d*x+c)-1/4*d^2*a*exp(c)
*Ei(1,-d*x)+1/2*b/d*exp(d*x+c)
```

Maxima [A]
time = 0.32, size = 87, normalized size = 1.26

$$\frac{1}{4}\left(ade^{(-c)}\Gamma(-1,dx)+ade^{c}\Gamma(-1,-dx)-\frac{2\,(dxe^{c}-e^{c})be^{(dx)}}{d^{2}}-\frac{2\,(dx+1)be^{(-dx-c)}}{d^{2}}\right)d+\frac{1}{2}\left(2\,bx-\frac{a}{x^{2}}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a*d*e^(-c)*gamma(-1, d*x) + a*d*e^c*gamma(-1, -d*x) - 2*(d*x*e^c - e^c
)*b*e^(d*x)/d^2 - 2*(d*x + 1)*b*e^(-d*x - c)/d^2)*d + 1/2*(2*b*x - a/x^2)*c
osh(d*x + c)
```

Fricas [A]
time = 0.36, size = 101, normalized size = 1.46

$$-\frac{2\,ad\cosh\left(dx+c\right)-\left(ad^{3}x^{2}\mathrm{Ei}(dx)+ad^{3}x^{2}\mathrm{Ei}(-dx)\right)\cosh\left(c\right)+2\,\left(ad^{2}x-2\,bx^{2}\right)\sinh\left(dx+c\right)-\left(ad^{3}x^{2}\mathrm{Ei}(dx)-ad^{3}x^{2}\mathrm{Ei}(-dx)\right)\sinh\left(c\right)}{4\,dx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d*cosh(d*x + c) - (a*d^3*x^2*Ei(d*x) + a*d^3*x^2*Ei(-d*x))*cosh(c
) + 2*(a*d^2*x - 2*b*x^2)*sinh(d*x + c) - (a*d^3*x^2*Ei(d*x) - a*d^3*x^2*Ei
(-d*x))*sinh(c))/(d*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^{3}\right)\cosh\left(c+dx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*cosh(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**3)*cosh(c + d*x)/x**3, x)
```

Giac [A]
time = 0.41, size = 118, normalized size = 1.71

$$\frac{ad^3x^2\mathrm{Ei}(-dx)\,e^{(-c)} + ad^3x^2\mathrm{Ei}(dx)\,e^c - ad^2xe^{(dx+c)} + ad^2xe^{(-dx-c)} + 2\,bx^2e^{(dx+c)} - 2\,bx^2e^{(-dx-c)} - ade^{(dx+c)} - ade^{(-dx-c)}}{4\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^3*x^2*Ei(-d*x)*e^(-c) + a*d^3*x^2*Ei(d*x)*e^c - a*d^2*x*e^(d*x + c
) + a*d^2*x*e^(-d*x - c) + 2*b*x^2*e^(d*x + c) - 2*b*x^2*e^(-d*x - c) - a*d
*e^(d*x + c) - a*d*e^(-d*x - c))/(d*x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^3))/x^3,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^3))/x^3, x)
```

3.86 $\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx$

Optimal. Leaf size=91

$$-\frac{a\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{6x}+b\cosh(c)\text{Chi}(dx)+\frac{1}{6}ad^3\text{Chi}(dx)\sinh(c)-\frac{ad\sinh(c+dx)}{6x^2}+\frac{1}{6}ad^3\cosh($$

```
[Out] b*Chi(d*x)*cosh(c)-1/3*a*cosh(d*x+c)/x^3-1/6*a*d^2*cosh(d*x+c)/x+1/6*a*d^3*
cosh(c)*Shi(d*x)+1/6*a*d^3*Chi(d*x)*sinh(c)+b*Shi(d*x)*sinh(c)-1/6*a*d*sinh
(d*x+c)/x^2
```

Rubi [A]
time = 0.15, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5395, 3378, 3384, 3379, 3382}

$$\frac{1}{6}ad^3\sinh(c)\text{Chi}(dx)+\frac{1}{6}ad^3\cosh(c)\text{Shi}(dx)-\frac{ad^2\cosh(c+dx)}{6x}-\frac{a\cosh(c+dx)}{3x^3}-\frac{ad\sinh(c+dx)}{6x^2}+b\cosh(c)\text{Chi}(dx)+b\sinh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*Cosh[c + d*x])/x^4,x]
```

```
[Out] -1/3*(a*Cosh[c + d*x])/x^3 - (a*d^2*Cosh[c + d*x])/(6*x) + b*Cosh[c]*CoshIn
tegral[d*x] + (a*d^3*CoshIntegral[d*x]*Sinh[c])/6 - (a*d*Sinh[c + d*x])/(6*
x^2) + (a*d^3*Cosh[c]*SinhIntegral[d*x])/6 + b*Sinh[c]*SinhIntegral[d*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx &= \int \left(\frac{a\cosh(c+dx)}{x^4}+\frac{b\cosh(c+dx)}{x}\right)dx\\ &= a\int \frac{\cosh(c+dx)}{x^4}\,dx+b\int \frac{\cosh(c+dx)}{x}\,dx\\ &= -\frac{a\cosh(c+dx)}{3x^3}+\frac{1}{3}(ad)\int \frac{\sinh(c+dx)}{x^3}\,dx+(b\cosh(c))\int \frac{\cosh(dx)}{x}\,dx+\\ &= -\frac{a\cosh(c+dx)}{3x^3}+b\cosh(c)\mathrm{Chi}(dx)-\frac{ad\sinh(c+dx)}{6x^2}+b\sinh(c)\mathrm{Shi}(dx)+\\ &= -\frac{a\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{6x}+b\cosh(c)\mathrm{Chi}(dx)-\frac{ad\sinh(c+dx)}{6x^2}+\\ &= -\frac{a\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{6x}+b\cosh(c)\mathrm{Chi}(dx)-\frac{ad\sinh(c+dx)}{6x^2}+\\ &= -\frac{a\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{6x}+b\cosh(c)\mathrm{Chi}(dx)+\frac{1}{6}ad^3\mathrm{Chi}(dx)\sinh\end{aligned}$$

Mathematica [A]
time = 0.19, size = 73, normalized size = 0.80

$$\frac{1}{6}\left(\mathrm{Chi}(dx)\left(6b\cosh(c)+ad^3\sinh(c)\right)-\frac{a((2+d^2x^2)\cosh(c+dx)+dx\sinh(c+dx))}{x^3}+\left(ad^3\cosh(c)+6b\sinh(c)\right)\mathrm{Shi}(dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*Cosh[c + d*x])/x^4,x]
```

```
[Out] (CoshIntegral[d*x]*(6*b*Cosh[c] + a*d^3*Sinh[c]) - (a*((2 + d^2*x^2)*Cosh[c
 + d*x] + d*x*Sinh[c + d*x]))/x^3 + (a*d^3*Cosh[c] + 6*b*Sinh[c])*SinhInteg
ral[d*x])/6
```

Maple [A]
time = 0.94, size = 143, normalized size = 1.57

method	result
risch	$-\frac{b\,\mathrm{e}^{-c}\,\mathrm{expIntegral}(1,dx)}{2}+\frac{d^3a\,\mathrm{e}^{-c}\,\mathrm{expIntegral}(1,dx)}{12}-\frac{d^2a\,\mathrm{e}^{-dx-c}}{12x}+\frac{da\,\mathrm{e}^{-dx-c}}{12x^2}-\frac{a\,\mathrm{e}^{-dx-c}}{6x^3}-\frac{da\,\mathrm{e}^{dx+c}}{12x^2}-\frac{d^3a\,\mathrm{e}^{c}\,\mathrm{expIn}$
meijerg	$\frac{b\cosh(c)\sqrt{\pi}\left(\frac{2\,\mathrm{hyperbolicCosineIntegral}(dx)-2\ln(dx)-2\gamma}{\sqrt{\pi}}+\frac{2\gamma+2\ln(x)+2\ln(id)}{\sqrt{\pi}}\right)}{2}+b\,\mathrm{hyperbolicSineIntegral}\,(dx)\sinh\,($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*cosh(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*exp(-c)*Ei(1,d*x)+1/12*d^3*a*exp(-c)*Ei(1,d*x)-1/12*d^2*a*exp(-d*x-c
)/x+1/12*d*a*exp(-d*x-c)/x^2-1/6*a*exp(-d*x-c)/x^3-1/12*d*a/x^2*exp(d*x+c)-
1/12*d^3*a*exp(c)*Ei(1,-d*x)-1/6*a/x^3*exp(d*x+c)-1/2*b*exp(c)*Ei(1,-d*x)-1
/12*d^2*a/x*exp(d*x+c)
```

Maxima [A]
time = 0.33, size = 95, normalized size = 1.04

$$\frac{1}{6}\left(\left(d^2e^{(-c)}\Gamma(-2,dx)-d^2e^c\Gamma(-2,-dx)\right)a-\frac{2\,b\cosh\left(dx+c\right)\log\left(x^3\right)}{d}+\frac{3\left(\mathrm{Ei}(-dx)\,e^{(-c)}+\mathrm{Ei}(dx)\,e^{c}\right)b}{d}\right)d+\frac{1}{3}\left(b\log\left(x^3\right)-\frac{a}{x^3}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((d^2*e^(-c)*gamma(-2, d*x) - d^2*e^c*gamma(-2, -d*x))*a - 2*b*cosh(d*x
 + c)*log(x^3)/d + 3*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b/d)*d + 1/3*(b*log(x^
3) - a/x^3)*cosh(d*x + c)
```

Fricas [A]
time = 0.41, size = 118, normalized size = 1.30

$$-\frac{2\,adx\sinh\left(dx+c\right)+2\left(ad^2x^2+2\,a\right)\cosh\left(dx+c\right)-\left(\left(ad^3+6\,b\right)x^3\mathrm{Ei}(dx)-\left(ad^3-6\,b\right)x^3\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-\left(\left(ad^3+6\,b\right)x^3\mathrm{Ei}(dx)+\left(ad^3-6\,b\right)x^3\mathrm{Ei}(-dx)\right)\sinh\left(c\right)}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*a*d*x*sinh(d*x + c) + 2*(a*d^2*x^2 + 2*a)*cosh(d*x + c) - ((a*d^3
+ 6*b)*x^3*Ei(d*x) - (a*d^3 - 6*b)*x^3*Ei(-d*x))*cosh(c) - ((a*d^3 + 6*b)*x
^3*Ei(d*x) + (a*d^3 - 6*b)*x^3*Ei(-d*x))*sinh(c))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^3\right)\cosh\left(c+dx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*cosh(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x**3)*cosh(c + d*x)/x**4, x)
```

Giac [A]
time = 0.40, size = 141, normalized size = 1.55

$$-\frac{ad^3x^3\mathrm{Ei}(-dx)\,e^{(-c)} - ad^3x^3\mathrm{Ei}(dx)\,e^{c} + ad^2x^2e^{(dx+c)} + ad^2x^2e^{(-dx-c)} - 6\,bx^3\mathrm{Ei}(-dx)\,e^{(-c)} - 6\,bx^3\mathrm{Ei}(dx)\,e^{c} + adxe^{(dx+c)} - adxe^{(-dx-c)} + 2\,ae^{(dx+c)} + 2\,ae^{(-dx-c)}}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(a*d^3*x^3*Ei(-d*x)*e^(-c) - a*d^3*x^3*Ei(d*x)*e^c + a*d^2*x^2*e^(d*x
 + c) + a*d^2*x^2*e^(-d*x - c) - 6*b*x^3*Ei(-d*x)*e^(-c) - 6*b*x^3*Ei(d*x)*
e^c + a*d*x*e^(d*x + c) - a*d*x*e^(-d*x - c) + 2*a*e^(d*x + c) + 2*a*e^(-d*
x - c))/x^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^3))/x^4,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^3))/x^4, x)
```

3.87 $\int x(a+bx^3)^2 \cosh(c+dx)\,dx$

Optimal. Leaf size=234

$$-\frac{5040b^2\cosh(c+dx)}{d^8}-\frac{a^2\cosh(c+dx)}{d^2}-\frac{48abx\cosh(c+dx)}{d^4}-\frac{2520b^2x^2\cosh(c+dx)}{d^6}-\frac{8abx^3\cosh(c+dx)}{d^2}$$

```
[Out] -5040*b^2*cosh(d*x+c)/d^8-a^2*cosh(d*x+c)/d^2-48*a*b*x*cosh(d*x+c)/d^4-2520
*b^2*x^2*cosh(d*x+c)/d^6-8*a*b*x^3*cosh(d*x+c)/d^2-210*b^2*x^4*cosh(d*x+c)/
d^4-7*b^2*x^6*cosh(d*x+c)/d^2+48*a*b*sinh(d*x+c)/d^5+5040*b^2*x*sinh(d*x+c)
/d^7+a^2*x*sinh(d*x+c)/d+24*a*b*x^2*sinh(d*x+c)/d^3+840*b^2*x^3*sinh(d*x+c)
/d^5+2*a*b*x^4*sinh(d*x+c)/d+42*b^2*x^5*sinh(d*x+c)/d^3+b^2*x^7*sinh(d*x+c)
/d
```

Rubi [A]
time = 0.28, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5395, 3377, 2718, 2717}

$$-\frac{a^2\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(c+dx)}{d}+\frac{48ab\sinh(c+dx)}{d^5}-\frac{48abx\cosh(c+dx)}{d^4}+\frac{24abx^2\sinh(c+dx)}{d^3}-\frac{8abx^3\cosh(c+dx)}{d^2}+\frac{2abx^4\sinh(c+dx)}{d}-\frac{5040b^2\cosh(c+dx)}{d^8}+\frac{5040b^2x\sinh(c+dx)}{d^7}-\frac{2520b^2x^2\cosh(c+dx)}{d^6}+\frac{840b^2x^3\sinh(c+dx)}{d^5}-\frac{210b^2x^4\cosh(c+dx)}{d^4}+\frac{42b^2x^5\sinh(c+dx)}{d^3}-\frac{7b^2x^6\cosh(c+dx)}{d^2}+\frac{b^2x^7\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^2*Cosh[c + d*x],x]
```

```
[Out] (-5040*b^2*Cosh[c + d*x])/d^8 - (a^2*Cosh[c + d*x])/d^2 - (48*a*b*x*Cosh[c
+ d*x])/d^4 - (2520*b^2*x^2*Cosh[c + d*x])/d^6 - (8*a*b*x^3*Cosh[c + d*x])/
d^2 - (210*b^2*x^4*Cosh[c + d*x])/d^4 - (7*b^2*x^6*Cosh[c + d*x])/d^2 + (48
*a*b*Sinh[c + d*x])/d^5 + (5040*b^2*x*Sinh[c + d*x])/d^7 + (a^2*x*Sinh[c +
d*x])/d + (24*a*b*x^2*Sinh[c + d*x])/d^3 + (840*b^2*x^3*Sinh[c + d*x])/d^5
+ (2*a*b*x^4*Sinh[c + d*x])/d + (42*b^2*x^5*Sinh[c + d*x])/d^3 + (b^2*x^7*S
inh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x\left(a+bx^3\right)^2\cosh(c+dx)\,dx &= \int \left(a^2x\cosh(c+dx)+2abx^4\cosh(c+dx)+b^2x^7\cosh(c+dx)\right)\,dx \\
&= a^2\int x\cosh(c+dx)\,dx+(2ab)\int x^4\cosh(c+dx)\,dx+b^2\int x^7\cosh(c+dx \\
&= \frac{a^2x\sinh(c+dx)}{d}+\frac{2abx^4\sinh(c+dx)}{d}+\frac{b^2x^7\sinh(c+dx)}{d}-\frac{a^2\int\sinh(c-}{d} \\
&= -\frac{a^2\cosh(c+dx)}{d^2}-\frac{8abx^3\cosh(c+dx)}{d^2}-\frac{7b^2x^6\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(}{d} \\
&= -\frac{a^2\cosh(c+dx)}{d^2}-\frac{8abx^3\cosh(c+dx)}{d^2}-\frac{7b^2x^6\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(}{d} \\
&= -\frac{a^2\cosh(c+dx)}{d^2}-\frac{48abx\cosh(c+dx)}{d^4}-\frac{8abx^3\cosh(c+dx)}{d^2}-\frac{210b^2x^4}{} \\
&= -\frac{a^2\cosh(c+dx)}{d^2}-\frac{48abx\cosh(c+dx)}{d^4}-\frac{8abx^3\cosh(c+dx)}{d^2}-\frac{210b^2x^4}{} \\
&= -\frac{a^2\cosh(c+dx)}{d^2}-\frac{48abx\cosh(c+dx)}{d^4}-\frac{2520b^2x^2\cosh(c+dx)}{d^6}-\frac{8abx^3}{} \\
&= -\frac{a^2\cosh(c+dx)}{d^2}-\frac{48abx\cosh(c+dx)}{d^4}-\frac{2520b^2x^2\cosh(c+dx)}{d^6}-\frac{8abx^3}{} \\
&= -\frac{5040b^2\cosh(c+dx)}{d^8}-\frac{a^2\cosh(c+dx)}{d^2}-\frac{48abx\cosh(c+dx)}{d^4}-\frac{2520b^2x}{}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 139, normalized size = 0.59

$$\frac{-\left(\left(a^2d^6+8abd^4x(6+d^2x^2)+7b^2(720+360d^2x^2+30d^4x^4+d^6x^6)\right)\cosh(c+dx)\right)+d\left(a^2d^6x+2abd^2(24+12d^2x^2+d^4x^4)+b^2x(5040+840d^2x^2+42d^4x^4+d^6x^6)\right)\sinh(c+dx)}{d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^2*Cosh[c + d*x],x]
[Out] (-((a^2*d^6 + 8*a*b*d^4*x*(6 + d^2*x^2) + 7*b^2*(720 + 360*d^2*x^2 + 30*d^4
*x^4 + d^6*x^6))*Cosh[c + d*x]) + d*(a^2*d^6*x + 2*a*b*d^2*(24 + 12*d^2*x^2
```

```
 + d^4*x^4) + b^2*x*(5040 + 840*d^2*x^2 + 42*d^4*x^4 + d^6*x^6))*Sinh[c + d
*x])/d^8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(234) = 468$.
time = 0.61, size = 818, normalized size = 3.50 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(a^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+7*b^2/d^6*c^6*((d*x+c)*sinh(d*
x+c)-cosh(d*x+c))+b^2/d^6*((d*x+c)^7*sinh(d*x+c)-7*(d*x+c)^6*cosh(d*x+c)+42
*(d*x+c)^5*sinh(d*x+c)-210*(d*x+c)^4*cosh(d*x+c)+840*(d*x+c)^3*sinh(d*x+c)-
2520*(d*x+c)^2*cosh(d*x+c)+5040*(d*x+c)*sinh(d*x+c)-5040*cosh(d*x+c))+12*b/
d^3*c^2*a*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-8*b/d
^3*c*a*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)
-6*cosh(d*x+c))+2*b/d^3*a*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12
*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))-7*b^2/d^6*c*(
(d*x+c)^6*sinh(d*x+c)-6*(d*x+c)^5*cosh(d*x+c)+30*(d*x+c)^4*sinh(d*x+c)-120*
(d*x+c)^3*cosh(d*x+c)+360*(d*x+c)^2*sinh(d*x+c)-720*(d*x+c)*cosh(d*x+c)+720
*sinh(d*x+c))+21*b^2/d^6*c^2*((d*x+c)^5*sinh(d*x+c)-5*(d*x+c)^4*cosh(d*x+c)
+20*(d*x+c)^3*sinh(d*x+c)-60*(d*x+c)^2*cosh(d*x+c)+120*(d*x+c)*sinh(d*x+c)-
120*cosh(d*x+c))+35*b^2/d^6*c^4*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x
+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))-35*b^2/d^6*c^3*((d*x+c)^4*sinh(d*x
+c)-4*(d*x+c)^3*cosh(d*x+c)+12*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)
+24*sinh(d*x+c))-8*b/d^3*c^3*a*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-21*b^2/d^6
*c^5*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-c*a^2*sinh
(d*x+c)-b^2/d^6*c^7*sinh(d*x+c)+2*b/d^3*c^4*a*sinh(d*x+c))
```

Maxima [A]
time = 0.27, size = 383, normalized size = 1.64

$$-\frac{1}{80}\,d\left(\frac{20\,(d^2x^2e^c-2\,dxe^c+2\,e^c)a^2e^{(dx)}}{d^3}+\frac{20\,(d^2x^2+2\,dx+2)a^2e^{(-dx-c)}}{d^3}+\frac{16\,(d^5x^5e^c-5\,d^4x^4e^c+20\,d^3x^3e^c-60\,d^2x^2e^c+120\,dxe^c-120\,e^c)abe^{(dx)}}{d^6}+\frac{16\,(d^5x^5+5\,d^4x^4+20\,d^3x^3+60\,d^2x^2+120\,dx+120)abe^{(-dx-c)}}{d^6}+\frac{5\,(d^8x^8e^c-8\,d^7x^7e^c+56\,d^6x^6e^c-336\,d^5x^5e^c+1680\,d^4x^4e^c-6720\,d^3x^3e^c+20160\,d^2x^2e^c-40320\,dxe^c+40320\,e^c)b^2e^{(dx)}}{d^9}+\frac{5\,(d^8x^8+8\,d^7x^7+56\,d^6x^6+336\,d^5x^5+1680\,d^4x^4+6720\,d^3x^3+20160\,d^2x^2+40320\,dx+40320)b^2e^{(-dx-c)}}{d^9}\right)+\frac{1}{40}\,(5\,b^2x^8+16\,abx^5+20\,a^2x^2)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/80*d*(20*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a^2*e^(d*x)/d^3 + 20*(d^2*x^2
 + 2*d*x + 2)*a^2*e^(-d*x - c)/d^3 + 16*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d
^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*a*b*e^(d*x)/d^6 + 16*(
d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*a*b*e^(-d*x
- c)/d^6 + 5*(d^8*x^8*e^c - 8*d^7*x^7*e^c + 56*d^6*x^6*e^c - 336*d^5*x^5*e^
c + 1680*d^4*x^4*e^c - 6720*d^3*x^3*e^c + 20160*d^2*x^2*e^c - 40320*d*x*e^c
 + 40320*e^c)*b^2*e^(d*x)/d^9 + 5*(d^8*x^8 + 8*d^7*x^7 + 56*d^6*x^6 + 336*d
^5*x^5 + 1680*d^4*x^4 + 6720*d^3*x^3 + 20160*d^2*x^2 + 40320*d*x + 40320)*b
^2*e^(-d*x - c)/d^9) + 1/40*(5*b^2*x^8 + 16*a*b*x^5 + 20*a^2*x^2)*cosh(d*x
+ c)
```

Fricas [A]
time = 0.38, size = 161, normalized size = 0.69

$$-\frac{\left(7 b^{2} d^{6} x^{6}+8 a b d^{6} x^{3}+210 b^{2} d^{4} x^{4}+a^{2} d^{6}+48 a b d^{4} x+2520 b^{2} d^{2} x^{2}+5040 b^{2}\right) \cosh \left(d x+c\right)-\left(b^{2} d^{7} x^{7}+2 a b d^{7} x^{4}+42 b^{2} d^{5} x^{5}+24 a b d^{5} x^{2}+840 b^{2} d^{3} x^{3}+48 a b d^{3}+\left(a^{2} d^{7}+5040 b^{2} d\right) x\right) \sinh \left(d x+c\right)}{d^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((7*b^2*d^6*x^6 + 8*a*b*d^6*x^3 + 210*b^2*d^4*x^4 + a^2*d^6 + 48*a*b*d^4*x
 + 2520*b^2*d^2*x^2 + 5040*b^2)*cosh(d*x + c) - (b^2*d^7*x^7 + 2*a*b*d^7*x^
4 + 42*b^2*d^5*x^5 + 24*a*b*d^5*x^2 + 840*b^2*d^3*x^3 + 48*a*b*d^3 + (a^2*d
^7 + 5040*b^2*d)*x)*sinh(d*x + c))/d^8
```

Sympy [A]
time = 0.98, size = 284, normalized size = 1.21

$$\begin{cases} \frac{a^{2} x \sinh{\left(c+dx \right)}}{d} - \frac{a^{2} \cosh{\left(c+dx \right)}}{d^{2}} + \frac{2 a b x^{4} \sinh{\left(c+dx \right)}}{d} - \frac{8 a b x^{3} \cosh{\left(c+dx \right)}}{d^{2}} + \frac{24 a b x^{2} \sinh{\left(c+dx \right)}}{d^{3}} - \frac{48 a b x \cosh{\left(c+dx \right)}}{d^{4}} + \frac{48 a b \sinh{\left(c+dx \right)}}{d^{5}} + \frac{b^{2} x^{7} \sinh{\left(c+dx \right)}}{d} - \frac{7 b^{2} x^{6} \cosh{\left(c+dx \right)}}{d^{2}} + \frac{42 b^{2} x^{5} \sinh{\left(c+dx \right)}}{d^{3}} - \frac{210 b^{2} x^{4} \cosh{\left(c+dx \right)}}{d^{4}} + \frac{840 b^{2} x^{3} \sinh{\left(c+dx \right)}}{d^{5}} - \frac{2520 b^{2} x^{2} \cosh{\left(c+dx \right)}}{d^{6}} + \frac{5040 b^{2} x \sinh{\left(c+dx \right)}}{d^{7}} - \frac{5040 b^{2} \cosh{\left(c+dx \right)}}{d^{8}} & \text{for}\: d \neq 0 \\ \left(\frac{a^{2} x^{2}}{2} + \frac{2 a b x^{5}}{5} + \frac{b^{2} x^{8}}{8}\right) \cosh{\left(c \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*x*sinh(c + d*x)/d - a**2*cosh(c + d*x)/d**2 + 2*a*b*x**4*si
nh(c + d*x)/d - 8*a*b*x**3*cosh(c + d*x)/d**2 + 24*a*b*x**2*sinh(c + d*x)/d
**3 - 48*a*b*x*cosh(c + d*x)/d**4 + 48*a*b*sinh(c + d*x)/d**5 + b**2*x**7*s
inh(c + d*x)/d - 7*b**2*x**6*cosh(c + d*x)/d**2 + 42*b**2*x**5*sinh(c + d*x
)/d**3 - 210*b**2*x**4*cosh(c + d*x)/d**4 + 840*b**2*x**3*sinh(c + d*x)/d**
5 - 2520*b**2*x**2*cosh(c + d*x)/d**6 + 5040*b**2*x*sinh(c + d*x)/d**7 - 50
40*b**2*cosh(c + d*x)/d**8, Ne(d, 0)), ((a**2*x**2/2 + 2*a*b*x**5/5 + b**2*
x**8/8)*cosh(c), True))
```

Giac [A]
time = 0.40, size = 303, normalized size = 1.29

$$\frac{\left(b^{2} d^{7} x^{7}-7 b^{2} d^{6} x^{6}+2 a b d^{7} x^{4}+42 b^{2} d^{5} x^{5}-8 a b d^{6} x^{3}+a^{2} d^{7} x-210 b^{2} d^{4} x^{4}+24 a b d^{5} x^{2}-a^{2} d^{6}+840 b^{2} d^{3} x^{3}-48 a b d^{4} x-2520 b^{2} d^{2} x^{2}+48 a b d^{3}+5040 b^{2} d x-5040 b^{2}\right) e^{\left(d x+c\right)}}{2 d^{8}}-\frac{\left(b^{2} d^{7} x^{7}+7 b^{2} d^{6} x^{6}+2 a b d^{7} x^{4}+42 b^{2} d^{5} x^{5}+8 a b d^{6} x^{3}+a^{2} d^{7} x+210 b^{2} d^{4} x^{4}+24 a b d^{5} x^{2}+a^{2} d^{6}+840 b^{2} d^{3} x^{3}+48 a b d^{4} x+2520 b^{2} d^{2} x^{2}+48 a b d^{3}+5040 b^{2} d x+5040 b^{2}\right) e^{\left(-d x-c\right)}}{2 d^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^7*x^7 - 7*b^2*d^6*x^6 + 2*a*b*d^7*x^4 + 42*b^2*d^5*x^5 - 8*a*b*d
^6*x^3 + a^2*d^7*x - 210*b^2*d^4*x^4 + 24*a*b*d^5*x^2 - a^2*d^6 + 840*b^2*d
^3*x^3 - 48*a*b*d^4*x - 2520*b^2*d^2*x^2 + 48*a*b*d^3 + 5040*b^2*d*x - 5040
*b^2)*e^(d*x + c)/d^8 - 1/2*(b^2*d^7*x^7 + 7*b^2*d^6*x^6 + 2*a*b*d^7*x^4 +
42*b^2*d^5*x^5 + 8*a*b*d^6*x^3 + a^2*d^7*x + 210*b^2*d^4*x^4 + 24*a*b*d^5*x
^2 + a^2*d^6 + 840*b^2*d^3*x^3 + 48*a*b*d^4*x + 2520*b^2*d^2*x^2 + 48*a*b*d
^3 + 5040*b^2*d*x + 5040*b^2)*e^(-d*x - c)/d^8
```

Mupad [B]
time = 0.35, size = 301, normalized size = 1.29

$$-e^{c+dx}\left(\frac{a^2 d^6-48 a b d^3+5040 b^2}{2 d^8}-\frac{x\left(a^2 d^7-48 a b d^4+5040 b^2 d\right)}{2 d^8}-\frac{b^2 x^7}{2 d}+\frac{7 b^2 x^6}{2 d^2}-\frac{21 b^2 x^5}{d^3}+\frac{b x^4\left(105 b-a d^3\right)}{d^4}-\frac{4 b x^3\left(105 b-a d^3\right)}{d^5}+\frac{12 b x^2\left(105 b-a d^3\right)}{d^6}\right)-e^{-c-dx}\left(\frac{a^2 d^6+48 a b d^3+5040 b^2}{2 d^8}+\frac{x\left(a^2 d^7+48 a b d^4+5040 b^2 d\right)}{2 d^8}+\frac{b^2 x^7}{2 d}+\frac{7 b^2 x^6}{2 d^2}+\frac{21 b^2 x^5}{d^3}+\frac{b x^4\left(a d^3+105 b\right)}{d^4}+\frac{4 b x^3\left(a d^3+105 b\right)}{d^5}+\frac{12 b x^2\left(a d^3+105 b\right)}{d^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(c + d*x)*(a + b*x^3)^2,x)
```

```
[Out] - exp(c + d*x)*((5040*b^2 + a^2*d^6 - 48*a*b*d^3)/(2*d^8) - (x*(5040*b^2*d
+ a^2*d^7 - 48*a*b*d^4))/(2*d^8) - (b^2*x^7)/(2*d) + (7*b^2*x^6)/(2*d^2) -
(21*b^2*x^5)/d^3 + (b*x^4*(105*b - a*d^3))/d^4 - (4*b*x^3*(105*b - a*d^3))/
d^5 + (12*b*x^2*(105*b - a*d^3))/d^6) - exp(- c - d*x)*((5040*b^2 + a^2*d^6
 + 48*a*b*d^3)/(2*d^8) + (x*(5040*b^2*d + a^2*d^7 + 48*a*b*d^4))/(2*d^8) +
(b^2*x^7)/(2*d) + (7*b^2*x^6)/(2*d^2) + (21*b^2*x^5)/d^3 + (b*x^4*(105*b +
a*d^3))/d^4 + (4*b*x^3*(105*b + a*d^3))/d^5 + (12*b*x^2*(105*b + a*d^3))/d^
6)
```

3.88 $\int (a+bx^3)^2 \cosh(c+dx)\,dx$

Optimal. Leaf size=186

$$-\frac{12ab\cosh(c+dx)}{d^4}-\frac{720b^2x\cosh(c+dx)}{d^6}-\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{120b^2x^3\cosh(c+dx)}{d^4}-\frac{6b^2x^5\cosh(c+dx)}{d^2}-$$

```
[Out] -12*a*b*cosh(d*x+c)/d^4-720*b^2*x*cosh(d*x+c)/d^6-6*a*b*x^2*cosh(d*x+c)/d^2
-120*b^2*x^3*cosh(d*x+c)/d^4-6*b^2*x^5*cosh(d*x+c)/d^2+720*b^2*sinh(d*x+c)/
d^7+a^2*sinh(d*x+c)/d+12*a*b*x*sinh(d*x+c)/d^3+360*b^2*x^2*sinh(d*x+c)/d^5+
2*a*b*x^3*sinh(d*x+c)/d+30*b^2*x^4*sinh(d*x+c)/d^3+b^2*x^6*sinh(d*x+c)/d
```

Rubi [A]

time = 0.21, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5385, 2717, 3377, 2718}

$$\frac{a^2\sinh(c+dx)}{d}-\frac{12ab\cosh(c+dx)}{d^4}+\frac{12abx\sinh(c+dx)}{d^3}-\frac{6abx^2\cosh(c+dx)}{d^2}+\frac{2abx^3\sinh(c+dx)}{d}+\frac{720b^2\sinh(c+dx)}{d^7}-\frac{720b^2x\cosh(c+dx)}{d^6}+\frac{360b^2x^2\sinh(c+dx)}{d^5}-\frac{120b^2x^3\cosh(c+dx)}{d^4}+\frac{30b^2x^4\sinh(c+dx)}{d^3}-\frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{b^2x^6\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*Cosh[c + d*x],x]
```

```
[Out] (-12*a*b*Cosh[c + d*x])/d^4 - (720*b^2*x*Cosh[c + d*x])/d^6 - (6*a*b*x^2*Co
sh[c + d*x])/d^2 - (120*b^2*x^3*Cosh[c + d*x])/d^4 - (6*b^2*x^5*Cosh[c + d*
x])/d^2 + (720*b^2*Sinh[c + d*x])/d^7 + (a^2*Sinh[c + d*x])/d + (12*a*b*x*S
inh[c + d*x])/d^3 + (360*b^2*x^2*Sinh[c + d*x])/d^5 + (2*a*b*x^3*Sinh[c + d
*x])/d + (30*b^2*x^4*Sinh[c + d*x])/d^3 + (b^2*x^6*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5385

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c,
```

```
d, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^2 \cosh(c+dx)\,dx = \int \left(a^2\cosh(c+dx)+2abx^3\cosh(c+dx)+b^2x^6\cosh(c+dx)\right)\,dx$$
$$= a^2\int \cosh(c+dx)\,dx+(2ab)\int x^3\cosh(c+dx)\,dx+b^2\int x^6\cosh(c+dx)$$
$$= \frac{a^2\sinh(c+dx)}{d}+\frac{2abx^3\sinh(c+dx)}{d}+\frac{b^2x^6\sinh(c+dx)}{d}-\frac{(6ab)\int x^2\sin}{d}$$
$$= -\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{a^2\sinh(c+dx)}{d}+\frac{2abx^3\sinh}{d}$$
$$= -\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{a^2\sinh(c+dx)}{d}+\frac{12abx\sinh}{d}$$
$$= -\frac{12ab\cosh(c+dx)}{d^4}-\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{120b^2x^3\cosh(c+dx)}{d^4}-6b^2x$$
$$= -\frac{12ab\cosh(c+dx)}{d^4}-\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{120b^2x^3\cosh(c+dx)}{d^4}-6b^2x$$
$$= -\frac{12ab\cosh(c+dx)}{d^4}-\frac{720b^2x\cosh(c+dx)}{d^6}-\frac{6abx^2\cosh(c+dx)}{d^2}-120b^2$$
$$= -\frac{12ab\cosh(c+dx)}{d^4}-\frac{720b^2x\cosh(c+dx)}{d^6}-\frac{6abx^2\cosh(c+dx)}{d^2}-120b^2$$

Mathematica [A]
time = 0.18, size = 111, normalized size = 0.60

$$\frac{-6bd(ad^2(2+d^2x^2)+bx(120+20d^2x^2+d^4x^4))\cosh(c+dx)+(a^2d^6+2abd^4x(6+d^2x^2)+b^2(720+360d^2x^2+30d^4x^4+d^6x^6))\sinh(c+dx)}{d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*Cosh[c + d*x],x]
```

```
[Out] (-6*b*d*(a*d^2*(2 + d^2*x^2) + b*x*(120 + 20*d^2*x^2 + d^4*x^4))*Cosh[c + d
*x] + (a^2*d^6 + 2*a*b*d^4*x*(6 + d^2*x^2) + b^2*(720 + 360*d^2*x^2 + 30*d^
4*x^4 + d^6*x^6))*Sinh[c + d*x])/d^7
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 591 vs. 2(186) = 372.
time = 0.62, size = 592, normalized size = 3.18 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*sinh(d*x+c)+15*b^2/d^6*c^2*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh
(d*x+c)+12*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))-2*b
/d^3*c^3*a*sinh(d*x+c)-6*b^2/d^6*c*((d*x+c)^5*sinh(d*x+c)-5*(d*x+c)^4*cosh(
d*x+c)+20*(d*x+c)^3*sinh(d*x+c)-60*(d*x+c)^2*cosh(d*x+c)+120*(d*x+c)*sinh(d
*x+c)-120*cosh(d*x+c))+15*b^2/d^6*c^4*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh
(d*x+c)+2*sinh(d*x+c))-20*b^2/d^6*c^3*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*co
sh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))-6*b^2/d^6*c^5*((d*x+c)*sinh(
d*x+c)-cosh(d*x+c))+6*b/d^3*c^2*a*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-6*b/d^3
*c*a*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+b^2/d^6*((
d*x+c)^6*sinh(d*x+c)-6*(d*x+c)^5*cosh(d*x+c)+30*(d*x+c)^4*sinh(d*x+c)-120*(
d*x+c)^3*cosh(d*x+c)+360*(d*x+c)^2*sinh(d*x+c)-720*(d*x+c)*cosh(d*x+c)+720*
sinh(d*x+c))+b^2/d^6*c^6*sinh(d*x+c)+2*b/d^3*a*((d*x+c)^3*sinh(d*x+c)-3*(d*
x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c)))
```

Maxima [A]
time = 0.29, size = 243, normalized size = 1.31

$$\frac{a^2e^{(dx+c)}}{2\,d} - \frac{a^2e^{(-dx-c)}}{2\,d} + \frac{(d^3x^3e^c - 3\,d^2x^2e^c + 6\,dxe^c - 6\,e^c)abe^{(dx)}}{d^4} - \frac{(d^3x^3 + 3\,d^2x^2 + 6\,dx + 6)abe^{(-dx-c)}}{d^4} + \frac{(d^6x^6e^c - 6\,d^5x^5e^c + 30\,d^4x^4e^c - 120\,d^3x^3e^c + 360\,d^2x^2e^c - 720\,dxe^c + 720\,e^c)b^2e^{(dx)}}{2\,d^7} - \frac{(d^6x^6 + 6\,d^5x^5 + 30\,d^4x^4 + 120\,d^3x^3 + 360\,d^2x^2 + 720\,dx + 720)b^2e^{(-dx-c)}}{2\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*e^(d*x + c)/d - 1/2*a^2*e^(-d*x - c)/d + (d^3*x^3*e^c - 3*d^2*x^2*e
^c + 6*d*x*e^c - 6*e^c)*a*b*e^(d*x)/d^4 - (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)
*a*b*e^(-d*x - c)/d^4 + 1/2*(d^6*x^6*e^c - 6*d^5*x^5*e^c + 30*d^4*x^4*e^c -
 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*d*x*e^c + 720*e^c)*b^2*e^(d*x)/d^7
 - 1/2*(d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 120*d^3*x^3 + 360*d^2*x^2 + 720*
d*x + 720)*b^2*e^(-d*x - c)/d^7
```

Fricas [A]
time = 0.40, size = 130, normalized size = 0.70

$$-\frac{6\,(b^2d^5x^5 + abd^5x^2 + 20\,b^2d^3x^3 + 2\,abd^3 + 120\,b^2dx)\cosh(dx+c) - (b^2d^6x^6 + 2\,abd^6x^3 + 30\,b^2d^4x^4 + a^2d^6 + 12\,abd^4x + 360\,b^2d^2x^2 + 720\,b^2)\sinh(dx+c)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(6*(b^2*d^5*x^5 + a*b*d^5*x^2 + 20*b^2*d^3*x^3 + 2*a*b*d^3 + 120*b^2*d*x)*
cosh(d*x + c) - (b^2*d^6*x^6 + 2*a*b*d^6*x^3 + 30*b^2*d^4*x^4 + a^2*d^6 + 1
2*a*b*d^4*x + 360*b^2*d^2*x^2 + 720*b^2)*sinh(d*x + c))/d^7
```

Sympy [A]
time = 0.69, size = 226, normalized size = 1.22

$$\begin{cases} \frac{a^2\sinh(c+dx)}{d} + \frac{2abx^3\sinh(c+dx)}{d} - \frac{6abx^2\cosh(c+dx)}{d^2} + \frac{12abx\sinh(c+dx)}{d^3} - \frac{12ab\cosh(c+dx)}{d^4} + \frac{b^2x^6\sinh(c+dx)}{d} - \frac{6b^2x^5\cosh(c+dx)}{d^2} + \frac{30b^2x^4\sinh(c+dx)}{d^3} - \frac{120b^2x^3\cosh(c+dx)}{d^4} + \frac{360b^2x^2\sinh(c+dx)}{d^5} - \frac{720b^2x\cosh(c+dx)}{d^6} + \frac{720b^2\sinh(c+dx)}{d^7} & \text{for}\ d \neq 0 \\ \left(a^2x + \frac{abx^4}{2} + \frac{b^2x^7}{7}\right)\cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*x**3*sinh(c + d*x)/d - 6*a*b*x**2*c
osh(c + d*x)/d**2 + 12*a*b*x*sinh(c + d*x)/d**3 - 12*a*b*cosh(c + d*x)/d**4
 + b**2*x**6*sinh(c + d*x)/d - 6*b**2*x**5*cosh(c + d*x)/d**2 + 30*b**2*x**
4*sinh(c + d*x)/d**3 - 120*b**2*x**3*cosh(c + d*x)/d**4 + 360*b**2*x**2*sin
h(c + d*x)/d**5 - 720*b**2*x*cosh(c + d*x)/d**6 + 720*b**2*sinh(c + d*x)/d*
*7, Ne(d, 0)), ((a**2*x + a*b*x**4/2 + b**2*x**7/7)*cosh(c), True))
```

Giac [A]
time = 0.40, size = 244, normalized size = 1.31

$$\frac{(b^2d^6x^6 - 6\,b^2d^5x^5 + 2\,abd^6x^3 + 30\,b^2d^4x^4 - 6\,abd^5x^2 + a^2d^6 - 120\,b^2d^3x^3 + 12\,abd^4x + 360\,b^2d^2x^2 - 12\,abd^3 - 720\,b^2dx + 720\,b^2)e^{(dx+c)}}{2\,d^7} - \frac{(b^2d^6x^6 + 6\,b^2d^5x^5 + 2\,abd^6x^3 + 30\,b^2d^4x^4 + 6\,abd^5x^2 + a^2d^6 + 120\,b^2d^3x^3 + 12\,abd^4x + 360\,b^2d^2x^2 + 12\,abd^3 + 720\,b^2dx + 720\,b^2)e^{(-dx-c)}}{2\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^6*x^6 - 6*b^2*d^5*x^5 + 2*a*b*d^6*x^3 + 30*b^2*d^4*x^4 - 6*a*b*d
^5*x^2 + a^2*d^6 - 120*b^2*d^3*x^3 + 12*a*b*d^4*x + 360*b^2*d^2*x^2 - 12*a*
b*d^3 - 720*b^2*d*x + 720*b^2)*e^(d*x + c)/d^7 - 1/2*(b^2*d^6*x^6 + 6*b^2*d
^5*x^5 + 2*a*b*d^6*x^3 + 30*b^2*d^4*x^4 + 6*a*b*d^5*x^2 + a^2*d^6 + 120*b^2
*d^3*x^3 + 12*a*b*d^4*x + 360*b^2*d^2*x^2 + 12*a*b*d^3 + 720*b^2*d*x + 720*
b^2)*e^(-d*x - c)/d^7
```

Mupad [B]
time = 0.24, size = 182, normalized size = 0.98

$$\frac{\sinh(c+d\,x)\,\left(a^2\,d^6+720\,b^2\right)}{d^7} - \frac{6\,b^2\,x^5\cosh(c+d\,x)}{d^2} - \frac{120\,b^2\,x^3\cosh(c+d\,x)}{d^4} + \frac{b^2\,x^6\sinh(c+d\,x)}{d} + \frac{30\,b^2\,x^4\sinh(c+d\,x)}{d^3} + \frac{360\,b^2\,x^2\sinh(c+d\,x)}{d^5} - \frac{12\,a\,b\cosh(c+d\,x)}{d^4} - \frac{720\,b^2\,x\cosh(c+d\,x)}{d^6} - \frac{6\,a\,b\,x^2\cosh(c+d\,x)}{d^2} + \frac{2\,a\,b\,x^3\sinh(c+d\,x)}{d} + \frac{12\,a\,b\,x\sinh(c+d\,x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b*x^3)^2,x)
```

```
[Out] (sinh(c + d*x)*(720*b^2 + a^2*d^6))/d^7 - (6*b^2*x^5*cosh(c + d*x))/d^2 - (
120*b^2*x^3*cosh(c + d*x))/d^4 + (b^2*x^6*sinh(c + d*x))/d + (30*b^2*x^4*si
nh(c + d*x))/d^3 + (360*b^2*x^2*sinh(c + d*x))/d^5 - (12*a*b*cosh(c + d*x))
/d^4 - (720*b^2*x*cosh(c + d*x))/d^6 - (6*a*b*x^2*cosh(c + d*x))/d^2 + (2*a
*b*x^3*sinh(c + d*x))/d + (12*a*b*x*sinh(c + d*x))/d^3
```

3.89 $\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x} \, dx$

Optimal. Leaf size=160

$$-\frac{120b^2\cosh(c+dx)}{d^6}-\frac{4abx\cosh(c+dx)}{d^2}-\frac{60b^2x^2\cosh(c+dx)}{d^4}-\frac{5b^2x^4\cosh(c+dx)}{d^2}+a^2\cosh(c)\text{Chi}(dx)+\frac{4a}{}$$

```
[Out] a^2*Chi(d*x)*cosh(c)-120*b^2*cosh(d*x+c)/d^6-4*a*b*x*cosh(d*x+c)/d^2-60*b^2
*x^2*cosh(d*x+c)/d^4-5*b^2*x^4*cosh(d*x+c)/d^2+a^2*Shi(d*x)*sinh(c)+4*a*b*s
inh(d*x+c)/d^3+120*b^2*x*sinh(d*x+c)/d^5+2*a*b*x^2*sinh(d*x+c)/d+20*b^2*x^3
*sinh(d*x+c)/d^3+b^2*x^5*sinh(d*x+c)/d
```

Rubi [A]

time = 0.21, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5395, 3384, 3379, 3382, 3377, 2717, 2718}

$$a^2\cosh(c)\text{Chi}(dx)+a^2\sinh(c)\text{Shi}(dx)+\frac{4ab\sinh(c+dx)}{d^3}-\frac{4abx\cosh(c+dx)}{d^2}+\frac{2abx^2\sinh(c+dx)}{d}-\frac{120b^2\cosh(c+dx)}{d^6}+\frac{120b^2x\sinh(c+dx)}{d^5}-\frac{60b^2x^2\cosh(c+dx)}{d^4}+\frac{20b^2x^3\sinh(c+dx)}{d^3}-\frac{5b^2x^4\cosh(c+dx)}{d^2}+\frac{b^2x^5\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Cosh[c + d*x])/x,x]
```

```
[Out] (-120*b^2*Cosh[c + d*x])/d^6 - (4*a*b*x*Cosh[c + d*x])/d^2 - (60*b^2*x^2*Co
sh[c + d*x])/d^4 - (5*b^2*x^4*Cosh[c + d*x])/d^2 + a^2*Cosh[c]*CoshIntegral
[d*x] + (4*a*b*Sinh[c + d*x])/d^3 + (120*b^2*x*Sinh[c + d*x])/d^5 + (2*a*b*
x^2*Sinh[c + d*x])/d + (20*b^2*x^3*Sinh[c + d*x])/d^3 + (b^2*x^5*Sinh[c + d
*x])/d + a^2*Sinh[c]*SinhIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x}\,dx = \int \left(\frac{a^2\cosh(c+dx)}{x} + 2abx^2\cosh(c+dx) + b^2x^5\cosh(c+dx)\right)dx$$

$$= a^2\int \frac{\cosh(c+dx)}{x}\,dx + (2ab)\int x^2\cosh(c+dx)\,dx + b^2\int x^5\cosh(c+dx$$

$$= \frac{2abx^2\sinh(c+dx)}{d} + \frac{b^2x^5\sinh(c+dx)}{d} - \frac{(4ab)\int x\sinh(c+dx)\,dx}{d} - \frac{(5b$$

$$= -\frac{4abx\cosh(c+dx)}{d^2} - \frac{5b^2x^4\cosh(c+dx)}{d^2} + a^2\cosh(c)\mathrm{Chi}(dx) + \frac{2abx^2\sin$$

$$= -\frac{4abx\cosh(c+dx)}{d^2} - \frac{5b^2x^4\cosh(c+dx)}{d^2} + a^2\cosh(c)\mathrm{Chi}(dx) + \frac{4ab\sinh}{d$$

$$= -\frac{4abx\cosh(c+dx)}{d^2} - \frac{60b^2x^2\cosh(c+dx)}{d^4} - \frac{5b^2x^4\cosh(c+dx)}{d^2} + a^2\cos$$

$$= -\frac{4abx\cosh(c+dx)}{d^2} - \frac{60b^2x^2\cosh(c+dx)}{d^4} - \frac{5b^2x^4\cosh(c+dx)}{d^2} + a^2\cos$$

$$= -\frac{120b^2\cosh(c+dx)}{d^6} - \frac{4abx\cosh(c+dx)}{d^2} - \frac{60b^2x^2\cosh(c+dx)}{d^4} - \frac{5b^2x^4}{}$$

Mathematica [A]
time = 0.35, size = 108, normalized size = 0.68

$$-\frac{b(4ad^4x+5b(24+12d^2x^2+d^4x^4))\cosh(c+dx)}{d^6}+a^2\cosh(c)\text{Chi}(dx)+\frac{b(2ad^2(2+d^2x^2)+bx(120+20d^2x^2+d^4x^4))\sinh(c+dx)}{d^5}+a^2\sinh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x,x]
```

```
[Out] -((b*(4*a*d^4*x + 5*b*(24 + 12*d^2*x^2 + d^4*x^4))*Cosh[c + d*x])/d^6) + a^
2*Cosh[c]*CoshIntegral[d*x] + (b*(2*a*d^2*(2 + d^2*x^2) + b*x*(120 + 20*d^2
*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5 + a^2*Sinh[c]*SinhIntegral[d*x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(160) = 320$.
time = 0.86, size = 335, normalized size = 2.09

method	result
risch	$\frac{ab\,\mathrm{e}^{dx+c}x^2}{d}-\frac{2ab\,\mathrm{e}^{dx+c}x}{d^2}-\frac{a^2\mathrm{e}^{-c}\,\text{expIntegral}(1,dx)}{2}-\frac{a^2\mathrm{e}^{c}\,\text{expIntegral}(1,-dx)}{2}-\frac{10b^2\mathrm{e}^{-dx-c}x^3}{d^3}-\frac{30b^2\mathrm{e}^{-dx-c}x^2}{d^4}-\frac{5b^2\mathrm{e}^{-}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*cosh(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a*b*exp(d*x+c)*x^2-2/d^2*a*b*exp(d*x+c)*x-1/2*a^2*exp(-c)*Ei(1,d*x)-1/2
*a^2*exp(c)*Ei(1,-d*x)-10/d^3*b^2*exp(-d*x-c)*x^3-30/d^4*b^2*exp(-d*x-c)*x^
2-5/2/d^2*b^2*exp(-d*x-c)*x^4-30/d^4*b^2*exp(d*x+c)*x^2+60/d^5*b^2*exp(d*x+
c)*x-2/d^2*a*b*exp(-d*x-c)*x-1/d*a*b*exp(-d*x-c)*x^2-60/d^5*b^2*exp(-d*x-c)
*x+1/2/d*b^2*exp(d*x+c)*x^5-5/2/d^2*b^2*exp(d*x+c)*x^4+10/d^3*b^2*exp(d*x+c
)*x^3-60/d^6*b^2*exp(d*x+c)-60/d^6*b^2*exp(-d*x-c)-2/d^3*a*b*exp(-d*x-c)-1/
2/d*b^2*exp(-d*x-c)*x^5+2/d^3*a*b*exp(d*x+c)
```

Maxima [A]
time = 0.32, size = 289, normalized size = 1.81

$$-\frac{1}{12}\left(4\,ab\left(\frac{(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c)e^{(dx)}}{d^4}+\frac{(d^3x^3+3\,d^2x^2+6\,dx+6)e^{(-dx-c)}}{d^4}\right)+b^2\left(\frac{(d^6x^6e^c-6\,d^5x^5e^c+30\,d^4x^4e^c-120\,d^3x^3e^c+360\,d^2x^2e^c-720\,dxe^c+720\,e^c)e^{(dx)}}{d^7}+\frac{(d^6x^6+6\,d^5x^5+30\,d^4x^4+120\,d^3x^3+360\,d^2x^2+720\,dx+720)e^{(-dx-c)}}{d^7}\right)+\frac{4\,a^2\cosh(dx+c)\log(x^3)}{d}-\frac{6\left(\text{Ei}(-dx)\,e^{(-c)}+\text{Ei}(dx)\,e^c\right)a^2}{d}\right)d+\frac{1}{6}\left(b^2x^6+4\,abx^3+2\,a^2\log(x^3)\right)\cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/12*(4*a*b*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4
 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4) + b^2*((d^6*x^6*e^c
- 6*d^5*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*
d*x*e^c + 720*e^c)*e^(d*x)/d^7 + (d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 120*d^
3*x^3 + 360*d^2*x^2 + 720*d*x + 720)*e^(-d*x - c)/d^7) + 4*a^2*cosh(d*x + c
)*log(x^3)/d - 6*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*a^2/d)*d + 1/6*(b^2*x^6 +
4*a*b*x^3 + 2*a^2*log(x^3))*cosh(d*x + c)
```

Fricas [A]
time = 0.37, size = 161, normalized size = 1.01

$$-\frac{2\,(5\,b^2d^4x^4+4\,abd^4x+60\,b^2d^2x^2+120\,b^2)\cosh\left(dx+c\right)-(a^2d^6\mathrm{Ei}(dx)+a^2d^6\mathrm{Ei}(-dx))\cosh\left(c\right)-2\,(b^2d^5x^5+2\,abd^5x^2+20\,b^2d^3x^3+4\,abd^3+120\,b^2dx)\sinh\left(dx+c\right)-(a^2d^6\mathrm{Ei}(dx)-a^2d^6\mathrm{Ei}(-dx))\sinh\left(c\right)}{2\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(5*b^2*d^4*x^4 + 4*a*b*d^4*x + 60*b^2*d^2*x^2 + 120*b^2)*cosh(d*x +
 c) - (a^2*d^6*Ei(d*x) + a^2*d^6*Ei(-d*x))*cosh(c) - 2*(b^2*d^5*x^5 + 2*a*b
*d^5*x^2 + 20*b^2*d^3*x^3 + 4*a*b*d^3 + 120*b^2*d*x)*sinh(d*x + c) - (a^2*d
^6*Ei(d*x) - a^2*d^6*Ei(-d*x))*sinh(c))/d^6
```

Sympy [A]
time = 3.19, size = 168, normalized size = 1.05

$$a^2\sinh\left(c\right)\mathrm{Shi}\left(dx\right)+a^2\cosh\left(c\right)\mathrm{Chi}\left(dx\right)+2ab\left(\begin{cases}\frac{x^2\sinh\left(c+dx\right)}{d}-\frac{2x\cosh\left(c+dx\right)}{d^2}+\frac{2\sinh\left(c+dx\right)}{d^3} & \text{for}\; d\neq 0\\ \frac{x^3\cosh\left(c\right)}{3} & \text{otherwise}\end{cases}\right)+b^2\left(\begin{cases}\frac{x^5\sinh\left(c+dx\right)}{d}-\frac{5x^4\cosh\left(c+dx\right)}{d^2}+\frac{20x^3\sinh\left(c+dx\right)}{d^3}-\frac{60x^2\cosh\left(c+dx\right)}{d^4}+\frac{120x\sinh\left(c+dx\right)}{d^5}-\frac{120\cosh\left(c+dx\right)}{d^6} & \text{for}\; d\neq 0\\ \frac{x^6\cosh\left(c\right)}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x,x)
```

```
[Out] a**2*sinh(c)*Shi(d*x) + a**2*cosh(c)*Chi(d*x) + 2*a*b*Piecewise((x**2*sinh(
c + d*x)/d - 2*x*cosh(c + d*x)/d**2 + 2*sinh(c + d*x)/d**3, Ne(d, 0)), (x**
3*cosh(c)/3, True)) + b**2*Piecewise((x**5*sinh(c + d*x)/d - 5*x**4*cosh(c
+ d*x)/d**2 + 20*x**3*sinh(c + d*x)/d**3 - 60*x**2*cosh(c + d*x)/d**4 + 120
*x*sinh(c + d*x)/d**5 - 120*cosh(c + d*x)/d**6, Ne(d, 0)), (x**6*cosh(c)/6,
 True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(160) = 320.
time = 0.41, size = 331, normalized size = 2.07

$$\frac{b^2d^5x^5e^{(dx+c)}-b^2d^5x^5e^{(-dx-c)}-5\,b^2d^4x^4e^{(dx+c)}-5\,b^2d^4x^4e^{(-dx-c)}+2\,abd^5x^2e^{(dx+c)}-2\,abd^5x^2e^{(-dx-c)}+a^2d^6\mathrm{Ei}(-dx)\,e^{(-c)}+a^2d^6\mathrm{Ei}(dx)\,e^c+20\,b^2d^3x^3e^{(dx+c)}-20\,b^2d^3x^3e^{(-dx-c)}-4\,abd^4xe^{(dx+c)}-4\,abd^4xe^{(-dx-c)}-60\,b^2d^2x^2e^{(dx+c)}-60\,b^2d^2x^2e^{(-dx-c)}+4\,abd^3e^{(dx+c)}-4\,abd^3e^{(-dx-c)}+120\,b^2dxe^{(dx+c)}-120\,b^2dxe^{(-dx-c)}-120\,b^2e^{(dx+c)}-120\,b^2e^{(-dx-c)}}{2\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^5*x^5*e^(d*x + c) - b^2*d^5*x^5*e^(-d*x - c) - 5*b^2*d^4*x^4*e^(
d*x + c) - 5*b^2*d^4*x^4*e^(-d*x - c) + 2*a*b*d^5*x^2*e^(d*x + c) - 2*a*b*d
^5*x^2*e^(-d*x - c) + a^2*d^6*Ei(-d*x)*e^(-c) + a^2*d^6*Ei(d*x)*e^c + 20*b^
2*d^3*x^3*e^(d*x + c) - 20*b^2*d^3*x^3*e^(-d*x - c) - 4*a*b*d^4*x*e^(d*x +
c) - 4*a*b*d^4*x*e^(-d*x - c) - 60*b^2*d^2*x^2*e^(d*x + c) - 60*b^2*d^2*x^2
*e^(-d*x - c) + 4*a*b*d^3*e^(d*x + c) - 4*a*b*d^3*e^(-d*x - c) + 120*b^2*d*
x*e^(d*x + c) - 120*b^2*d*x*e^(-d*x - c) - 120*b^2*e^(d*x + c) - 120*b^2*e^
(-d*x - c))/d^6
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + d\,x)\,\left(b\,x^3 + a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^3)^2)/x,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^3)^2)/x, x)
```

3.90 $\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^2}\,dx$

Optimal. Leaf size=143

$$-\frac{2ab\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{x}-\frac{24b^2x\cosh(c+dx)}{d^4}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+a^2d\text{Chi}(dx)\sinh(c)+\frac{24b^2\,\text{s}}{}$$

```
[Out] -2*a*b*cosh(d*x+c)/d^2-a^2*cosh(d*x+c)/x-24*b^2*x*cosh(d*x+c)/d^4-4*b^2*x^3
*cosh(d*x+c)/d^2+a^2*d*cosh(c)*Shi(d*x)+a^2*d*Chi(d*x)*sinh(c)+24*b^2*sinh(
d*x+c)/d^5+2*a*b*x*sinh(d*x+c)/d+12*b^2*x^2*sinh(d*x+c)/d^3+b^2*x^4*sinh(d*
x+c)/d
```

Rubi [A]
time = 0.20, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {5395, 3378, 3384, 3379, 3382, 3377, 2718, 2717}

$$a^2d\sinh(c)\text{Chi}(dx)+a^2d\cosh(c)\text{Shi}(dx)-\frac{a^2\cosh(c+dx)}{x}-\frac{2ab\cosh(c+dx)}{d^2}+\frac{2abx\sinh(c+dx)}{d}+\frac{24b^2\sinh(c+dx)}{d^5}-\frac{24b^2x\cosh(c+dx)}{d^4}+\frac{12b^2x^2\sinh(c+dx)}{d^3}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+\frac{b^2x^4\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Cosh[c + d*x])/x^2,x]
```

```
[Out] (-2*a*b*Cosh[c + d*x])/d^2 - (a^2*Cosh[c + d*x])/x - (24*b^2*x*Cosh[c + d*x
])/d^4 - (4*b^2*x^3*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral[d*x]*Sinh[c] +
(24*b^2*Sinh[c + d*x])/d^5 + (2*a*b*x*Sinh[c + d*x])/d + (12*b^2*x^2*Sinh[c
 + d*x])/d^3 + (b^2*x^4*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2}\,dx = \int \left(\frac{a^2\cosh(c+dx)}{x^2} + 2abx\cosh(c+dx) + b^2x^4\cosh(c+dx)\right)dx$$

$$= a^2\int \frac{\cosh(c+dx)}{x^2}\,dx + (2ab)\int x\cosh(c+dx)\,dx + b^2\int x^4\cosh(c+dx)$$

$$= -\frac{a^2\cosh(c+dx)}{x} + \frac{2abx\sinh(c+dx)}{d} + \frac{b^2x^4\sinh(c+dx)}{d} - \frac{(2ab)\int \sinh}{d}$$

$$= -\frac{2ab\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{x} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + \frac{2abx\sinh(c}{d}$$

$$= -\frac{2ab\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{x} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + a^2d\text{Chi}(dx)$$

$$= -\frac{2ab\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{x} - \frac{24b^2x\cosh(c+dx)}{d^4} - \frac{4b^2x^3\cosh(}{d^2}$$

$$= -\frac{2ab\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{x} - \frac{24b^2x\cosh(c+dx)}{d^4} - \frac{4b^2x^3\cosh(}{d^2}$$

Mathematica [A]
time = 0.23, size = 143, normalized size = 1.00

$$-\frac{2ab\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{x} - \frac{24b^2x\cosh(c+dx)}{d^4} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + a^2d\text{Chi}(dx)\sinh(c) + \frac{24b^2\sinh(c+dx)}{d^5} + \frac{2abx\sinh(c+dx)}{d} + \frac{12b^2x^2\sinh(c+dx)}{d^3} + \frac{b^2x^4\sinh(c+dx)}{d} + a^2d\cosh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^2,x]
```

```
[Out] (-2*a*b*Cosh[c + d*x])/d^2 - (a^2*Cosh[c + d*x])/x - (24*b^2*x*Cosh[c + d*x
])/d^4 - (4*b^2*x^3*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral[d*x]*Sinh[c] +
(24*b^2*Sinh[c + d*x])/d^5 + (2*a*b*x*Sinh[c + d*x])/d + (12*b^2*x^2*Sinh[c
 + d*x])/d^3 + (b^2*x^4*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(143) = 286$.
time = 0.89, size = 296, normalized size = 2.07

method	result
risch	$\frac{d\,a^2\mathrm{e}^{-c}\,\text{expIntegral}(1,dx)}{2} - \frac{2b^2\mathrm{e}^{-dx-c}x^3}{d^2} - \frac{6b^2\mathrm{e}^{-dx-c}x^2}{d^3} - \frac{b^2\mathrm{e}^{-dx-c}x^4}{2d} - \frac{ab\,\mathrm{e}^{-dx-c}x}{d} - \frac{12b^2\mathrm{e}^{-dx-c}x}{d^4} - \frac{12b^2\mathrm{e}^{-dx-c}}{d^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*cosh(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d*a^2*exp(-c)*Ei(1,d*x)-2/d^2*b^2*exp(-d*x-c)*x^3-6/d^3*b^2*exp(-d*x-c)
*x^2-1/2/d*b^2*exp(-d*x-c)*x^4-1/d*a*b*exp(-d*x-c)*x-12/d^4*b^2*exp(-d*x-c)
```

```
*x-12/d^5*b^2*exp(-d*x-c)-1/d^2*a*b*exp(-d*x-c)-1/2*a^2*exp(-d*x-c)/x+1/d*a
*b*exp(d*x+c)*x-1/2*d*a^2*exp(c)*Ei(1,-d*x)+6/d^3*b^2*exp(d*x+c)*x^2-12/d^4
*b^2*exp(d*x+c)*x+1/2/d*b^2*exp(d*x+c)*x^4-2/d^2*b^2*exp(d*x+c)*x^3+12/d^5*
b^2*exp(d*x+c)-1/d^2*a*b*exp(d*x+c)-1/2*a^2/x*exp(d*x+c)
```

Maxima [**A**]
time = 0.33, size = 235, normalized size = 1.64

$$-\frac{1}{10}\left(5\,a^2\mathrm{Ei}(-dx)\,e^{(-c)} - 5\,a^2\mathrm{Ei}(dx)\,e^c + \frac{5\,(d^2x^2e^c - 2\,dxe^c + 2\,e^c)abe^{(dx)}}{d^3} + \frac{5\,(d^2x^2 + 2\,dx + 2)abe^{(-dx-c)}}{d^3} + \frac{(d^5x^5e^c - 5\,d^4x^4e^c + 20\,d^3x^3e^c - 60\,d^2x^2e^c + 120\,dxe^c - 120\,e^c)b^2e^{(dx)}}{d^6} + \frac{(d^5x^5 + 5\,d^4x^4 + 20\,d^3x^3 + 60\,d^2x^2 + 120\,dx + 120)b^2e^{(-dx-c)}}{d^6}\right)d + \frac{1}{5}\left(b^2x^5 + 5\,abx^2 - \frac{5\,a^2}{x}\right)\cosh\left(dx + c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] -1/10*(5*a^2*Ei(-d*x)*e^(-c) - 5*a^2*Ei(d*x)*e^c + 5*(d^2*x^2*e^c - 2*d*x*e
^c + 2*e^c)*a*b*e^(d*x)/d^3 + 5*(d^2*x^2 + 2*d*x + 2)*a*b*e^(-d*x - c)/d^3
+ (d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*
e^c - 120*e^c)*b^2*e^(d*x)/d^6 + (d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2
*x^2 + 120*d*x + 120)*b^2*e^(-d*x - c)/d^6)*d + 1/5*(b^2*x^5 + 5*a*b*x^2 -
5*a^2/x)*cosh(d*x + c)
```

Fricas [**A**]
time = 0.41, size = 160, normalized size = 1.12

$$-\frac{2\,(4\,b^2d^3x^4 + a^2d^5 + 2\,abd^3x + 24\,b^2dx^2)\cosh\left(dx + c\right) - (a^2d^6x\mathrm{Ei}(dx) - a^2d^6x\mathrm{Ei}(-dx))\cosh\left(c\right) - 2\,(b^2d^4x^5 + 2\,abd^4x^2 + 12\,b^2d^2x^3 + 24\,b^2x)\sinh\left(dx + c\right) - (a^2d^6x\mathrm{Ei}(dx) + a^2d^6x\mathrm{Ei}(-dx))\sinh\left(c\right)}{2\,d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(4*b^2*d^3*x^4 + a^2*d^5 + 2*a*b*d^3*x + 24*b^2*d*x^2)*cosh(d*x + c
) - (a^2*d^6*x*Ei(d*x) - a^2*d^6*x*Ei(-d*x))*cosh(c) - 2*(b^2*d^4*x^5 + 2*a
*b*d^4*x^2 + 12*b^2*d^2*x^3 + 24*b^2*x)*sinh(d*x + c) - (a^2*d^6*x*Ei(d*x)
+ a^2*d^6*x*Ei(-d*x))*sinh(c))/(d^5*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^3\right)^2\cosh\left(c + dx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**3)**2*cosh(c + d*x)/x**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(143) = 286.
time = 0.41, size = 308, normalized size = 2.15

$$\frac{b^2d^4x^5e^{(dx+c)} - b^2d^4x^5e^{(-dx-c)} - a^2d^6x\mathrm{Ei}(-dx)\,e^{(-c)} + a^2d^6x\mathrm{Ei}(dx)\,e^c - 4\,b^2d^3x^4e^{(dx+c)} - 4\,b^2d^3x^4e^{(-dx-c)} + 2\,abd^4x^2e^{(dx+c)} - 2\,abd^4x^2e^{(-dx-c)} - a^2d^5e^{(dx+c)} + 12\,b^2d^2x^3e^{(dx+c)} - a^2d^5e^{(-dx-c)} - 12\,b^2d^2x^3e^{(-dx-c)} - 2\,abd^3xe^{(dx+c)} - 2\,abd^3xe^{(-dx-c)} - 24\,b^2dx^2e^{(dx+c)} - 24\,b^2dx^2e^{(-dx-c)} + 24\,b^2xe^{(dx+c)} - 24\,b^2xe^{(-dx-c)}}{2\,d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^4*x^5*e^(d*x + c) - b^2*d^4*x^5*e^(-d*x - c) - a^2*d^6*x*Ei(-d*x
)*e^(-c) + a^2*d^6*x*Ei(d*x)*e^c - 4*b^2*d^3*x^4*e^(d*x + c) - 4*b^2*d^3*x^
4*e^(-d*x - c) + 2*a*b*d^4*x^2*e^(d*x + c) - 2*a*b*d^4*x^2*e^(-d*x - c) - a
^2*d^5*e^(d*x + c) + 12*b^2*d^2*x^3*e^(d*x + c) - a^2*d^5*e^(-d*x - c) - 12
*b^2*d^2*x^3*e^(-d*x - c) - 2*a*b*d^3*x*e^(d*x + c) - 2*a*b*d^3*x*e^(-d*x -
 c) - 24*b^2*d*x^2*e^(d*x + c) - 24*b^2*d*x^2*e^(-d*x - c) + 24*b^2*x*e^(d*
x + c) - 24*b^2*x*e^(-d*x - c))/(d^5*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^3)^2)/x^2,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^3)^2)/x^2, x)
```

3.91 $\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^3}\,dx$

Optimal. Leaf size=141

$$-\frac{6b^2\cosh(c+dx)}{d^4}-\frac{a^2\cosh(c+dx)}{2x^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{1}{2}a^2d^2\cosh(c)\mathrm{Chi}(dx)+\frac{2ab\sinh(c+dx)}{d}-\frac{a^2d\sin$$

```
[Out] 1/2*a^2*d^2*Chi(d*x)*cosh(c)-6*b^2*cosh(d*x+c)/d^4-1/2*a^2*cosh(d*x+c)/x^2-
3*b^2*x^2*cosh(d*x+c)/d^2+1/2*a^2*d^2*Shi(d*x)*sinh(c)+2*a*b*sinh(d*x+c)/d-
1/2*a^2*d*sinh(d*x+c)/x+6*b^2*x*sinh(d*x+c)/d^3+b^2*x^3*sinh(d*x+c)/d
```

Rubi [A]
time = 0.16, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$,
Rules used = {5395, 2717, 3378, 3384, 3379, 3382, 3377, 2718}

$$\frac{1}{2}a^2d^2\cosh(c)\mathrm{Chi}(dx)+\frac{1}{2}a^2d^2\sinh(c)\mathrm{Shi}(dx)-\frac{a^2\cosh(c+dx)}{2x^2}-\frac{a^2d\sinh(c+dx)}{2x}+\frac{2ab\sinh(c+dx)}{d}-\frac{6b^2\cosh(c+dx)}{d^4}+\frac{6b^2x\sinh(c+dx)}{d^3}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{b^2x^3\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Cosh[c + d*x])/x^3,x]
```

```
[Out] (-6*b^2*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/(2*x^2) - (3*b^2*x^2*Cosh[
c + d*x])/d^2 + (a^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (2*a*b*Sinh[c + d*x
])/d - (a^2*d*Sinh[c + d*x])/(2*x) + (6*b^2*x*Sinh[c + d*x])/d^3 + (b^2*x^3
*Sinh[c + d*x])/d + (a^2*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^3}\,dx = \int \left(2ab\cosh(c+dx)+\frac{a^2\cosh(c+dx)}{x^3}+b^2x^3\cosh(c+dx)\right)\,dx$$

$$= a^2\int \frac{\cosh(c+dx)}{x^3}\,dx+(2ab)\int \cosh(c+dx)\,dx+b^2\int x^3\cosh(c+dx)\,dx$$

$$= -\frac{a^2\cosh(c+dx)}{2x^2}+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2x^3\sinh(c+dx)}{d}-\frac{(3b^2)\int x^2\sin\text{h}}{d}$$

$$= -\frac{a^2\cosh(c+dx)}{2x^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{2ab\sinh(c+dx)}{d}-\frac{a^2d\sinh(c+}{2x}$$

$$= -\frac{a^2\cosh(c+dx)}{2x^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{2ab\sinh(c+dx)}{d}-\frac{a^2d\sinh(c+}{2x}$$

$$= -\frac{6b^2\cosh(c+dx)}{d^4}-\frac{a^2\cosh(c+dx)}{2x^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{1}{2}a^2d^2\cosh($$

Mathematica [A]
time = 0.23, size = 136, normalized size = 0.96

$$\frac{1}{2}\left(-\frac{12b^2\cosh(c+dx)}{d^4}-\frac{a^2\cosh(c+dx)}{x^2}-\frac{6b^2x^2\cosh(c+dx)}{d^2}+a^2d^2\cosh(c)\text{Chi}(dx)+\frac{4ab\sinh(c+dx)}{d}-\frac{a^2d\sinh(c+dx)}{x}+\frac{12b^2x\sinh(c+dx)}{d^3}+\frac{2b^2x^3\sinh(c+dx)}{d}+a^2d^2\sinh(c)\text{Shi}(dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^3,x]
```

```
[Out] ((-12*b^2*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/x^2 - (6*b^2*x^2*Cosh[c
+ d*x])/d^2 + a^2*d^2*Cosh[c]*CoshIntegral[d*x] + (4*a*b*Sinh[c + d*x])/d -
 (a^2*d*Sinh[c + d*x])/x + (12*b^2*x*Sinh[c + d*x])/d^3 + (2*b^2*x^3*Sinh[c
 + d*x])/d + a^2*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Maple [A]
time = 0.90, size = 265, normalized size = 1.88

method	result
risch	$-\frac{d^2a^2\mathrm{e}^{-c}\,\text{expIntegral}(1,dx)}{4}-\frac{b^2\mathrm{e}^{-dx-c}x^3}{2d}-\frac{3b^2\mathrm{e}^{-dx-c}x^2}{2d^2}-\frac{3b^2\mathrm{e}^{-dx-c}x}{d^3}+\frac{d\,a^2\mathrm{e}^{-dx-c}}{4x}-\frac{a^2\mathrm{e}^{-dx-c}}{4x^2}-\frac{3b^2\mathrm{e}^{-dx-c}}{d^4}-$ a

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*cosh(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d^2*a^2*exp(-c)*Ei(1,d*x)-1/2/d*b^2*exp(-d*x-c)*x^3-3/2/d^2*b^2*exp(-d
*x-c)*x^2-3/d^3*b^2*exp(-d*x-c)*x+1/4*d*a^2*exp(-d*x-c)/x-1/4*a^2*exp(-d*x-
c)/x^2-3/d^4*b^2*exp(-d*x-c)-1/d*a*b*exp(-d*x-c)-1/4*d^2*a^2*exp(c)*Ei(1,-d
*x)-3/2/d^2*b^2*exp(d*x+c)*x^2+3/d^3*b^2*exp(d*x+c)*x+1/2/d*b^2*exp(d*x+c)*
x^3-3/d^4*b^2*exp(d*x+c)+a*b/d*exp(d*x+c)-1/4*a^2/x^2*exp(d*x+c)-1/4*d*a^2/
x*exp(d*x+c)
```

Maxima [A]
time = 0.32, size = 203, normalized size = 1.44

$$\frac{1}{8}\left(2\,a^2de^{(-c)}\Gamma(-1,dx)+2\,a^2de^c\Gamma(-1,-dx)-\frac{8\,(dxe^c-e^c)abe^{(dx)}}{d^2}-\frac{8\,(dx+1)abe^{(-dx-c)}}{d^2}-\frac{(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2e^c-24\,dxe^c+24\,e^c)b^2e^{(dx)}}{d^5}-\frac{(d^4x^4+4\,d^3x^3+12\,d^2x^2+24\,dx+24)b^2e^{(-dx-c)}}{d^5}\right)d+\frac{1}{4}\left(b^2x^4+8\,abx-\frac{2\,a^2}{x^2}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(2*a^2*d*e^(-c)*gamma(-1, d*x) + 2*a^2*d*e^c*gamma(-1, -d*x) - 8*(d*x*e
^c - e^c)*a*b*e^(d*x)/d^2 - 8*(d*x + 1)*a*b*e^(-d*x - c)/d^2 - (d^4*x^4*e^c
 - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*b^2*e^(d*x)/d^5 -
(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*b^2*e^(-d*x - c)/d^5)*d +
1/4*(b^2*x^4 + 8*a*b*x - 2*a^2/x^2)*cosh(d*x + c)
```

Fricas [A]
time = 0.36, size = 161, normalized size = 1.14

$$-\frac{2\,(6\,b^2d^2x^4+a^2d^4+12\,b^2x^2)\cosh\left(dx+c\right)-(a^2d^6x^2\text{Ei}(dx)+a^2d^6x^2\text{Ei}(-dx))\cosh\left(c\right)-2\,(2\,b^2d^3x^5-a^2d^5x+4\,abd^3x^2+12\,b^2dx^3)\sinh\left(dx+c\right)-(a^2d^6x^2\text{Ei}(dx)-a^2d^6x^2\text{Ei}(-dx))\sinh\left(c\right)}{4\,d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(6*b^2*d^2*x^4 + a^2*d^4 + 12*b^2*x^2)*cosh(d*x + c) - (a^2*d^6*x^2
*Ei(d*x) + a^2*d^6*x^2*Ei(-d*x))*cosh(c) - 2*(2*b^2*d^3*x^5 - a^2*d^5*x + 4
*a*b*d^3*x^2 + 12*b^2*d*x^3)*sinh(d*x + c) - (a^2*d^6*x^2*Ei(d*x) - a^2*d^6
*x^2*Ei(-d*x))*sinh(c))/(d^4*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{3}\right)^{2}\cosh\left(c + dx\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**3)**2*cosh(c + d*x)/x**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(133) = 266.
time = 0.40, size = 280, normalized size = 1.99

$$\frac{a^{2}d^{6}x^{2}\mathrm{Ei}\left(-dx\right)e^{\left(-c\right)} + a^{2}d^{6}x^{2}\mathrm{Ei}\left(dx\right)e^{c} + 2\,b^{2}d^{3}x^{5}e^{\left(dx+c\right)} - 2\,b^{2}d^{3}x^{5}e^{\left(-dx-c\right)} - a^{2}d^{5}xe^{\left(dx+c\right)} - 6\,b^{2}d^{2}x^{4}e^{\left(dx+c\right)} + a^{2}d^{5}xe^{\left(-dx-c\right)} - 6\,b^{2}d^{2}x^{4}e^{\left(-dx-c\right)} + 4\,abd^{3}x^{2}e^{\left(dx+c\right)} - 4\,abd^{3}x^{2}e^{\left(-dx-c\right)} - a^{2}d^{4}e^{\left(dx+c\right)} + 12\,b^{2}dx^{3}e^{\left(dx+c\right)} - a^{2}d^{4}e^{\left(-dx-c\right)} - 12\,b^{2}dx^{3}e^{\left(-dx-c\right)} - 12\,b^{2}x^{2}e^{\left(dx+c\right)} - 12\,b^{2}x^{2}e^{\left(-dx-c\right)}}{4\,d^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^6*x^2*Ei(-d*x)*e^(-c) + a^2*d^6*x^2*Ei(d*x)*e^c + 2*b^2*d^3*x^5*
e^(d*x + c) - 2*b^2*d^3*x^5*e^(-d*x - c) - a^2*d^5*x*e^(d*x + c) - 6*b^2*d^
2*x^4*e^(d*x + c) + a^2*d^5*x*e^(-d*x - c) - 6*b^2*d^2*x^4*e^(-d*x - c) + 4
*a*b*d^3*x^2*e^(d*x + c) - 4*a*b*d^3*x^2*e^(-d*x - c) - a^2*d^4*e^(d*x + c)
 + 12*b^2*d*x^3*e^(d*x + c) - a^2*d^4*e^(-d*x - c) - 12*b^2*d*x^3*e^(-d*x -
 c) - 12*b^2*x^2*e^(d*x + c) - 12*b^2*x^2*e^(-d*x - c))/(d^4*x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{cosh}\left(c+d\,x\right)\,{\left(b\,x^3+a\right)}^2}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^3)^2)/x^3,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^3)^2)/x^3, x)
```

3.92 $\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^4} \, dx$

Optimal. Leaf size=150

$$-\frac{a^2\cosh(c+dx)}{3x^3}-\frac{a^2d^2\cosh(c+dx)}{6x}-\frac{2b^2x\cosh(c+dx)}{d^2}+2ab\cosh(c)\mathrm{Chi}(dx)+\frac{1}{6}a^2d^3\mathrm{Chi}(dx)\sinh(c)+\frac{2b^2\,\mathrm{s}}{}$$

```
[Out] 2*a*b*Chi(d*x)*cosh(c)-1/3*a^2*cosh(d*x+c)/x^3-1/6*a^2*d^2*cosh(d*x+c)/x-2*
b^2*x*cosh(d*x+c)/d^2+1/6*a^2*d^3*cosh(c)*Shi(d*x)+1/6*a^2*d^3*Chi(d*x)*sin
h(c)+2*a*b*Shi(d*x)*sinh(c)+2*b^2*sinh(d*x+c)/d^3-1/6*a^2*d*sinh(d*x+c)/x^2
+b^2*x^2*sinh(d*x+c)/d
```

Rubi [A]
time = 0.21, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5395, 3378, 3384, 3379, 3382, 3377, 2717}

$$\frac{1}{6}a^2d^3\sinh(c)\mathrm{Chi}(dx)+\frac{1}{6}a^2d^3\cosh(c)\mathrm{Shi}(dx)-\frac{a^2d^2\cosh(c+dx)}{6x}-\frac{a^2\cosh(c+dx)}{3x^3}-\frac{a^2d\sinh(c+dx)}{6x^2}+2ab\cosh(c)\mathrm{Chi}(dx)+2ab\sinh(c)\mathrm{Shi}(dx)+\frac{2b^2\sinh(c+dx)}{d^3}-\frac{2b^2x\cosh(c+dx)}{d^2}+\frac{b^2x^2\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Cosh[c + d*x])/x^4,x]
```

```
[Out] -1/3*(a^2*Cosh[c + d*x])/x^3 - (a^2*d^2*Cosh[c + d*x])/(6*x) - (2*b^2*x*Cos
h[c + d*x])/d^2 + 2*a*b*Cosh[c]*CoshIntegral[d*x] + (a^2*d^3*CoshIntegral[d
*x]*Sinh[c])/6 + (2*b^2*Sinh[c + d*x])/d^3 - (a^2*d*Sinh[c + d*x])/(6*x^2)
+ (b^2*x^2*Sinh[c + d*x])/d + (a^2*d^3*Cosh[c]*SinhIntegral[d*x])/6 + 2*a*b
*Sinh[c]*SinhIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^4}\,dx = \int \left(\frac{a^2\cosh(c+dx)}{x^4}+\frac{2ab\cosh(c+dx)}{x}+b^2x^2\cosh(c+dx)\right)dx$$

$$= a^2\int \frac{\cosh(c+dx)}{x^4}\,dx+(2ab)\int \frac{\cosh(c+dx)}{x}\,dx+b^2\int x^2\cosh(c+dx)\,d$$

$$= -\frac{a^2\cosh(c+dx)}{3x^3}+\frac{b^2x^2\sinh(c+dx)}{d}-\frac{(2b^2)\int x\sinh(c+dx)\,dx}{d}+\frac{1}{3}(a^2$$

$$= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{2b^2x\cosh(c+dx)}{d^2}+2ab\cosh(c)\text{Chi}(dx)-\frac{a^2d\sinh(c}{6x^2}$$

$$= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{a^2d^2\cosh(c+dx)}{6x}-\frac{2b^2x\cosh(c+dx)}{d^2}+2ab\cosh(c)$$

$$= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{a^2d^2\cosh(c+dx)}{6x}-\frac{2b^2x\cosh(c+dx)}{d^2}+2ab\cosh(c)$$

$$= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{a^2d^2\cosh(c+dx)}{6x}-\frac{2b^2x\cosh(c+dx)}{d^2}+2ab\cosh(c)$$

Mathematica [A]
time = 0.41, size = 135, normalized size = 0.90

$$\frac{1}{6}\left(-\frac{2a^2\cosh(c+dx)}{x^3}-\frac{a^2d^2\cosh(c+dx)}{x}-\frac{12b^2x\cosh(c+dx)}{d^2}+a\text{Chi}(dx)\left(12b\cosh(c)+ad^3\sinh(c)\right)+\frac{12b^2\sinh(c+dx)}{d^3}-\frac{a^2d\sinh(c+dx)}{x^2}+\frac{6b^2x^2\sinh(c+dx)}{d}+a\left(ad^3\cosh(c)+12b\sinh(c)\right)\text{Shi}(dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^4,x]
```

```
[Out] ((-2*a^2*Cosh[c + d*x])/x^3 - (a^2*d^2*Cosh[c + d*x])/x - (12*b^2*x*Cosh[c
+ d*x])/d^2 + a*CoshIntegral[d*x]*(12*b*Cosh[c] + a*d^3*Sinh[c]) + (12*b^2*
Sinh[c + d*x])/d^3 - (a^2*d*Sinh[c + d*x])/x^2 + (6*b^2*x^2*Sinh[c + d*x])/
d + a*(a*d^3*Cosh[c] + 12*b*Sinh[c])*SinhIntegral[d*x])/6
```

Maple [A]
time = 0.91, size = 261, normalized size = 1.74

method	result
risch	$\frac{d^3a^2\mathrm{e}^{-c}\operatorname{expIntegral}(1,dx)}{12}-\frac{b^2\mathrm{e}^{-dx-c}x^2}{2d}-\frac{b^2\mathrm{e}^{-dx-c}x}{d^2}-ab\,\mathrm{e}^{-c}\operatorname{expIntegral}(1,dx)-\frac{d^2a^2\mathrm{e}^{-dx-c}}{12x}+\frac{d\,a^2\mathrm{e}^{-dx-c}}{12x^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*cosh(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*d^3*a^2*exp(-c)*Ei(1,d*x)-1/2/d*b^2*exp(-d*x-c)*x^2-1/d^2*b^2*exp(-d*x
-c)*x-a*b*exp(-c)*Ei(1,d*x)-1/12*d^2*a^2*exp(-d*x-c)/x+1/12*d*a^2*exp(-d*x-
c)/x^2-1/d^3*b^2*exp(-d*x-c)-1/6*a^2*exp(-d*x-c)/x^3-1/12*d^3*a^2*exp(c)*Ei
(1,-d*x)+1/2/d*b^2*exp(d*x+c)*x^2-1/d^2*b^2*exp(d*x+c)*x+1/d^3*b^2*exp(d*x+
c)-1/6*a^2/x^3*exp(d*x+c)-1/12*d*a^2/x^2*exp(d*x+c)-1/12*d^2*a^2/x*exp(d*x+
c)-a*b*exp(c)*Ei(1,-d*x)
```

Maxima [A]
time = 0.33, size = 188, normalized size = 1.25

$$\frac{1}{6}\left(\left(d^2e^{(-c)}\Gamma(-2,dx)-d^2e^c\Gamma(-2,-dx)\right)a^2-b^2\left(\frac{(d^3x^3e^c-3d^2x^2e^c+6dxe^c-6e^c)e^{(dx)}}{d^4}+\frac{(d^3x^3+3d^2x^2+6dx+6)e^{(-dx-c)}}{d^4}\right)-\frac{4ab\cosh(dx+c)\log(x^3)}{d}+\frac{6\left(\text{Ei}(-dx)e^{(-c)}+\text{Ei}(dx)e^c\right)ab}{d}\right)d+\frac{1}{3}\left(b^2x^3+2ab\log(x^3)-\frac{a^2}{x^3}\right)\cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((d^2*e^(-c)*gamma(-2, d*x) - d^2*e^c*gamma(-2, -d*x))*a^2 - b^2*((d^3*
x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4 + (d^3*x^3 + 3*d^2
*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4) - 4*a*b*cosh(d*x + c)*log(x^3)/d + 6*(E
i(-d*x)*e^(-c) + Ei(d*x)*e^c)*a*b/d)*d + 1/3*(b^2*x^3 + 2*a*b*log(x^3) - a^
2/x^3)*cosh(d*x + c)
```

Fricas [A]
time = 0.40, size = 187, normalized size = 1.25

$$-\frac{2\left(a^2d^5x^2+12\,b^2dx^4+2\,a^2d^3\right)\cosh(dx+c)-\left(\left(a^2d^6+12\,abd^3\right)x^3\text{Ei}(dx)-\left(a^2d^6-12\,abd^3\right)x^3\text{Ei}(-dx)\right)\cosh(c)-2\left(6\,b^2d^2x^5-a^2d^4x+12\,b^2x^3\right)\sinh(dx+c)-\left(\left(a^2d^6+12\,abd^3\right)x^3\text{Ei}(dx)+\left(a^2d^6-12\,abd^3\right)x^3\text{Ei}(-dx)\right)\sinh(c)}{12\,d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(a^2*d^5*x^2 + 12*b^2*d*x^4 + 2*a^2*d^3)*cosh(d*x + c) - ((a^2*d^6
 + 12*a*b*d^3)*x^3*Ei(d*x) - (a^2*d^6 - 12*a*b*d^3)*x^3*Ei(-d*x))*cosh(c) -
 2*(6*b^2*d^2*x^5 - a^2*d^4*x + 12*b^2*x^3)*sinh(d*x + c) - ((a^2*d^6 + 12*
a*b*d^3)*x^3*Ei(d*x) + (a^2*d^6 - 12*a*b*d^3)*x^3*Ei(-d*x))*sinh(c))/(d^3*x
^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^2 \cosh{(c + dx)}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x**3)**2*cosh(c + d*x)/x**4, x)
```

Giac [A]
time = 0.41, size = 279, normalized size = 1.86

$$-\frac{a^2 d^6 x^3 \mathrm{Ei}(-dx)\, e^{(-c)} - a^2 d^6 x^3 \mathrm{Ei}(dx)\, e^{c} + a^2 d^5 x^2 e^{(dx+c)} - 6\, b^2 d^2 x^5 e^{(dx+c)} + a^2 d^5 x^2 e^{(-dx-c)} + 6\, b^2 d^2 x^5 e^{(-dx-c)} - 12\, abd^3 x^3 \mathrm{Ei}(-dx)\, e^{(-c)} - 12\, abd^3 x^3 \mathrm{Ei}(dx)\, e^{c} + a^2 d^4 x e^{(dx+c)} + 12\, b^2 d x^4 e^{(dx+c)} - a^2 d^4 x e^{(-dx-c)} + 12\, b^2 d x^4 e^{(-dx-c)} + 2\, a^2 d^3 e^{(dx+c)} - 12\, b^2 x^3 e^{(dx+c)} + 2\, a^2 d^3 e^{(-dx-c)} + 12\, b^2 x^3 e^{(-dx-c)}}{12\, d^3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(a^2*d^6*x^3*Ei(-d*x)*e^(-c) - a^2*d^6*x^3*Ei(d*x)*e^c + a^2*d^5*x^2*
e^(d*x + c) - 6*b^2*d^2*x^5*e^(d*x + c) + a^2*d^5*x^2*e^(-d*x - c) + 6*b^2*
d^2*x^5*e^(-d*x - c) - 12*a*b*d^3*x^3*Ei(-d*x)*e^(-c) - 12*a*b*d^3*x^3*Ei(d
*x)*e^c + a^2*d^4*x*e^(d*x + c) + 12*b^2*d*x^4*e^(d*x + c) - a^2*d^4*x*e^(-
d*x - c) + 12*b^2*d*x^4*e^(-d*x - c) + 2*a^2*d^3*e^(d*x + c) - 12*b^2*x^3*e
^(d*x + c) + 2*a^2*d^3*e^(-d*x - c) + 12*b^2*x^3*e^(-d*x - c))/(d^3*x^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + d\,x)\,\left(b\,x^3 + a\right)^2}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^3)^2)/x^4,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^3)^2)/x^4, x)
```

3.93 $\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^5} \, dx$

Optimal. Leaf size=167

$$-\frac{b^2 \cosh(c+dx)}{d^2}-\frac{a^2 \cosh(c+dx)}{4x^4}-\frac{a^2 d^2 \cosh(c+dx)}{24x^2}-\frac{2ab \cosh(c+dx)}{x}+\frac{1}{24}a^2 d^4 \cosh(c)\text{Chi}(dx)+2abd\text{Chi}$$

```
[Out] 1/24*a^2*d^4*Chi(d*x)*cosh(c)-b^2*cosh(d*x+c)/d^2-1/4*a^2*cosh(d*x+c)/x^4-1
/24*a^2*d^2*cosh(d*x+c)/x^2-2*a*b*cosh(d*x+c)/x+2*a*b*d*cosh(c)*Shi(d*x)+2*
a*b*d*Chi(d*x)*sinh(c)+1/24*a^2*d^4*Shi(d*x)*sinh(c)-1/12*a^2*d*sinh(d*x+c)
/x^3-1/24*a^2*d^3*sinh(d*x+c)/x+b^2*x*sinh(d*x+c)/d
```

Rubi [A]
time = 0.25, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5395, 3378, 3384, 3379, 3382, 3377, 2718}

$$\frac{1}{24}a^2 d^4 \cosh(c)\text{Chi}(dx)+\frac{1}{24}a^2 d^4 \sinh(c)\text{Shi}(dx)-\frac{a^2 d^3 \sinh(c+dx)}{24x}-\frac{a^2 d^2 \cosh(c+dx)}{24x^2}-\frac{a^2 \cosh(c+dx)}{4x^4}-\frac{a^2 d \sinh(c+dx)}{12x^3}+2abd\sinh(c)\text{Chi}(dx)+2abd\cosh(c)\text{Shi}(dx)-\frac{2ab\cosh(c+dx)}{x}-\frac{b^2\cosh(c+dx)}{d^2}+\frac{b^2 x\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Cosh[c + d*x])/x^5,x]
```

```
[Out] -((b^2*Cosh[c + d*x])/d^2) - (a^2*Cosh[c + d*x])/(4*x^4) - (a^2*d^2*Cosh[c
+ d*x])/(24*x^2) - (2*a*b*Cosh[c + d*x])/x + (a^2*d^4*Cosh[c]*CoshIntegral[
d*x])/24 + 2*a*b*d*CoshIntegral[d*x]*Sinh[c] - (a^2*d*Sinh[c + d*x])/(12*x^
3) - (a^2*d^3*Sinh[c + d*x])/(24*x) + (b^2*x*Sinh[c + d*x])/d + 2*a*b*d*Cos
h[c]*SinhIntegral[d*x] + (a^2*d^4*Sinh[c]*SinhIntegral[d*x])/24
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^5}\,dx = \int \left(\frac{a^2\cosh(c+dx)}{x^5}+\frac{2ab\cosh(c+dx)}{x^2}+b^2x\cosh(c+dx)\right)dx$$

$$= a^2\int \frac{\cosh(c+dx)}{x^5}\,dx+(2ab)\int \frac{\cosh(c+dx)}{x^2}\,dx+b^2\int x\cosh(c+dx)\,dx$$

$$= -\frac{a^2\cosh(c+dx)}{4x^4}-\frac{2ab\cosh(c+dx)}{x}+\frac{b^2x\sinh(c+dx)}{d}-\frac{b^2\int \sinh(c+}{d}$$

$$= -\frac{b^2\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{4x^4}-\frac{2ab\cosh(c+dx)}{x}-\frac{a^2d\sinh(c+dx}{12x^3}$$

$$= -\frac{b^2\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{4x^4}-\frac{a^2d^2\cosh(c+dx)}{24x^2}-\frac{2ab\cosh(c+a}{x}$$

$$= -\frac{b^2\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{4x^4}-\frac{a^2d^2\cosh(c+dx)}{24x^2}-\frac{2ab\cosh(c+a}{x}$$

$$= -\frac{b^2\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{4x^4}-\frac{a^2d^2\cosh(c+dx)}{24x^2}-\frac{2ab\cosh(c+a}{x}$$

$$= -\frac{b^2\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{4x^4}-\frac{a^2d^2\cosh(c+dx)}{24x^2}-\frac{2ab\cosh(c+a}{x}$$

Mathematica [A]
time = 0.36, size = 150, normalized size = 0.90

$$\frac{1}{24}\left(-\frac{24b^2\cosh(c+dx)}{d^2}-\frac{6a^2\cosh(c+dx)}{x^4}-\frac{a^2d^2\cosh(c+dx)}{x^2}-\frac{48ab\cosh(c+dx)}{x}+ad\mathrm{Chi}(dx)\left(ad^3\cosh(c)+48b\sinh(c)\right)-\frac{2a^2d\sinh(c+dx)}{x^3}-\frac{a^2d^3\sinh(c+dx)}{x}+\frac{24b^2x\sinh(c+dx)}{d}+ad(48b\cosh(c)+ad^3\sinh(c))\,\mathrm{Shi}(dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^5,x]
```

```
[Out] ((-24*b^2*Cosh[c + d*x])/d^2 - (6*a^2*Cosh[c + d*x])/x^4 - (a^2*d^2*Cosh[c
+ d*x])/x^2 - (48*a*b*Cosh[c + d*x])/x + a*d*CoshIntegral[d*x]*(a*d^3*Cosh[
c] + 48*b*Sinh[c]) - (2*a^2*d*Sinh[c + d*x])/x^3 - (a^2*d^3*Sinh[c + d*x])/
x + (24*b^2*x*Sinh[c + d*x])/d + a*d*(48*b*Cosh[c] + a*d^3*Sinh[c])*SinhInt
egral[d*x])/24
```

Maple [A]
time = 0.93, size = 292, normalized size = 1.75

method	result
risch	$-\frac{d^4a^2\mathrm{e}^{-c}\,\mathrm{expIntegral}(1,dx)}{48}-\frac{b^2\mathrm{e}^{-dx-c}x}{2d}-\frac{b^2\mathrm{e}^{-dx-c}}{2d^2}-\frac{ab\,\mathrm{e}^{-dx-c}}{x}+dab\,\mathrm{e}^{-c}\,\mathrm{expIntegral}\left(1,dx\right)+\frac{d^3a^2\mathrm{e}^{-dx-c}}{48x}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*cosh(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*d^4*a^2*exp(-c)*Ei(1,d*x)-1/2/d*b^2*exp(-d*x-c)*x-1/2/d^2*b^2*exp(-d*
x-c)-a*b*exp(-d*x-c)/x+d*a*b*exp(-c)*Ei(1,d*x)+1/48*d^3*a^2*exp(-d*x-c)/x-1
/48*d^2*a^2*exp(-d*x-c)/x^2+1/24*d*a^2*exp(-d*x-c)/x^3-1/8*a^2*exp(-d*x-c)/
x^4-1/48*d^4*a^2*exp(c)*Ei(1,-d*x)-a*b/x*exp(d*x+c)-d*a*b*exp(c)*Ei(1,-d*x)
+1/2/d*b^2*exp(d*x+c)*x-1/2/d^2*b^2*exp(d*x+c)-1/8*a^2/x^4*exp(d*x+c)-1/24*
d*a^2/x^3*exp(d*x+c)-1/48*d^2*a^2/x^2*exp(d*x+c)-1/48*d^3*a^2/x*exp(d*x+c)
```

Maxima [A]
time = 0.34, size = 154, normalized size = 0.92

$$\frac{1}{8}\left(a^2d^3e^{(-c)}\Gamma(-3,dx)+a^2d^3e^c\Gamma(-3,-dx)-8\,ab\mathrm{Ei}(-dx)\,e^{(-c)}+8\,ab\mathrm{Ei}(dx)\,e^c-\frac{2\,(d^2x^2e^c-2\,dxe^c+2\,e^c)b^2e^{(dx)}}{d^3}-\frac{2\,(d^2x^2+2\,dx+2)b^2e^{(-dx-c)}}{d^3}\right)d+\frac{1}{4}\left(2\,b^2x^2-\frac{8\,abx^3+a^2}{x^4}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*d^3*e^(-c)*gamma(-3, d*x) + a^2*d^3*e^c*gamma(-3, -d*x) - 8*a*b*Ei
(-d*x)*e^(-c) + 8*a*b*Ei(d*x)*e^c - 2*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*b^2
*e^(d*x)/d^3 - 2*(d^2*x^2 + 2*d*x + 2)*b^2*e^(-d*x - c)/d^3)*d + 1/4*(2*b^2
*x^2 - (8*a*b*x^3 + a^2)/x^4)*cosh(d*x + c)
```

Fricas [A]
time = 0.37, size = 196, normalized size = 1.17

$$-\frac{2\,(a^2d^4x^2+48\,abd^2x^3+24\,b^2x^4+6\,a^2d^2)\cosh\left(dx+c\right)-((a^2d^6+48\,abd^3)x^4\mathrm{Ei}(dx)+(a^2d^6-48\,abd^3)x^4\mathrm{Ei}(-dx))\cosh\left(c\right)+2\,(a^2d^5x^3-24\,b^2dx^5+2\,a^2d^3x)\sinh\left(dx+c\right)-((a^2d^6+48\,abd^3)x^4\mathrm{Ei}(dx)-(a^2d^6-48\,abd^3)x^4\mathrm{Ei}(-dx))\sinh\left(c\right)}{48\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(2*(a^2*d^4*x^2 + 48*a*b*d^2*x^3 + 24*b^2*x^4 + 6*a^2*d^2)*cosh(d*x +
 c) - ((a^2*d^6 + 48*a*b*d^3)*x^4*Ei(d*x) + (a^2*d^6 - 48*a*b*d^3)*x^4*Ei(-
d*x))*cosh(c) + 2*(a^2*d^5*x^3 - 24*b^2*d*x^5 + 2*a^2*d^3*x)*sinh(d*x + c)
- ((a^2*d^6 + 48*a*b*d^3)*x^4*Ei(d*x) - (a^2*d^6 - 48*a*b*d^3)*x^4*Ei(-d*x)
)*sinh(c))/(d^2*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^2\cosh\left(c+dx\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x**3)**2*cosh(c + d*x)/x**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(155) = 310.
time = 0.42, size = 315, normalized size = 1.89

$$\frac{a^2d^6x^4\mathrm{Ei}(-dx)\,e^{(-c)}+a^2d^6x^4\mathrm{Ei}(dx)\,e^{c}-a^2d^5x^3e^{(dx+c)}+a^2d^5x^3e^{(-dx-c)}-48\,abd^3x^4\mathrm{Ei}(-dx)\,e^{(-c)}+48\,abd^3x^4\mathrm{Ei}(dx)\,e^{c}-a^2d^4x^2e^{(dx+c)}+24\,b^2dx^5e^{(dx+c)}-a^2d^4x^2e^{(-dx-c)}-24\,b^2dx^5e^{(-dx-c)}-48\,abd^2x^3e^{(dx+c)}-48\,abd^2x^3e^{(-dx-c)}-2\,a^2d^3xe^{(dx+c)}-24\,b^2x^4e^{(dx+c)}+2\,a^2d^3xe^{(-dx-c)}-24\,b^2x^4e^{(-dx-c)}-6\,a^2d^2e^{(dx+c)}-6\,a^2d^2e^{(-dx-c)}}{48\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] 1/48*(a^2*d^6*x^4*Ei(-d*x)*e^(-c) + a^2*d^6*x^4*Ei(d*x)*e^c - a^2*d^5*x^3*e
^(d*x + c) + a^2*d^5*x^3*e^(-d*x - c) - 48*a*b*d^3*x^4*Ei(-d*x)*e^(-c) + 48
*a*b*d^3*x^4*Ei(d*x)*e^c - a^2*d^4*x^2*e^(d*x + c) + 24*b^2*d*x^5*e^(d*x +
c) - a^2*d^4*x^2*e^(-d*x - c) - 24*b^2*d*x^5*e^(-d*x - c) - 48*a*b*d^2*x^3*
e^(d*x + c) - 48*a*b*d^2*x^3*e^(-d*x - c) - 2*a^2*d^3*x*e^(d*x + c) - 24*b^
2*x^4*e^(d*x + c) + 2*a^2*d^3*x*e^(-d*x - c) - 24*b^2*x^4*e^(-d*x - c) - 6*
a^2*d^2*e^(d*x + c) - 6*a^2*d^2*e^(-d*x - c))/(d^2*x^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{cosh}\left(c+d\,x\right)\,{\left(b\,x^3+a\right)}^2}{x^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(a + b*x^3)^2)/x^5,x)
```

```
[Out] int((cosh(c + d*x)*(a + b*x^3)^2)/x^5, x)
```

3.94 $\int \frac{x^4 \cosh(c+dx)}{a+bx^3}\,dx$

Optimal. Leaf size=373

$$-\frac{\cosh(c+dx)}{bd^2}+\frac{(-1)^{2/3}a^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}}-\frac{\sqrt[3]{-1}\,a^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{}}{\sqrt[3]{b}}\right.}{3}$$

```
[Out] 1/3*a^(2/3)*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/b^(5/3)+1/
3*(-1)^(2/3)*a^(2/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3
)*a^(1/3)*d/b^(1/3))/b^(5/3)-1/3*(-1)^(1/3)*a^(2/3)*Chi(-(-1)^(2/3)*a^(1/3)
*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b^(5/3)-cosh(d*x+c)/b/
d^2+1/3*a^(2/3)*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/b^(5/3
)+1/3*(-1)^(2/3)*a^(2/3)*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)
^(1/3)*a^(1/3)*d/b^(1/3))/b^(5/3)-1/3*(-1)^(1/3)*a^(2/3)*Shi((-1)^(2/3)*a^(
1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b^(5/3)+x*sinh(d*x
+c)/b/d
```

Rubi [A]

time = 0.72, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5401, 3377, 2718, 3384, 3379, 3382}

$$\frac{(-1)^{2/3}a^{2/3}\cosh\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}}-\frac{\sqrt[3]{-1}\,a^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b^{5/3}}+\frac{a^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b^{5/3}}-\frac{(-1)^{2/3}a^{2/3}\sinh\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}}+\frac{a^{2/3}\sinh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b^{5/3}}-\frac{\sqrt[3]{-1}\,a^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b^{5/3}}-\frac{\cosh(c+dx)}{bd^2}+\frac{x\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] -(Cosh[c + d*x]/(b*d^2)) + ((-1)^(2/3)*a^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)
*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(5/3)
) - ((-1)^(1/3)*a^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegr
al[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*b^(5/3)) + (a^(2/3)*Cosh[c
- (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(5/3))
 + (x*Sinh[c + d*x])/(b*d) - ((-1)^(2/3)*a^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/
3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(5/
3)) + (a^(2/3)*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/
3) + d*x])/(3*b^(5/3)) - ((-1)^(1/3)*a^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d
)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(5/3))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^4\cosh(c+dx)}{a+bx^3}\,dx = \int \left(\frac{x\cosh(c+dx)}{b} - \frac{ax\cosh(c+dx)}{b(a+bx^3)}\right)dx$$

$$= \frac{\int x\cosh(c+dx)\,dx}{b} - \frac{a\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx}{b}$$

$$= \frac{x\sinh(c+dx)}{bd} - \frac{a\int \left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \cdots\right.}{b}$$

$$= -\frac{\cosh(c+dx)}{bd^2} + \frac{x\sinh(c+dx)}{bd} + \frac{a^{2/3}\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}\,a^{2/3}\right)\int \frac{\cosh(c+\cdots}{\sqrt[3]{a}+(-1)^{2}\cdots}}{3b^{4/3}}$$

$$= -\frac{\cosh(c+dx)}{bd^2} + \frac{x\sinh(c+dx)}{bd} + \frac{\left(a^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{4/3}} - \cdots$$

$$= -\frac{\cosh(c+dx)}{bd^2} + \frac{(-1)^{2/3}a^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}} - \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.37, size = 213, normalized size = 0.57

$$-\frac{ad^2\text{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1))-\text{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\text{Shi}(d(x-\#1))+\sinh(c+d\#1)\text{Shi}(d(x-\#1))}{\#1}\&\right]+ad^2\text{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1))+\text{Chi}(d(x-\#1))\sinh(c+d\#1)+\cosh(c+d\#1)\text{Shi}(d(x-\#1))+\sinh(c+d\#1)\text{Shi}(d(x-\#1))}{\#1}\&\right]+6b(\cosh(c+dx)-dx\sinh(c+dx))}{6b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] -1/6*(a*d^2*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]
 - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*
(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 & ] + a*d^2*RootSum
[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(
x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c
+ d*#1]*SinhIntegral[d*(x - #1)])/#1 & ] + 6*b*(Cosh[c + d*x] - d*x*Sinh[c
+ d*x]))/(b^2*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.16, size = 925, normalized size = 2.48

method	result	size
risch	Expression too large to display	925

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cosh(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*exp(-d*x-c)/b*x-1/2/d^2*exp(-d*x-c)/b-1/6/d^2/b^2*sum((6*_R1^2*b*c^2
-_R1*a*d^3-8*_R1*b*c^3-3*a*c*d^3+3*b*c^4)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(
1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d^2*
c^4/b*sum(1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b
-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+2/3/d^2*c^3/b*sum(_R1/(_R1^2-2*_R1*c+c
^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3
-b*c^3))-1/d^2*c^2/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c
),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+2/3/d^2*c/b^2*sum((
3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x
-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d/b*exp(d
*x+c)*x-1/2/d^2/b*exp(d*x+c)-1/6/d^2/b^2*sum((6*_R1^2*b*c^2-_R1*a*d^3-8*_R1
*b*c^3-3*a*c*d^3+3*b*c^4)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1
=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d^2*c^4/b*sum(1/(_R1
^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z
*b*c^2+a*d^3-b*c^3))+2/3/d^2*c^3/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(
1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/d^2*c
^2/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^
3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+2/3/d^2*c/b^2*sum((3*_R1^2*b*c-3*_R
1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=Root
Of(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 989 vs. $2(265) = 530$.
time = 0.43, size = 989, normalized size = 2.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*((a*d^3/b)^(2/3)*((sqrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*sinh
(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b
)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*((sqrt(-3) - 1)*cosh(d*x + c
)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(
-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(2/3)*(
(sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(d*x + 1
/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
- c) - (-a*d^3/b)^(2/3)*((sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*si
nh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*
d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(2/3)*(cosh(d*x + c)^2 - si
nh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*(
a*d^3/b)^(2/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x + (a*d^3/b)^(1/3)
)*cosh(-c + (a*d^3/b)^(1/3)) + (a*d^3/b)^(2/3)*((sqrt(-3) - 1)*cosh(d*x + c
)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*((s
qrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(-d*x - 1/
2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1)
 - c) + (a*d^3/b)^(2/3)*((sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*si
nh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) - 1) - c) + (-a*d^3/b)^(2/3)*((sqrt(-3) + 1)*cosh(d*x +
 c)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqr
t(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(-a*d^3/b)^(2
/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c
+ (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(2/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)
*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) + 12*d*x*sinh(d*x + c
) - 12*cosh(d*x + c))/(b*d^2*cosh(d*x + c)^2 - b*d^2*sinh(d*x + c)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \cosh(c + dx)}{a + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cosh(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x**4*cosh(c + d*x)/(a + b*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^4*cosh(d*x + c)/(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \cosh(c + d\,x)}{b\,x^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*cosh(c + d*x))/(a + b*x^3),x)
```

```
[Out] int((x^4*cosh(c + d*x))/(a + b*x^3), x)
```

3.95 $\int \frac{x^3 \cosh(c+dx)}{a+bx^3} \, dx$

Optimal. Leaf size=358

$$\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{a}\,\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}}{\cdots}\right.}{3b^{4/3}}$$

```
[Out] -1/3*a^(1/3)*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/b^(4/3)+1
/3*(-1)^(1/3)*a^(1/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/
3)*a^(1/3)*d/b^(1/3))/b^(4/3)-1/3*(-1)^(2/3)*a^(1/3)*Chi(-(-1)^(2/3)*a^(1/3
)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b^(4/3)-1/3*a^(1/3)*S
hi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/b^(4/3)+1/3*(-1)^(1/3)*
a^(1/3)*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/
b^(1/3))/b^(4/3)-1/3*(-1)^(2/3)*a^(1/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*
x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b^(4/3)+sinh(d*x+c)/b/d
```

Rubi [A]

time = 0.48, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5401, 2717, 5389, 3384, 3379, 3382}

$$\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,\cosh\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{a}\,\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b^{4/3}}-\frac{\sqrt[3]{a}\,\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b^{4/3}}-\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,\sinh\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}}-\frac{\sqrt[3]{a}\,\sinh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{a}\,\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b^{4/3}}+\frac{\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] ((-1)^(1/3)*a^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(4/3)) - ((-1)^(2/3)*a^(1/3)*Cos
h[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)) - d*x])/(3*b^(4/3)) - (a^(1/3)*Cosh[c - (a^(1/3)*d)/b^(1/3)]*Cosh
Integral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3)) + Sinh[c + d*x]/(b*d) - ((
-1)^(1/3)*a^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-
1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(4/3)) - (a^(1/3)*Sinh[c - (a^(1/3
)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3)) - ((-1)^
(2/3)*a^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^3\cosh(c+dx)}{a+bx^3}\,dx = \int \left(\frac{\cosh(c+dx)}{b} - \frac{a\cosh(c+dx)}{b\left(a+bx^3\right)}\right)dx$$
$$= \frac{\int \cosh(c+dx)\,dx}{b} - \frac{a\int \frac{\cosh(c+dx)}{a+bx^3}\,dx}{b}$$
$$= \frac{\sinh(c+dx)}{bd} - \frac{a\int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)} - \frac{\cosh\cdots}{3a^{2/3}\left(-\sqrt[3]{a}\cdots\right.}\cdots}{b}$$
$$= \frac{\sinh(c+dx)}{bd} + \frac{\sqrt[3]{a}\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{\cdots}{-\sqrt[3]{a}\cdots}}{\cdots}$$
$$= \frac{\sinh(c+dx)}{bd} + \frac{\left(\sqrt[3]{a}\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3b} + \frac{\left(\sqrt[3]{a}\cosh\left(c+\cdots\right.\right.}{\cdots}$$
$$= \frac{\sqrt[3]{-1}\,\sqrt[3]{a}\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}} - \frac{(-1)^{2/3}\sqrt[3]{a}\cosh\left(c-\cdots\right.}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.20, size = 198, normalized size = 0.55

$$-\frac{ad\,\text{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1))-\text{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\text{Shi}(d(x-\#1))+\sinh(c+d\#1)\text{Shi}(d(x-\#1))}{\#1^2}\&\right]+ad\,\text{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1))+\text{Chi}(d(x-\#1))\sinh(c+d\#1)+\cosh(c+d\#1)\text{Shi}(d(x-\#1))+\sinh(c+d\#1)\text{Shi}(d(x-\#1))}{\#1^2}\&\right]-6b\sinh(c+dx)}{6b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] -1/6*(a*d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] -
 CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x
 - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ] + a*d*RootSum[a
 + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x
- #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c +
d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ] - 6*b*Sinh[c + d*x])/(b^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.07, size = 671, normalized size = 1.87

method	result

risch	$-\frac{e^{-dx-c}}{2bd}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{\left(3\_R1^2bc-3\_R1b\,c^2-a\,d^3+b\,c^3\right)e^{-\_R1}\,\mathrm{expIntegral}\left(1,dx-\_R1\right.}{\_R1^2-2\_R1c+c^2}}{6d\,b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/d*exp(-d*x-c)-1/6/d/b^2*sum((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_
R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*
_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c^3/b*sum(1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1
,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/d*c^2
/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-
3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d*c/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)
*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*
c^3))+1/2/b/d*exp(d*x+c)-1/6/d/b^2*sum((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3
)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*
c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c^3/b*sum(1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*E
i(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/d
*c^2/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^
3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d*c/b*sum(_R1^2/(_R1^2-2*_R1*c+
c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^
3-b*c^3))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(250) = 500$.
time = 0.40, size = 977, normalized size = 2.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*((a*d^3/b)^(1/3)*((sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*sinh
(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b
```

```
)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(1/3)*((sqrt(-3) + 1)*cosh(d*x + c
)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(
-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(1/3)*(
(sqrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(d*x + 1
/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
- c) + (-a*d^3/b)^(1/3)*((sqrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*si
nh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*
d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(1/3)*(cosh(d*x + c)^2 - si
nh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) - 2*(
a*d^3/b)^(1/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x + (a*d^3/b)^(1/3)
)*cosh(-c + (a*d^3/b)^(1/3)) + (a*d^3/b)^(1/3)*((sqrt(-3) + 1)*cosh(d*x + c
)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(1/3)*((s
qrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(-d*x - 1/
2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1)
 - c) + (a*d^3/b)^(1/3)*((sqrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*si
nh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) - 1) - c) - (-a*d^3/b)^(1/3)*((sqrt(-3) - 1)*cosh(d*x +
 c)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqr
t(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(-a*d^3/b)^(1
/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c
+ (-a*d^3/b)^(1/3)) + 2*(a*d^3/b)^(1/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)
*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) + 12*sinh(d*x + c))/(
b*d*cosh(d*x + c)^2 - b*d*sinh(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \cosh (c + dx)}{a + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**3+a),x)
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(x^3*cosh(d*x + c)/(b*x^3 + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\cosh(c+d\,x)}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*cosh(c + d*x))/(a + b*x^3),x)
[Out] int((x^3*cosh(c + d*x))/(a + b*x^3), x)
```

3.96 $\int \frac{x^2 \cosh(c+dx)}{a+bx^3} \, dx$

Optimal. Leaf size=283

$$\frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3b}+\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3b}+\frac{\cosh\left(\right.}{}$$

```
[Out] 1/3*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/b+1/3*Chi((-1)^(1/
3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/b+1/3*Chi(-(
-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b+1/3
*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/b+1/3*Shi(-(-1)^(1/3)
*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/b+1/3*Shi((-1)
^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b
```

Rubi [A]
time = 0.36, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5401, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3b}+\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b}-\frac{\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3b}+\frac{\sinh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b}+\frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*
d)/b^(1/3) - d*x])/(3*b) + (Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIn
tegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*b) + (Cosh[c - (a^(1/3)
*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b) - (Sinh[c + ((-
1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x])/(3*b) + (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/
3) + d*x])/(3*b) + (Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[(
(-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\cosh(c+dx)}{a+bx^3}\,dx &= \int \left(\frac{\cosh(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\cosh(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\cosh(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\right.}\right.\\ &= \frac{\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{2/3}}+\frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{2/3}}+\frac{\int \frac{\cosh(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{2/3}}\\ &= \frac{\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{2/3}}+\frac{\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right.}{-\sqrt[3]{-1}\,\sqrt[3]{a}+}}{3b^{2/3}}\\ &= \frac{\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3b}+\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-\right.}{3b}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.17, size = 170, normalized size = 0.60

$$\frac{\mathrm{RootSum}\left[a+b\#1^3\&,\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1))-\mathrm{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\mathrm{Shi}(d(x-\#1))+\sinh(c+d\#1)\mathrm{Shi}(d(x-\#1))\&\right]+\mathrm{RootSum}\left[a+b\#1^3\&,\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1))+\mathrm{Chi}(d(x-\#1))\sinh(c+d\#1)+\cosh(c+d\#1)\mathrm{Shi}(d(x-\#1))+\sinh(c+d\#1)\mathrm{Shi}(d(x-\#1))\&\right]}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] (RootSum[a + b*#1^3 & , Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshInteg
ral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] +
Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] & ] + RootSum[a + b*#1^3 & , Cosh[c
```

```
 + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]
 + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*
(x - #1)] & ])/(6*b)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.01, size = 423, normalized size = 1.49

method	result
risch	$-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3bc^2\_Z+ad^3-bc^3\right)}\frac{\_R1^2 e^{-\_R1}\,\mathrm{expIntegral}\left(1,dx-\_R1+c\right)}{\_R1^2-2\_R1c+c^2}}{6b}-\frac{c^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_
Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6*c^2/b*sum(1/(_R1^2-2*_R1*c+c^
2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-
b*c^3))+1/3*c/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=R
ootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/b*sum(_R1^2/(_R1^2-2*_
R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2
+a*d^3-b*c^3))-1/6*c^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c
),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/3*c/b*sum(_R1/(_R
1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_
Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*((d*x^2*e^(2*c) + x*e^(2*c))*e^(d*x) - (d*x^2 - x)*e^(-d*x))/(b*d^2*x^3
*e^c + a*d^2*e^c) + 1/2*integrate((2*b*x^3*e^c - 3*a*d*x*e^c - a*e^c)*e^(d*
x)/(b^2*d^2*x^6 + 2*a*b*d^2*x^3 + a^2*d^2), x) + 1/2*integrate((2*b*x^3 + 3
*a*d*x - a)*e^(-d*x)/(b^2*d^2*x^6*e^c + 2*a*b*d^2*x^3*e^c + a^2*d^2*e^c), x
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 500 vs. 2(207) = 414.
time = 0.41, size = 500, normalized size = 1.77

$$\frac{\mathrm{Ei}\left(dx-\frac{1}{2}\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3}+1)\right)\cosh\left(\frac{1}{2}\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3}+1)+c\right)+\mathrm{Ei}\left(-dx-\frac{1}{2}\left(-\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3}+1)\right)\cosh\left(\frac{1}{2}\left(-\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3}+1)-c\right)+\mathrm{Ei}\left(dx+\frac{1}{2}\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3}-1)\right)\cosh\left(\frac{1}{2}\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3}-1)-c\right)+\mathrm{Ei}\left(-dx+\frac{1}{2}\left(-\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3}-1)\right)\cosh\left(\frac{1}{2}\left(-\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3}-1)+c\right)+\mathrm{Ei}\left(-dx+\left(-\frac{ad^3}{b}\right)^{\frac{1}{3}}\right)\cosh\left(c+\left(-\frac{ad^3}{b}\right)^{\frac{1}{3}}\right)+\mathrm{Ei}\left(dx+\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}\right)\cosh\left(-c+\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}\right)+\mathrm{Ei}\left(dx-\frac{1}{2}\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3}+1)\right)\sinh\left(\frac{1}{2}\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3}+1)+c\right)+\mathrm{Ei}\left(-dx-\frac{1}{2}\left(-\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3}+1)\right)\sinh\left(\frac{1}{2}\left(-\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3}+1)-c\right)-\mathrm{Ei}\left(dx+\frac{1}{2}\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3}-1)\right)\sinh\left(\frac{1}{2}\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3}-1)-c\right)-\mathrm{Ei}\left(-dx+\frac{1}{2}\left(-\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3}-1)\right)\sinh\left(\frac{1}{2}\left(-\frac{ad^3}{b}\right)^{\frac{1}{3}}(\sqrt{-3}-1)+c\right)-\mathrm{Ei}\left(-dx+\left(-\frac{ad^3}{b}\right)^{\frac{1}{3}}\right)\sinh\left(c+\left(-\frac{ad^3}{b}\right)^{\frac{1}{3}}\right)-\mathrm{Ei}\left(dx+\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}\right)\sinh\left(-c+\left(\frac{ad^3}{b}\right)^{\frac{1}{3}}\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*
(sqrt(-3) + 1) + c) + Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1
/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqr
t(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + Ei(-d*x + 1/2*(-
a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c
) + Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + Ei(d*x + (a*d^
3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt
(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + Ei(-d*x - 1/2*(-a
*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c)
 - Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(s
qrt(-3) - 1) - c) - Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2
*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c
+ (-a*d^3/b)^(1/3)) - Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)))
/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cosh (c + dx)}{a + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(d*x + c)/(b*x^3 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \cosh(c + d\,x)}{b\,x^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(c + d*x))/(a + b*x^3),x)
```

```
[Out] int((x^2*cosh(c + d*x))/(a + b*x^3), x)
```

3.97 $\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx$

Optimal. Leaf size=345

$$-\frac{(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}\,b^{2/3}}+\frac{\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right.}{3\sqrt[3]{a}\,b^{2/3}}$$

```
[Out] -1/3*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(1/3)/b^(2/3)-1
/3*(-1)^(2/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/
3)*d/b^(1/3))/a^(1/3)/b^(2/3)+1/3*(-1)^(1/3)*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1
/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^(2/3)-1/3*Shi(a^(1/
3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(1/3)/b^(2/3)-1/3*(-1)^(2/3)*
Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))
/a^(1/3)/b^(2/3)+1/3*(-1)^(1/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(
c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^(2/3)
```

Rubi [A]
time = 0.30, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5401, 3384, 3379, 3382}

$$-\frac{(-1)^{2/3}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}\,b^{2/3}}+\frac{\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}\,b^{2/3}}+\frac{(-1)^{2/3}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\sinh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}\,b^{2/3}}+\frac{\sqrt[3]{-1}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] -1/3*((-1)^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1
)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(a^(1/3)*b^(2/3)) + ((-1)^(1/3)*Cosh[c -
 ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1
/3)) - d*x])/(3*a^(1/3)*b^(2/3)) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshInteg
ral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^(2/3)*Sinh[c +
((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
 - d*x])/(3*a^(1/3)*b^(2/3)) - (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[
(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^(1/3)*Sinh[c - ((-1
)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d
*x])/(3*a^(1/3)*b^(2/3))
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx = \int \left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{(-1)^{2/3}\cosh(c+dx)}{3\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)} + \frac{\sqrt[3]{-1}\,\cdots}{3\sqrt[3]{a}\,\sqrt[3]{b}\left(\cdots\right.}\cdots\right.$$

$$= -\frac{\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}} + \frac{\sqrt[3]{-1}\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}} - \frac{(-1)^{2/3}\int \frac{\cosh(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\,\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}}$$

$$= -\frac{\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}} + \frac{\left(\sqrt[3]{-1}\,\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\cdots}{\cdots}}{3\sqrt[3]{a}\,\sqrt[3]{b}}\cdots$$

$$= -\frac{(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}\,b^{2/3}} + \frac{\sqrt[3]{-1}\,\cosh\left(c-\frac{(-1)^{2/\cdots}}{\cdots}\right.\cdots}{\cdots}$$

<u>Mathematica</u> [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.17, size = 180, normalized size = 0.52

$$\frac{\operatorname{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\operatorname{Chi}(d(x-\#1))-\operatorname{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\operatorname{Shi}(d(x-\#1))+\sinh(c+d\#1)\operatorname{Shi}(d(x-\#1))}{\#1}\&\right]+\operatorname{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\operatorname{Chi}(d(x-\#1))+\operatorname{Chi}(d(x-\#1))\sinh(c+d\#1)+\cosh(c+d\#1)\operatorname{Shi}(d(x-\#1))+\sinh(c+d\#1)\operatorname{Shi}(d(x-\#1))}{\#1}\&\right]}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] (RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshInte
gral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] +
 Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 & ] + RootSum[a + b*#1^3 & , (
Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c +
 d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhInteg
ral[d*(x - #1)])/#1 & ])/(6*b)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.01, size = 280, normalized size = 0.81

method	result
risch	$-\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3bc^2\_Z+a\,d^3-b\,c^3\right)}\frac{\_R1\,\mathrm{e}^{-\_R1}\,\mathrm{expIntegral}\left(1,dx-\_R1+c\right)}{\_R1^2-2\_R1c+c^2}\right)}{6b}+\frac{dc\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\ldots\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*d/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_
Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6*d*c/b*sum(1/(_R1^2-2*_R1*c+c^
2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-
b*c^3))-1/6*d/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=R
ootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6*d*c/b*sum(1/(_R1^2-2*_
R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2
+a*d^3-b*c^3))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 671 vs. 2(237) = 474.
time = 0.44, size = 671, normalized size = 1.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*((a*d^3/b)^(2/3)*(sqrt(-3) - 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*(sq
rt(-3) - 1)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(d*x + 1/2
*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) -
c) - (-a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-
3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + (a*d^3/b)^(2/3)*(s
qrt(-3) - 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b
)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*(sqrt(-3) - 1)*Ei(-d*x - 1/2
*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1)
- c) + (a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (-a*d^3/b)^(2/3)*(sq
rt(-3) + 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(2/3)*Ei(-d*x + (-a*d^3/b)^(1/
3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*(a*d^3/b)^(2/3)*Ei(d*x + (a*d^3/b)^(1/3)
)*cosh(-c + (a*d^3/b)^(1/3)) - 2*(-a*d^3/b)^(2/3)*Ei(-d*x + (-a*d^3/b)^(1/3
))*sinh(c + (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(2/3)*Ei(d*x + (a*d^3/b)^(1/3))
*sinh(-c + (a*d^3/b)^(1/3)))/(a*d^2)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh{(c+dx)}}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x*cosh(c + d*x)/(a + b*x**3), x)
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(d*x + c)/(b*x^3 + a), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\cosh(c+d\,x)}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(c + d*x))/(a + b*x^3),x)
```

```
[Out] int((x*cosh(c + d*x))/(a + b*x^3), x)
```

3.98 $\int \frac{\cosh(c+dx)}{a+bx^3}\,dx$

Optimal. Leaf size=345

$$-\frac{\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right.}{3a^{2/3}\sqrt[3]{b}}$$

```
[Out] 1/3*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(1/3)-1/
3*(-1)^(1/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3
)*d/b^(1/3))/a^(2/3)/b^(1/3)+1/3*(-1)^(2/3)*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/
3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(1/3)+1/3*Shi(a^(1/3
)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(1/3)-1/3*(-1)^(1/3)*S
hi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/
a^(2/3)/b^(1/3)+1/3*(-1)^(2/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c
-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(1/3)
```

Rubi [A]
time = 0.30, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5389, 3384, 3379, 3382}

$$-\frac{\sqrt[3]{-1}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\sqrt[3]{-1}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\sinh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*x^3),x]
```

```
[Out] -1/3*((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1
)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Cosh[c -
 ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1
/3)) - d*x])/(3*a^(2/3)*b^(1/3)) + (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshInteg
ral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(1/3)*Sinh[c +
((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
 - d*x])/(3*a^(2/3)*b^(1/3)) + (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[
(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Sinh[c - ((-1
)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d
*x])/(3*a^(2/3)*b^(1/3))
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{a+bx^3}\,dx = \int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1\cdots\right.}\cdots\right.$$

$$= -\frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3a^{2/3}} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x}\,dx}{3a^{2/3}} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}\,dx}{3a^{2/3}}$$

$$= -\frac{\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3a^{2/3}} - \frac{\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-\cdots\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{\cdots}}}{3a^{2/3}}\cdots$$

$$= -\frac{\sqrt[3]{-1}\,\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\cdots}{\sqrt[3]{b}}\cdots\right.}{3\cdots}\cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.11, size = 180, normalized size = 0.52

$$\frac{\mathrm{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1))-\mathrm{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\mathrm{Shi}(d(x-\#1))+\sinh(c+d\#1)\mathrm{Shi}(d(x-\#1))}{\#1^2}\&\right]+\mathrm{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1))+\mathrm{Chi}(d(x-\#1))\sinh(c+d\#1)+\cosh(c+d\#1)\mathrm{Shi}(d(x-\#1))+\sinh(c+d\#1)\mathrm{Shi}(d(x-\#1))}{\#1^2}\&\right]}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x^3),x]
```

```
[Out] (RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshInte
gral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] +
 Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ] + RootSum[a + b*#1^3 & ,
 (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c
 + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhInt
egral[d*(x - #1)])/#1^2 & ])/(6*b)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.00, size = 143, normalized size = 0.41

method	result
risch	$-\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3bc^2\_Z+ad^3-bc^3\right)}\frac{\mathrm{e}^{-\_R1}\,\mathrm{expIntegral}\left(1,dx-\_R1+c\right)}{\_R1^2-2\_R1c+c^2}\right)}{6b}-\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*d^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_
Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6*d^2/b*sum(1/(_R1^2-2*_R1*c+c^
2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-
b*c^3))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 673 vs. 2(237) = 474.
time = 0.45, size = 673, normalized size = 1.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*((a*d^3/b)^(1/3)*(sqrt(-3) + 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(1/3)*(sq
rt(-3) + 1)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3
```

```
/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(1/3)*(sqrt(-3) - 1)*Ei(d*x + 1/2
*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) -
c) + (-a*d^3/b)^(1/3)*(sqrt(-3) - 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-
3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + (a*d^3/b)^(1/3)*(s
qrt(-3) + 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b
)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(1/3)*(sqrt(-3) + 1)*Ei(-d*x - 1/2
*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1)
- c) + (a*d^3/b)^(1/3)*(sqrt(-3) - 1)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - (-a*d^3/b)^(1/3)*(sq
rt(-3) - 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(1/3)*Ei(-d*x + (-a*d^3/b)^(1/
3))*cosh(c + (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(1/3)*Ei(d*x + (a*d^3/b)^(1/3)
)*cosh(-c + (a*d^3/b)^(1/3)) - 2*(-a*d^3/b)^(1/3)*Ei(-d*x + (-a*d^3/b)^(1/3
))*sinh(c + (-a*d^3/b)^(1/3)) + 2*(a*d^3/b)^(1/3)*Ei(d*x + (a*d^3/b)^(1/3))
*sinh(-c + (a*d^3/b)^(1/3)))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c+dx)}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(a + b*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*x^3),x)
```

```
[Out] int(cosh(c + d*x)/(a + b*x^3), x)
```

3.99 $\int \frac{\cosh(c+dx)}{x(a+bx^3)}\,dx$

Optimal. Leaf size=303

$$\frac{\cosh(c)\text{Chi}(dx)}{a}-\frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}}{\sqrt[3]{b}}\right.}{3a}$$

```
[Out] Chi(d*x)*cosh(c)/a-1/3*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))
/a-1/3*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^
(1/3))/a-1/3*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/
3)*d/b^(1/3))/a+Shi(d*x)*sinh(c)/a-1/3*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^
(1/3)*d/b^(1/3))/a-1/3*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(
1/3)*a^(1/3)*d/b^(1/3))/a-1/3*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-
(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a
```

Rubi [A]
time = 0.37, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5401, 3384, 3379, 3382}

$$-\frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a}-\frac{\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a}+\frac{\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\sinh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a}-\frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a}+\frac{\cosh(c)\text{Chi}(dx)}{a}+\frac{\sinh(c)\text{Shi}(dx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x^3)),x]
```

```
[Out] (Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*C
oshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a) - (Cosh[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) -
d*x])/(3*a) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/
3) + d*x])/(3*a) + (Sinh[c]*SinhIntegral[d*x])/a + (Sinh[c + ((-1)^(1/3)*a^
(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a)
 - (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/
(3*a) - (Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x])/(3*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x\left(a+bx^3\right)}\,dx &= \int \left(\frac{\cosh(c+dx)}{ax}-\frac{bx^2\cosh(c+dx)}{a\left(a+bx^3\right)}\right)dx\\
&= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a}-\frac{b\int \frac{x^2\cosh(c+dx)}{a+bx^3}\,dx}{a}\\
&= -\frac{b\int \left(\frac{\cosh(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\cosh(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\cosh(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}\right)dx}{a}+\cdots\\
&= \frac{\cosh(c)\mathrm{Chi}(dx)}{a}+\frac{\sinh(c)\mathrm{Shi}(dx)}{a}-\frac{\sqrt[3]{b}\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a}-\frac{\sqrt[3]{b}\int \frac{\cosh(c+dx)}{-\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{b}\,x}\cdots}{3a}\\
&= \frac{\cosh(c)\mathrm{Chi}(dx)}{a}+\frac{\sinh(c)\mathrm{Shi}(dx)}{a}-\frac{\left(\sqrt[3]{b}\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a}-\cdots\\
&= \frac{\cosh(c)\mathrm{Chi}(dx)}{a}-\frac{\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\cosh\left(c-\frac{(-1)^{\cdots}}{\cdots}\right.\cdots}{\cdots}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.21, size = 186, normalized size = 0.61

$$-\frac{-6\cosh(c)\mathrm{Chi}(dx)+\mathrm{RootSum}\left[a+b\#1^3\&,\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1))-\mathrm{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\mathrm{Shi}(d(x-\#1))+\sinh(c+d\#1)\mathrm{Shi}(d(x-\#1))\&\right]+\mathrm{RootSum}\left[a+b\#1^3\&,\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1))+\mathrm{Chi}(d(x-\#1))\sinh(c+d\#1)+\cosh(c+d\#1)\mathrm{Shi}(d(x-\#1))+\sinh(c+d\#1)\mathrm{Shi}(d(x-\#1))\&\right]-6\sinh(c)\mathrm{Shi}(dx)}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^3)),x]
```

```
[Out] -1/6*(-6*Cosh[c]*CoshIntegral[d*x] + RootSum[a + b*#1^3 & , Cosh[c + d*#1]*
CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c
 + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]
 & ] + RootSum[a + b*#1^3 & , Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + Cos
hIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #
1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] & ] - 6*Sinh[c]*SinhIntegral[
d*x])/a
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.99, size = 138, normalized size = 0.46

method	result
risch	$-\frac{e^{-c}\,\mathrm{expIntegral}(1,dx)}{2a}+\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3bc^2\_Z+a\,d^3-b\,c^3\right)} e^{-\_R1}\,\mathrm{expIntegral}\left(1,dx-\_R1+c\right)}{6a}-\frac{e^{c}\,\mathrm{expIntegr}}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*exp(-c)*Ei(1,d*x)+1/6/a*sum(exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^
3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/a*exp(c)*Ei(1,-d*x)+1/6/a*sum(e
xp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^
3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^3 + a)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(227) = 454.
time = 0.41, size = 530, normalized size = 1.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*(Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)
*(sqrt(-3) + 1) + c) + Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(
1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sq
rt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + Ei(-d*x + 1/2*(
-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) +
c) + Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) - 3*(Ei(d*x) +
Ei(-d*x))*cosh(c) + Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) +
Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt
(-3) + 1) + c) + Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-
a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3)
 - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - Ei(-d*x + 1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - E
i(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 3*(Ei(d*x) - Ei(-d*
x))*sinh(c) - Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)))/a
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c+dx)}{x(a+bx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x**3+a),x)
[Out] Integral(cosh(c + d*x)/(x*(a + b*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^3+a),x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/((b*x^3 + a)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)}{x\,\left(b\,x^3+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x*(a + b*x^3)),x)
[Out] int(cosh(c + d*x)/(x*(a + b*x^3)), x)
```

3.100 $\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=381

$$-\frac{\cosh(c+dx)}{ax}+\frac{(-1)^{2/3}\sqrt[3]{b}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{\ }}{\sqrt[3]{b}}\right.}{3a}$$

```
[Out] 1/3*b^(1/3)*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(4/3)+1/
3*(-1)^(2/3)*b^(1/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3
)*a^(1/3)*d/b^(1/3))/a^(4/3)-1/3*(-1)^(1/3)*b^(1/3)*Chi(-(-1)^(2/3)*a^(1/3)
*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)-cosh(d*x+c)/a/
x+d*cosh(c)*Shi(d*x)/a+d*Chi(d*x)*sinh(c)/a+1/3*b^(1/3)*Shi(a^(1/3)*d/b^(1/
3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(4/3)+1/3*(-1)^(2/3)*b^(1/3)*Shi(-(-1)^
(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(4/3)-1
/3*(-1)^(1/3)*b^(1/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/
3)*a^(1/3)*d/b^(1/3))/a^(4/3)
```

Rubi [A]

time = 0.44, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5401, 3378, 3384, 3379, 3382}

$$\frac{(-1)^{2/3}\sqrt[3]{b}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{\sqrt[3]{b}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{b}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}+\frac{\sqrt[3]{b}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{d\sinh(c)\mathrm{Chi}(dx)}{a}+\frac{d\cosh(c)\mathrm{Shi}(dx)}{a}-\frac{\cosh(c+dx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x^3)),x]
```

```
[Out] -(Cosh[c + d*x]/(a*x)) + ((-1)^(2/3)*b^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d
)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(4/3))
- ((-1)^(1/3)*b^(1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral
[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*a^(4/3)) + (b^(1/3)*Cosh[c -
(a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) +
 (d*CoshIntegral[d*x]*Sinh[c])/a + (d*Cosh[c]*SinhIntegral[d*x])/a - ((-1)^
(2/3)*b^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(4/3)) + (b^(1/3)*Sinh[c - (a^(1/3)*d)
/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) - ((-1)^(1/3
)*b^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)
*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^3\right)}\,dx = \int \left(\frac{\cosh(c+dx)}{ax^2} - \frac{bx\cosh(c+dx)}{a\left(a+bx^3\right)}\right) dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a} - \frac{b\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx}{a}$$

$$= -\frac{\cosh(c+dx)}{ax} - \frac{b\int \left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{}{3\sqrt[3]{a}}\right.}{a}$$

$$= -\frac{\cosh(c+dx)}{ax} + \frac{b^{2/3}\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}} - \frac{\left(\sqrt[3]{-1}\,b^{2/3}\right)\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{4/3}} + \frac{\left((-1)^{2/3}\right.}{}$$

$$= -\frac{\cosh(c+dx)}{ax} + \frac{d\text{Chi}(dx)\sinh(c)}{a} + \frac{d\cosh(c)\text{Shi}(dx)}{a} + \frac{\left(b^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int}{3a^{4/3}}$$

$$= -\frac{\cosh(c+dx)}{ax} + \frac{(-1)^{2/3}\sqrt[3]{b}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}} - \frac{\sqrt[3]{-1}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.32, size = 215, normalized size = 0.56

$$-\frac{6\cosh(c+dx) + x\text{RootSum}\left[a+b\#1^3\&, \frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1)) - \text{Chi}(d(x-\#1))\sinh(c+d\#1) - \cosh(c+d\#1)\text{Shi}(d(x-\#1)) + \sinh(c+d\#1)\text{Shi}(d(x-\#1))}{\#1}\&\right] + x\text{RootSum}\left[a+b\#1^3\&, \frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1)) + \text{Chi}(d(x-\#1))\sinh(c+d\#1) + \cosh(c+d\#1)\text{Shi}(d(x-\#1)) + \sinh(c+d\#1)\text{Shi}(d(x-\#1))}{\#1}\&\right] - 6dx\text{Chi}(dx)\sinh(c) - 6dx\cosh(c)\text{Shi}(dx)}{6ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x^3)),x]
```

```
[Out] -1/6*(6*Cosh[c + d*x] + x*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegr
al[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*S
inhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 & ] +
 x*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIn
tegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]
 + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 & ] - 6*d*x*CoshIntegral[d*x
]*Sinh[c] - 6*d*x*Cosh[c]*SinhIntegral[d*x])/(a*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.02, size = 187, normalized size = 0.49

method	result

risch	$-\frac{e^{-dx-c}}{2ax}+\frac{d\,e^{-c}\,\mathrm{expIntegral}(1,dx)}{2a}+\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{e^{-\_R1}\,\mathrm{expIntegral}\left(1,dx-\_R1+c\right)}{\_R1-c}\right.}{6a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] -1/2*exp(-d*x-c)/a/x+1/2*d/a*exp(-c)*Ei(1,d*x)+1/6*d/a*sum(1/(_R1-c)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2*exp(d*x+c)/a/x-1/2*d/a*exp(c)*Ei(1,-d*x)+1/6*d/a*sum(1/(_R1-c)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x^3+a),x, algorithm="maxima")

[Out] Timed out

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1154 vs. $2(273) = 546$.
time = 0.48, size = 1154, normalized size = 3.03

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")

[Out] -1/12*(12*a*d^2*cosh(d*x + c) - (a*d^3/b)^(2/3)*((sqrt(-3)*b*x - b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x - b*x)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(2/3)*((sqrt(-3)*b*x - b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x - b*x)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + (a*d^3/b)^(2/3)*((sqrt(-3)*b*x + b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x + b*x)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (-a*d^3/b)^(2/3)*((sqrt(-3)*b*x + b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x + b*x)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*(-a*d^3/b)^(2/3)*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) - 2*(b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*(a*d^3/b)^(2/3)*Ei(d*x +

```
(a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) - (a*d^3/b)^(2/3)*((sqrt(-3)*b*
x - b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x - b*x)*sinh(d*x + c)^2)*Ei(d*x - 1
/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1)
+ c) - (-a*d^3/b)^(2/3)*((sqrt(-3)*b*x - b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b
*x - b*x)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*s
inh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(2/3)*((sqrt(-3)*b
*x + b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x + b*x)*sinh(d*x + c)^2)*Ei(d*x +
1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
 - c) - (-a*d^3/b)^(2/3)*((sqrt(-3)*b*x + b*x)*cosh(d*x + c)^2 - (sqrt(-3)*
b*x + b*x)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*
sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(b*x*cosh(d*x + c)^2 - b*
x*sinh(d*x + c)^2)*(-a*d^3/b)^(2/3)*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-
a*d^3/b)^(1/3)) + 2*(b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*(a*d^3/b)^(
2/3)*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) - 6*(a*d^3*x*Ei(d
*x) - a*d^3*x*Ei(-d*x))*cosh(c) - 6*(a*d^3*x*Ei(d*x) + a*d^3*x*Ei(-d*x))*si
nh(c))/(a^2*d^2*x*cosh(d*x + c)^2 - a^2*d^2*x*sinh(d*x + c)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**2/(b*x**3+a),x)
[Out] Integral(cosh(c + d*x)/(x**2*(a + b*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^3+a),x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/((b*x^3 + a)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)}{x^2\,\left(b\,x^3+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x^2*(a + b*x^3)),x)
[Out] int(cosh(c + d*x)/(x^2*(a + b*x^3)), x)
```

3.101 $\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx$

Optimal. Leaf size=410

$$-\frac{\cosh(c+dx)}{2ax^2}+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a}+\frac{\sqrt[3]{-1}\,b^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2}}{}$$

```
[Out] 1/2*d^2*Chi(d*x)*cosh(c)/a-1/3*b^(2/3)*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^
(1/3)*d/b^(1/3))/a^(5/3)+1/3*(-1)^(1/3)*b^(2/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^
(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)-1/3*(-1)^(2/3)*b^(2
/3)*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1
/3))/a^(5/3)-1/2*cosh(d*x+c)/a/x^2+1/2*d^2*Shi(d*x)*sinh(c)/a-1/3*b^(2/3)*S
hi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(5/3)+1/3*(-1)^(1/3)*
b^(2/3)*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/
b^(1/3))/a^(5/3)-1/3*(-1)^(2/3)*b^(2/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*
x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)-1/2*d*sinh(d*x+c)/a/x
```

Rubi [A]
time = 0.48, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5401, 3378, 3384, 3379, 3382, 5389}

$$\frac{\sqrt[3]{-1}\,b^{2/3}\cosh\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{5/3}}-\frac{b^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{5/3}}-\frac{\sqrt[3]{-1}\,b^{2/3}\sinh\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}-\frac{b^{2/3}\sinh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a}+\frac{d^2\sinh(c)\text{Shi}(dx)}{2a}-\frac{\cosh(c+dx)}{2ax^2}-\frac{d\sinh(c+dx)}{2ax}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/2*Cosh[c + d*x]/(a*x^2) + (d^2*Cosh[c]*CoshIntegral[d*x])/(2*a) + ((-1)^
(1/3)*b^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Cosh[c -
((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/
3)) - d*x])/(3*a^(5/3)) - (b^(2/3)*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegr
al[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(5/3)) - (d*Sinh[c + d*x])/(2*a*x) + (d
^2*Sinh[c]*SinhIntegral[d*x])/(2*a) - ((-1)^(1/3)*b^(2/3)*Sinh[c + ((-1)^(1
/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])
/(3*a^(5/3)) - (b^(2/3)*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)
*d)/b^(1/3) + d*x])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Sinh[c - ((-1)^(2/3)*
a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*
a^(5/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx = \int \left(\frac{\cosh(c+dx)}{ax^3} - \frac{b\cosh(c+dx)}{a(a+bx^3)}\right) dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^3}\,dx}{a} - \frac{b\int \frac{\cosh(c+dx)}{a+bx^3}\,dx}{a}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b\int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)} - \frac{\cos}{3a^{2/3}\left(-\sqrt[3]{a}\right.}\right.}{a}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} - \frac{d\sinh(c+dx)}{2ax} + \frac{b\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3a^{5/3}} + \frac{b\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x}\,dx}{3a^{5/3}} +$$

$$= -\frac{\cosh(c+dx)}{2ax^2} - \frac{d\sinh(c+dx)}{2ax} + \frac{(d^2\cosh(c))\int \frac{\cosh(dx)}{x}\,dx}{2a} + \frac{\left(b\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\right.}{3a^5}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} + \frac{d^2\cosh(c)\mathrm{Chi}(dx)}{2a} + \frac{\sqrt[3]{-1}\,b^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}}{\sqrt[3]{}}\right.}{3a^{5/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.30, size = 237, normalized size = 0.58

$$-\frac{3\cosh(c+dx) - 3d^2x^2\cosh(c)\mathrm{Chi}(dx) + x^2\mathrm{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1)) - \mathrm{Chi}(d(x-\#1))\sinh(c+d\#1) - \cosh(c+d\#1)\mathrm{Shi}(d(x-\#1)) + \sinh(c+d\#1)\mathrm{Shi}(d(x-\#1))}{\#1^2}\&\right] + x^2\mathrm{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1)) + \mathrm{Chi}(d(x-\#1))\sinh(c+d\#1) + \cosh(c+d\#1)\mathrm{Shi}(d(x-\#1)) + \sinh(c+d\#1)\mathrm{Shi}(d(x-\#1))}{\#1^2}\&\right] + 3dx\sinh(c+dx) - 3d^2x^2\sinh(c)\mathrm{Shi}(dx)}{6ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/6*(3*Cosh[c + d*x] - 3*d^2*x^2*Cosh[c]*CoshIntegral[d*x] + x^2*RootSum[a
 + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x
- #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c +
d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ] + x^2*RootSum[a + b*#1^3 & , (Cosh
[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#
1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[
d*(x - #1)])/#1^2 & ] + 3*d*x*Sinh[c + d*x] - 3*d^2*x^2*Sinh[c]*SinhIntegra
l[d*x])/(a*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.04, size = 240, normalized size = 0.59

method	result

risch	$\frac{d\,e^{-dx-c}}{4ax}-\frac{e^{-dx-c}}{4a\,x^2}-\frac{d^2e^{-c}\,\mathrm{expIntegral}(1,dx)}{4a}+\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{e^{-\_R1}\,\mathrm{expIntegral}\left(1,dx-\right.}{\_R1^2-2\_R1c}\right.}{6a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d*exp(-d*x-c)/a/x-1/4*exp(-d*x-c)/a/x^2-1/4*d^2/a*exp(-c)*Ei(1,d*x)+1/6
*d^2/a*sum(1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*
b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/4*d*exp(d*x+c)/a/x-1/4*exp(d*x+c)/a
/x^2-1/4*d^2/a*exp(c)*Ei(1,-d*x)+1/6*d^2/a*sum(1/(_R1^2-2*_R1*c+c^2)*exp(_R
1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1251 vs. 2(294) = 588.
time = 0.44, size = 1251, normalized size = 3.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(6*a*d^2*x*sinh(d*x + c) - (a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 + b*x^2)*
cosh(d*x + c)^2 - (sqrt(-3)*b*x^2 + b*x^2)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a
*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c)
+ (-a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 + b*x^2)*cosh(d*x + c)^2 - (sqrt(-3)*b*
x^2 + b*x^2)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1)
)*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + (a*d^3/b)^(1/3)*((sqrt(-3
)*b*x^2 - b*x^2)*cosh(d*x + c)^2 - (sqrt(-3)*b*x^2 - b*x^2)*sinh(d*x + c)^2
)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sq
rt(-3) - 1) - c) - (-a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 - b*x^2)*cosh(d*x + c)
^2 - (sqrt(-3)*b*x^2 - b*x^2)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/
3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(b*x^2
*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*(-a*d^3/b)^(1/3)*Ei(-d*x + (-a*d^
```

```
3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*(b*x^2*cosh(d*x + c)^2 - b*x^2*s
inh(d*x + c)^2)*(a*d^3/b)^(1/3)*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/
b)^(1/3)) - (a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 + b*x^2)*cosh(d*x + c)^2 - (sq
rt(-3)*b*x^2 + b*x^2)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-
3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(1/3)*((
sqrt(-3)*b*x^2 + b*x^2)*cosh(d*x + c)^2 - (sqrt(-3)*b*x^2 + b*x^2)*sinh(d*x
 + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)
^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 - b*x^2)*cosh
(d*x + c)^2 - (sqrt(-3)*b*x^2 - b*x^2)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (-
a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 - b*x^2)*cosh(d*x + c)^2 - (sqrt(-3)*b*x^2
- b*x^2)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*si
nh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(b*x^2*cosh(d*x + c)^2 - b*
x^2*sinh(d*x + c)^2)*(-a*d^3/b)^(1/3)*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c +
(-a*d^3/b)^(1/3)) - 2*(b*x^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*(a*d^
3/b)^(1/3)*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) + 6*a*d*cos
h(d*x + c) - 3*(a*d^3*x^2*Ei(d*x) + a*d^3*x^2*Ei(-d*x))*cosh(c) - 3*(a*d^3*
x^2*Ei(d*x) - a*d^3*x^2*Ei(-d*x))*sinh(c))/(a^2*d*x^2*cosh(d*x + c)^2 - a^2
*d*x^2*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**3/(b*x**3+a),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/((b*x^3 + a)*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)}{x^3\,\left(b\,x^3+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x^3*(a + b*x^3)),x)
[Out] int(cosh(c + d*x)/(x^3*(a + b*x^3)), x)
```

3.102 $\int \frac{x^3 \cosh(c+dx)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=718

$$-\frac{x\cosh(c+dx)}{3b\left(a+bx^3\right)}-\frac{\sqrt[3]{-1}\,\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{C}}{9a^{2/3}b^{4/3}}$$

```
[Out] 1/9*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)-1/
9*(-1)^(1/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3
)*d/b^(1/3))/a^(2/3)/b^(4/3)+1/9*(-1)^(2/3)*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/
3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)-1/3*x*cosh(d*x
+c)/b/(b*x^3+a)-1/9*(-1)^(2/3)*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-
(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(1/3)/b^(5/3)-1/9*d*cosh(c-a^(1/3)*d/b^
(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(1/3)/b^(5/3)+1/9*(-1)^(1/3)*d*cosh(c-(
-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(1/3)/
b^(5/3)-1/9*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(1/3)/
b^(5/3)+1/9*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^
(4/3)-1/9*(-1)^(2/3)*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c+(-1)^(1
/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^(5/3)-1/9*(-1)^(1/3)*Shi(-(-1)^(1/3)*a^(1/
3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)+1/9*
(-1)^(1/3)*d*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/
3)*d/b^(1/3))/a^(1/3)/b^(5/3)+1/9*(-1)^(2/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/
3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)
```

Rubi [A]
time = 0.79, antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5399, 5389, 3384, 3379, 3382, 5400}

$$-\frac{\sqrt[3]{-1}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}+\frac{\sqrt[3]{-1}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}+\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}+\frac{(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}-\frac{d\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{5/3}}-\frac{(-1)^{2/3}d\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9\sqrt[3]{a}b^{5/3}}+\frac{\sqrt[3]{-1}d\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{5/3}}+\frac{(-1)^{2/3}d\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9\sqrt[3]{a}b^{5/3}}-\frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{5/3}}+\frac{\sqrt[3]{-1}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9\sqrt[3]{a}b^{5/3}}-\frac{x\cosh(c+dx)}{3b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] -1/3*(x*Cosh[c + d*x])/(b*(a + b*x^3)) - ((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a
^(2/3)*b^(4/3)) + ((-1)^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Cosh
Integral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(9*a^(2/3)*b^(4/3)) + (C
osh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^
(2/3)*b^(4/3)) - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/
3)*d)/b^(1/3)])/(9*a^(1/3)*b^(5/3)) - ((-1)^(2/3)*d*CoshIntegral[((-1)^(1/3
)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a^
(1/3)*b^(5/3)) + ((-1)^(1/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3
```

```
)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(1/3)*b^(5/3)) + (
(-1)^(2/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1
/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(1/3)*b^(5/3)) + ((-1)^(1/3)*Sinh[c + (
(-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
- d*x])/(9*a^(2/3)*b^(4/3)) - (d*Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral
[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + (Sinh[c - (a^(1/3)*d)/b^
(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3)) + ((-1)
^(1/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + ((-1)^(2/3)*Sinh[c - ((-1)
^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*
x])/(9*a^(2/3)*b^(4/3))
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^3)^2}\,dx = -\frac{x\cosh(c+dx)}{3b(a+bx^3)} + \frac{\int \frac{\cosh(c+dx)}{a+bx^3}\,dx}{3b} + \frac{d\int \frac{x\sinh(c+dx)}{a+bx^3}\,dx}{3b}$$

$$= -\frac{x\cosh(c+dx)}{3b(a+bx^3)} + \frac{\int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\,x\right)} - \frac{c}{3a^{2/3}\left(-\sqrt[3]{\right.}}\right.}{3b}$$

$$= -\frac{x\cosh(c+dx)}{3b(a+bx^3)} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{9a^{2/3}b} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\,x}\,dx}{9a^{2/3}b} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{\right.}}}{9a^{2/3}b}$$

$$= -\frac{x\cosh(c+dx)}{3b(a+bx^3)} - \frac{\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{9a^{2/3}b} - \frac{\left(d\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\right)}{9\sqrt[3]{a}}$$

$$= -\frac{x\cosh(c+dx)}{3b(a+bx^3)} - \frac{\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}} + \frac{(-1)^{2}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.17, size = 363, normalized size = 0.51

$$\frac{-\frac{6bx\cosh(c+dx)}{a+bx^3} - \text{RootSum}\left[a+b\#1^3\&, \frac{-\cosh(c+d\#1)\text{Chi}(d(x-\#1))+\text{Chi}(d(x-\#1))\sinh(c+d\#1)+\cosh(c+d\#1)\text{Shi}(d(x-\#1))-\sinh(c+d\#1)\text{Shi}(d(x-\#1))+d\cosh(c+d\#1)\text{Chi}(d(x-\#1))\#1-d\text{Chi}(d(x-\#1))\sinh(c+d\#1)\#1-d\cosh(c+d\#1)\text{Shi}(d(x-\#1))\#1+d\sinh(c+d\#1)\text{Shi}(d(x-\#1))\#1}{\#1^2}\&\right] + \text{RootSum}\left[a+b\#1^3\&, \frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1))+\text{Chi}(d(x-\#1))\sinh(c+d\#1)+\cosh(c+d\#1)\text{Shi}(d(x-\#1))+\sinh(c+d\#1)\text{Shi}(d(x-\#1))+d\cosh(c+d\#1)\text{Chi}(d(x-\#1))\#1+d\text{Chi}(d(x-\#1))\sinh(c+d\#1)\#1+d\cosh(c+d\#1)\text{Shi}(d(x-\#1))\#1+d\sinh(c+d\#1)\text{Shi}(d(x-\#1))\#1}{\#1^2}\&\right]}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] ((-6*b*x*Cosh[c + d*x])/(a + b*x^3) - RootSum[a + b*#1^3 & , (-(Cosh[c + d*
#1]*CoshIntegral[d*(x - #1)]) + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + C
osh[c + d*#1]*SinhIntegral[d*(x - #1)] - Sinh[c + d*#1]*SinhIntegral[d*(x -
 #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x
- #1)]*Sinh[c + d*#1]*#1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d
*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] + RootSum[a + b*#1^3
& , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sin
```

```
h[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*Sinh
Integral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 + d*Cos
hIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhIntegral[d*(
x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ])/(18*b
^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.15, size = 877, normalized size = 1.22

method	result	size
risch	Expression too large to display	877

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*d^3*exp(-d*x-c)/b/(b*d^3*x^3+a*d^3)*x-1/18/d/a/b^2*sum((3*_R1^2*b*c^2-
_R1*a*d^3-5*_R1*b*c^3-2*a*c*d^3+2*b*c^4+3*_R1*b*c^2+a*d^3-b*c^3)/(_R1^2-2*_
R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2
+a*d^3-b*c^3))+1/18/d*c^3/b/a*sum((_R1-c+2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*E
i(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d*
c^2/b/a*sum((_R1^2-_R1*c+_R1+c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+
c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c/a/b^2*sum(
(2*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3+2*_R1*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R
1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/
6*d^3*exp(d*x+c)/b/(b*d^3*x^3+a*d^3)*x+1/18/d/a/b^2*sum((3*_R1^2*b*c^2-_R1*
a*d^3-5*_R1*b*c^3-2*a*c*d^3+2*b*c^4-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c
+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d
^3-b*c^3))-1/18/d*c^3/b/a*sum((_R1-c-2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-
d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c^2/
b/a*sum((_R1^2-_R1*c-_R1-c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_
R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d*c/a/b^2*sum((2*_
R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3-2*_R1*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei
(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((d^2*x^3*e^(2*c) + 3*d*x^2*e^(2*c) + 12*x*e^(2*c))*e^(d*x) - (d^2*x^3
- 3*d*x^2 + 12*x)*e^(-d*x))/(b^2*d^3*x^6*e^c + 2*a*b*d^3*x^3*e^c + a^2*d^3*
e^c) + 1/2*integrate(-6*(a*d^2*x^2*e^c - 10*b*x^3*e^c + 3*a*d*x*e^c + 2*a*e
```

```
^c)*e^(d*x)/(b^3*d^3*x^9 + 3*a*b^2*d^3*x^6 + 3*a^2*b*d^3*x^3 + a^3*d^3), x)
 - 1/2*integrate(-6*(a*d^2*x^2 - 10*b*x^3 - 3*a*d*x + 2*a)*e^(-d*x)/(b^3*d^
3*x^9*e^c + 3*a*b^2*d^3*x^6*e^c + 3*a^2*b*d^3*x^3*e^c + a^3*d^3*e^c), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2050 vs. $2(500) = 1000$.
time = 0.52, size = 2050, normalized size = 2.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*a*d*x*cosh(d*x + c) - ((a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3
+ a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c
)^2) - (a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2
- (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((-
a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3
 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*((b*x^3 +
sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) +
 a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1
/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - ((a*d^3/b)^(2/3)*((b*x^3 + sqrt(-
3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*si
nh(d*x + c)^2) - (a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d
*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x + 1
/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
- c) + ((-a*d^3/b)^(2/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^
2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*
((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*
x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) -
1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*((-a*d^3/b)^(2/3)*((b
*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)
*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(-d*x + (-a
*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*((a*d^3/b)^(2/3)*((b*x^3 + a)
*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((b*x^3 +
 a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3
))*cosh(-c + (a*d^3/b)^(1/3)) - ((a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3
+ a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c
)^2) - (a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2
- (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((-
a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3
 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*((b*x^3 +
sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) +
```

```
 a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1
/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + ((a*d^3/b)^(2/3)*((b*x^3 + sqrt(-
3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*si
nh(d*x + c)^2) - (a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d
*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x + 1
/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
- c) - ((-a*d^3/b)^(2/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^
2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*
((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*
x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) -
1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*((-a*d^3/b)^(2/3)*((b
*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)
*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(-d*x + (-a
*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 2*((a*d^3/b)^(2/3)*((b*x^3 + a)
*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((b*x^3 +
 a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3
))*sinh(-c + (a*d^3/b)^(1/3)))/((a*b^2*d*x^3 + a^2*b*d)*cosh(d*x + c)^2 - (
a*b^2*d*x^3 + a^2*b*d)*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**3+a)**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(x^3*cosh(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\cosh(c+d\,x)}{{\left(b\,x^3+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*cosh(c + d*x))/(a + b*x^3)^2,x)
[Out] int((x^3*cosh(c + d*x))/(a + b*x^3)^2, x)
```

3.103 $\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2} \, dx$

Optimal. Leaf size=373

$$-\frac{\cosh(c+dx)}{3b\,(a+bx^3)}+\frac{d\text{Chi}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}-\frac{\sqrt[3]{-1}\,d\text{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}}{\sqrt[3]{b}}\right.}{9a^{2/3}b^{4/3}}$$

```
[Out] -1/3*cosh(d*x+c)/b/(b*x^3+a)-1/9*(-1)^(1/3)*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b
^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(4/3)+1/9*d*cosh(c
-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(4/3)+1/9*(-1)^(2/
3)*d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+
d*x)/a^(2/3)/b^(4/3)+1/9*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1
/3))/a^(2/3)/b^(4/3)-1/9*(-1)^(1/3)*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)
*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)+1/9*(-1)^(2/3)*d*Chi(
-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(
2/3)/b^(4/3)
```

Rubi [A]
time = 0.44, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5397, 5388, 3384, 3379, 3382}

$$\frac{d\sinh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}-\frac{\sqrt[3]{-1}\,d\sinh\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}+\frac{(-1)^{2/3}d\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}+\frac{\sqrt[3]{-1}\,d\cosh\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}+\frac{d\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}+\frac{(-1)^{2/3}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}-\frac{\cosh(c+dx)}{3b\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] -1/3*Cosh[c + d*x]/(b*(a + b*x^3)) + (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) +
d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(2/3)*b^(4/3)) - ((-1)^(1/3)*d*Cos
hIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3
)*d)/b^(1/3)])/(9*a^(2/3)*b^(4/3)) + ((-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/
3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*
a^(2/3)*b^(4/3)) + ((-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*S
inhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(2/3)*b^(4/3)) + (d
*Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*
a^(2/3)*b^(4/3)) + ((-1)^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*S
inhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3))
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2}\,dx = -\frac{\cosh(c+dx)}{3b\,(a+bx^3)} + \frac{d\int \frac{\sinh(c+dx)}{a+bx^3}\,dx}{3b}$$

$$= -\frac{\cosh(c+dx)}{3b\,(a+bx^3)} + \frac{d\int \left(-\frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} - \frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)} - \frac{\mathrm{s}}{3a^{2/3}\left(-\sqrt[3]{}\right.}\right.}{3b}$$

$$= -\frac{\cosh(c+dx)}{3b\,(a+bx^3)} - \frac{d\int \frac{\sinh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{9a^{2/3}b} - \frac{d\int \frac{\sinh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x}\,dx}{9a^{2/3}b} - \frac{d\int \frac{\sinh(c+d}{-\sqrt[3]{a}-(-1)^{2/}}}{9a^{2/3}b}$$

$$= -\frac{\cosh(c+dx)}{3b\,(a+bx^3)} - \frac{\left(d\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{9a^{2/3}b} - \frac{\left(id\cosh\left(c+\frac{\sqrt[3]{-}}{}\right.\right.}{}$$

$$= -\frac{\cosh(c+dx)}{3b\,(a+bx^3)} + \frac{d\mathrm{Chi}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} - \frac{\sqrt[3]{-1}\,d\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right.}{9a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.13, size = 203, normalized size = 0.54

$$\frac{-\frac{6b\cosh(c+dx)}{a+bx^3} - d\,\mathrm{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1))-\mathrm{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\mathrm{Shi}(d(x-\#1))+\sinh(c+d\#1)\mathrm{Shi}(d(x-\#1))}{\#1^2}\&\right] + d\,\mathrm{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1))+\mathrm{Chi}(d(x-\#1))\sinh(c+d\#1)+\cosh(c+d\#1)\mathrm{Shi}(d(x-\#1))+\sinh(c+d\#1)\mathrm{Shi}(d(x-\#1))}{\#1^2}\&\right]}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] ((-6*b*Cosh[c + d*x])/(a + b*x^3) - d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1
]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh
[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1
)])/#1^2 & ] + d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x -
 #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegr
al[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ])/(18*b^2
)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.10, size = 594, normalized size = 1.59

method	result

risch	$-\frac{d^3 e^{-dx-c}}{6b(b\,d^3x^3+a\,d^3)}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{\left(2\_R1^2bc-3\_R1b\,c^2-a\,d^3+b\,c^3+2\_R1bc\right)e^{-\_R1}\,\mathrm{expI}}{\_R1^2-2\_R1c+c^2}}{18a\,b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*d^3*exp(-d*x-c)/b/(b*d^3*x^3+a*d^3)-1/18/a/b^2*sum((2*_R1^2*b*c-3*_R1*
b*c^2-a*d^3+b*c^3+2*_R1*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),
_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18*c^2/b/a*sum((_R1
-c+2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^
2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9*c/b/a*sum((_R1^2-_R1*c+_R1+c)/(_R1^2-2*_
R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2
+a*d^3-b*c^3))-1/6*d^3*exp(d*x+c)/b/(b*d^3*x^3+a*d^3)+1/18/a/b^2*sum((2*_R1
^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3-2*_R1*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1
,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/18*c^2
/b/a*sum((_R1-c-2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf
(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/9*c/b/a*sum((_R1^2-_R1*c-_R1-
c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b
*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((d*x^2*e^(2*c) + 4*x*e^(2*c))*e^(d*x) - (d*x^2 - 4*x)*e^(-d*x))/(b^2*d
^2*x^6*e^c + 2*a*b*d^2*x^3*e^c + a^2*d^2*e^c) + 1/2*integrate(2*(10*b*x^3*e
^c - 3*a*d*x*e^c - 2*a*e^c)*e^(d*x)/(b^3*d^2*x^9 + 3*a*b^2*d^2*x^6 + 3*a^2*
b*d^2*x^3 + a^3*d^2), x) + 1/2*integrate(2*(10*b*x^3 + 3*a*d*x - 2*a)*e^(-d
*x)/(b^3*d^2*x^9*e^c + 3*a*b^2*d^2*x^6*e^c + 3*a^2*b*d^2*x^3*e^c + a^3*d^2*
e^c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1276 vs. 2(263) = 526.
time = 0.41, size = 1276, normalized size = 3.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*((a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2
- (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b
)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*
d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 +
 sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*
(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + (a*d^3/b)^(
1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3
)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3)
- 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (-a*d^3/b)^(1/3)*((b*x
^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 +
 a) + a)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*co
sh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(-a*d^3/b)^(1/3)*((b*x^3 +
a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3
))*cosh(c + (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c
)^2 - (b*x^3 + a)*sinh(d*x + c)^2)*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d
^3/b)^(1/3)) + (a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x
 + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(d*x - 1/2*
(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c
) + (-a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 -
(b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b
)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*
d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 -
 sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(s
qrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - (-a*d^3/b)^(1/
3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*
(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3)
- 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(1/3)*((
b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2)*Ei(-d*x + (-a*d^3
/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) + 2*(a*d^3/b)^(1/3)*((b*x^3 + a)*cosh
(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2)*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-
c + (a*d^3/b)^(1/3)) + 12*a*cosh(d*x + c))/((a*b^2*x^3 + a^2*b)*cosh(d*x +
c)^2 - (a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\cosh(c+d\,x)}{{\left(b\,x^3+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(c + d*x))/(a + b*x^3)^2,x)
```

```
[Out] int((x^2*cosh(c + d*x))/(a + b*x^3)^2, x)
```

3.104 $\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=695

$$\frac{\cosh(c+dx)}{3abx}-\frac{\cosh(c+dx)}{3bx\,(a+bx^3)}-\frac{(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}}+\frac{\sqrt[3]{-1}\,\cosh\left(c-\frac{(-}{}\right.}{}$$

```
[Out] -1/9*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)-1
/9*(-1)^(2/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/
3)*d/b^(1/3))/a^(4/3)/b^(2/3)+1/9*(-1)^(1/3)*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1
/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)+1/3*cosh(d*x+
c)/a/b/x-1/3*cosh(d*x+c)/b/x/(b*x^3+a)-1/9*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^
(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a/b-1/9*d*cosh(c-a^(1/3)*d/b^
(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a/b-1/9*d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(
1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a/b-1/9*d*Chi(a^(1/3)*d/b^(1/3)
+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a/b-1/9*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a
^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)-1/9*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*
x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a/b-1/9*(-1)^(2/3)*Shi(-(-1)^(1/3)*
a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)
-1/9*d*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b
^(1/3))/a/b+1/9*(-1)^(1/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1
)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)
```

Rubi [A]
time = 0.93, antiderivative size = 695, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5399, 5401, 3378, 3384, 3379, 3382, 5400}

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] Cosh[c + d*x]/(3*a*b*x) - Cosh[c + d*x]/(3*b*x*(a + b*x^3)) - ((-1)^(2/3)*C
osh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)
/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) + ((-1)^(1/3)*Cosh[c - ((-1)^(2/3)*a^(
1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(9*
a^(4/3)*b^(2/3)) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/
b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) +
 d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(9*a*b) - (d*CoshIntegral[((-1)^(1/3)*
a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a*b)
 - (d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a*b) + (d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(
```

```
1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a*b) + ((-1)^(
2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1
/3)*d)/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) - (d*Cosh[c - (a^(1/3)*d)/b^(1/3
)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b) - (Sinh[c - (a^(1/3)*d)
/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) - (d
*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*
d)/b^(1/3) + d*x])/(9*a*b) + ((-1)^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^
(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/
3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx &= -\frac{\cosh(c+dx)}{3bx\,(a+bx^3)} - \frac{\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx}{3b} + \frac{d\int \frac{\sinh(c+dx)}{x(a+bx^3)}\,dx}{3b}\\
&= -\frac{\cosh(c+dx)}{3bx\,(a+bx^3)} - \frac{\int \left(\frac{\cosh(c+dx)}{ax^2} - \frac{bx\cosh(c+dx)}{a(a+bx^3)}\right)dx}{3b} + \frac{d\int \left(\frac{\sinh(c+dx)}{ax} - \frac{bx^2\sinh(c+dx)}{a(a+bx^3)}\right) d\ldots}{3b}\\
&= -\frac{\cosh(c+dx)}{3bx\,(a+bx^3)} + \frac{\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx}{3a} - \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{3ab} - \frac{d\int \frac{x^2\sinh(c+dx)}{a+bx^3}\,dx}{3a} + \frac{d\int \frac{\sinh(c+\ldots}{x}}{3ab\ldots}\\
&= \frac{\cosh(c+dx)}{3abx} - \frac{\cosh(c+dx)}{3bx\,(a+bx^3)} + \frac{\int \left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{(-1)^{2/3}\cosh(c+dx\ldots}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\ldots\right.}\ldots\right.}{3a}\\
&= \frac{\cosh(c+dx)}{3abx} - \frac{\cosh(c+dx)}{3bx\,(a+bx^3)} + \frac{d\mathrm{Chi}(dx)\sinh(c)}{3ab} + \frac{d\cosh(c)\mathrm{Shi}(dx)}{3ab} - \frac{\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}\ldots}}{9a^{4/3}\sqrt[3]{b}\ldots}\\
&= \frac{\cosh(c+dx)}{3abx} - \frac{\cosh(c+dx)}{3bx\,(a+bx^3)} - \frac{\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{4/3}\sqrt[3]{b}} - \frac{\left(d\cosh\left(\ldots\right.\right.}{\ldots}\\
&= \frac{\cosh(c+dx)}{3abx} - \frac{\cosh(c+dx)}{3bx\,(a+bx^3)} - \frac{(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-\ldots\right.}{9a^{4/3}b^{2/3}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.15, size = 387, normalized size = 0.56

$$
\frac{6bx^2\cosh(c+dx)+(a+bx^3)\,\mathrm{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1))-\mathrm{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\mathrm{Shi}(d(x-\#1))+\sinh(c+d\#1)\mathrm{Shi}(d(x-\#1))+d\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1))\#1-d\mathrm{Chi}(d(x-\#1))\sinh(c+d\#1)\#1-d\cosh(c+d\#1)\mathrm{Shi}(d(x-\#1))\#1+d\sinh(c+d\#1)\mathrm{Shi}(d(x-\#1))\#1}{\#1}\&\right]-(a+bx^3)\,\mathrm{RootSum}\left[a+b\#1^3\&,\frac{-\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1))-\mathrm{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\mathrm{Shi}(d(x-\#1))-\sinh(c+d\#1)\mathrm{Shi}(d(x-\#1))+d\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1))\#1+d\mathrm{Chi}(d(x-\#1))\sinh(c+d\#1)\#1+d\cosh(c+d\#1)\mathrm{Shi}(d(x-\#1))\#1+d\sinh(c+d\#1)\mathrm{Shi}(d(x-\#1))\#1}{\#1}\&\right]}{18ab\,(a+bx^3)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] (6*b*x^2*Cosh[c + d*x] + (a + b*x^3)*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]
*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[
c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)
] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x - #1
)]*Sinh[c + d*#1]*#1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sin
h[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1 & ] - (a + b*x^3)*RootSum[a + b
*#1^3 & , (-(Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]) - CoshIntegral[d*(x -
 #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - Sinh[c + d
*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#
1 + d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhInt
egral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1 & ]
)/(18*a*b*(a + b*x^3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.06, size = 395, normalized size = 0.57

method	result
risch	$\frac{d^{3} \mathrm{e}^{-dx-c} x^{2}}{6a\left(b\,d^{3} x^{3}+a\,d^{3}\right)}-\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{3}-3bc\_Z^{2}+3b\,c^{2}\_Z+a\,d^{3}-b\,c^{3}\right)}\frac{\left(\_R1^{2}-\_R1c+\_R1+c\right)\mathrm{e}^{-\_R1}\,\mathrm{expIntegral}\left(1,dx-\_\right.}{\_R1^{2}-2\_R1c+c^{2}}\right.}{18ba}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*d^3*exp(-d*x-c)*x^2/a/(b*d^3*x^3+a*d^3)-1/18*d/b/a*sum((_R1^2-_R1*c+_R1
+c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*
b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/18*d*c/b/a*sum((_R1-c+2)/(_R1^2-2*_R1*c+c^2)
*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*
c^3))+1/6*d^3*exp(d*x+c)*x^2/a/(b*d^3*x^3+a*d^3)+1/18*d/b/a*sum((_R1^2-_R1*
c-_R1-c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*
_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18*d*c/b/a*sum((_R1-c-2)/(_R1^2-2*_R1*c
+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d
^3-b*c^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b^2*d*x^6*e^c + 2*a*b*d*x^3*e^c + a^2*d
*e^c) + 1/2*integrate((5*b*x^3*e^c - a*e^c)*e^(d*x)/(b^3*d*x^9 + 3*a*b^2*d*
x^6 + 3*a^2*b*d*x^3 + a^3*d), x) - 1/2*integrate((5*b*x^3 - a)*e^(-d*x)/(b^
3*d*x^9*e^c + 3*a*b^2*d*x^6*e^c + 3*a^2*b*d*x^3*e^c + a^3*d*e^c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2135 vs. $2(507) = 1014$.
time = 0.52, size = 2135, normalized size = 3.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*b*d^2*x^2*cosh(d*x + c) - (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x +
c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (a*d^3/b)^(2/3)*((b^2*x^
3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b - sqrt
(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(
-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (2*(a*b*d^3*x^3 +
a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (-a*
d^3/b)^(2/3)*((b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 -
(b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*
(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) -
 c) - (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3
)*sinh(d*x + c)^2 - (a*d^3/b)^(2/3)*((b^2*x^3 + a*b + sqrt(-3)*(b^2*x^3 + a
*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b + sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x
+ c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1
/3)*(sqrt(-3) - 1) - c) + (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a
*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (-a*d^3/b)^(2/3)*((b^2*x^3 + a*b +
sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b + sqrt(-3)*(b^2*
x^3 + a*b))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1)
)*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*((a*b*d^3*x^3 + a^2*d^3
)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (-a*d^3/b)^(2
/3)*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a*b)*sinh(d*x + c)^2))*Ei
(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) - 2*((a*b*d^3*x^3 + a^
2*d^3)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (a*d^3/b
)^(2/3)*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a*b)*sinh(d*x + c)^2)
)*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) - (2*(a*b*d^3*x^3 +
a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (a*d
^3/b)^(2/3)*((b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (
b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a
*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c)
+ (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*si
nh(d*x + c)^2 + (-a*d^3/b)^(2/3)*((b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b)
)*cosh(d*x + c)^2 - (b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c
```

```
)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1
/3)*(sqrt(-3) + 1) - c) + (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a
*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (a*d^3/b)^(2/3)*((b^2*x^3 + a*b + s
qrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b + sqrt(-3)*(b^2*x
^3 + a*b))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*s
inh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - (2*(a*b*d^3*x^3 + a^2*d^3)*co
sh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (-a*d^3/b)^(2/3
)*((b^2*x^3 + a*b + sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 +
a*b + sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^
(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*((a
*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d*x +
c)^2 - (-a*d^3/b)^(2/3)*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a*b)*
sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) +
2*((a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d
*x + c)^2 + (a*d^3/b)^(2/3)*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a
*b)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)))
/((a^2*b^2*d^2*x^3 + a^3*b*d^2)*cosh(d*x + c)^2 - (a^2*b^2*d^2*x^3 + a^3*b*
d^2)*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x**3+a)**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(x*cosh(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\cosh(c+d\,x)}{{\left(b\,x^3+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(c + d*x))/(a + b*x^3)^2,x)
[Out] int((x*cosh(c + d*x))/(a + b*x^3)^2, x)
```

3.105 $\int \frac{\cosh(c+dx)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=739

$$\frac{\cosh(c+dx)}{3abx^2}-\frac{\cosh(c+dx)}{3bx^2\,(a+bx^3)}-\frac{2\sqrt[3]{-1}\,\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}}+\frac{2(-1)^{2/3}\cosh\left(c-\right.}{}$$

[Out] 2/9*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)-2/9*(-1)^(1/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)+2/9*(-1)^(2/3)*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)+1/3*cosh(d*x+c)/a/b/x^2-1/3*cosh(d*x+c)/b/x^2/(b*x^3+a)+1/9*(-1)^(2/3)*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/3)+1/9*d*cosh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/3)-1/9*(-1)^(1/3)*d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/3)+1/9*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)+2/9*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)+1/9*(-1)^(2/3)*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)-2/9*(-1)^(1/3)*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)-1/9*(-1)^(1/3)*d*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)+2/9*(-1)^(2/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)

Rubi [A]
time = 1.00, antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5387, 5401, 3378, 3384, 3379, 3382, 5389, 5400}

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*x^3)^2,x]

[Out] Cosh[c + d*x]/(3*a*b*x^2) - Cosh[c + d*x]/(3*b*x^2*(a + b*x^3)) - (2*(-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/3)*b^(1/3)) + (2*(-1)^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(9*a^(5/3)*b^(1/3)) + (2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) + (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) + ((-1)^(2/3)*d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(

```
1/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) - ((-1)^(1/3)*d*CoshIntegral[
-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^
(1/3)])/(9*a^(4/3)*b^(2/3)) - ((-1)^(2/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)
/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(4/3)*b^
(2/3)) + (2*(-1)^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/3)*b^(1/3)) + (d*Cosh[c -
(a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^
(2/3)) + (2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3)
+ d*x])/(9*a^(5/3)*b^(1/3)) - ((-1)^(1/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^
(2/3)) + (2*(-1)^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegra
l[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5387

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Si
mp[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1))), x] + (-Dis
t[(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Cosh[c + d*x])/x^n, x],
x] - Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x]
, x], x]) /; FreeQ[{a, b, c, d}, x] && IntegerQ[p] && IGtQ[n, 0] && LtQ[p,
```

```
-1] && GtQ[n, 2]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{(a+bx^3)^2}\,dx = -\frac{\cosh(c+dx)}{3bx^2(a+bx^3)} - \frac{2\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx}{3b} + \frac{d\int \frac{\sinh(c+dx)}{x^2(a+bx^3)}\,dx}{3b}$$

$$= -\frac{\cosh(c+dx)}{3bx^2(a+bx^3)} - \frac{2\int \left(\frac{\cosh(c+dx)}{ax^3} - \frac{b\cosh(c+dx)}{a(a+bx^3)}\right)dx}{3b} + \frac{d\int \left(\frac{\sinh(c+dx)}{ax^2} - \frac{bx\sinh(c+dx)}{a(a+bx^3)}\right) \cdots}{3b}$$

$$= -\frac{\cosh(c+dx)}{3bx^2(a+bx^3)} + \frac{2\int \frac{\cosh(c+dx)}{a+bx^3}\,dx}{3a} - \frac{2\int \frac{\cosh(c+dx)}{x^3}\,dx}{3ab} - \frac{d\int \frac{x\sinh(c+dx)}{a+bx^3}\,dx}{3a} + \frac{d\int \frac{\sinh(\cdots}{\cdots}}{3\cdots}$$

$$= \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2(a+bx^3)} - \frac{d\sinh(c+dx)}{3abx} + \frac{2\int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} - \frac{\cdots}{3a^{2/3}\cdots}\right.}{\cdots}$$

$$= \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2(a+bx^3)} - \frac{2\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{9a^{5/3}} - \frac{2\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\,x}\,dx}{9a^{5/3}} - \cdots$$

$$= \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2(a+bx^3)} + \frac{d^2\cosh(c)\text{Chi}(dx)}{3ab} + \frac{d^2\sinh(c)\text{Shi}(dx)}{3ab} - \frac{(d^2\cosh(\cdots}{\cdots}$$

$$= \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2(a+bx^3)} - \frac{2\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}} - \cdots\right.}{9a^{5/3}\sqrt[3]{b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.15, size = 387, normalized size = 0.52

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x^3)^2,x]
```

```
[Out] (6*b*x*Cosh[c + d*x] + (a + b*x^3)*RootSum[a + b*#1^3 & , (2*Cosh[c + d*#1]
*CoshIntegral[d*(x - #1)] - 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - 2*C
osh[c + d*#1]*SinhIntegral[d*(x - #1)] + 2*Sinh[c + d*#1]*SinhIntegral[d*(x
 - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(
x - #1)]*Sinh[c + d*#1]*#1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 +
 d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] - (a + b*x^3)*RootS
um[a + b*#1^3 & , (-2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - 2*CoshInteg
ral[d*(x - #1)]*Sinh[c + d*#1] - 2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]
- 2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral
```

```
[d*(x - #1)]*#1 + d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c +
 d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #
1)]*#1)/#1^2 & ])/(18*a*b*(a + b*x^3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.02, size = 226, normalized size = 0.31

method	result
risch	$\frac{d^{3} \mathrm{e}^{-d x -c} x}{6 a \left(b \,d^{3} x^{3}+a \,d^{3}\right)}-\frac{d^{2} \left(\sum_{\_R1 =\mathrm{RootOf}\left(b \,\_Z^{3}-3 b c \,\_Z^{2}+3 b \,c^{2} \_Z +a \,d^{3}-b \,c^{3}\right)}\frac{\left(\_R1 -c +2\right) \mathrm{e}^{-\_R1} \mathrm{expIntegral}\left(1, d x -\_R1 +c \right)}{\_R1^{2}-2 \_R1 c +c^{2}}\right)}{18 b a}+\frac{}{6a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*d^3*exp(-d*x-c)*x/a/(b*d^3*x^3+a*d^3)-1/18*d^2/b/a*sum((_R1-c+2)/(_R1^2
-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b
*c^2+a*d^3-b*c^3))+1/6*d^3*exp(d*x+c)*x/a/(b*d^3*x^3+a*d^3)+1/18*d^2/b/a*su
m((_R1-c-2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b
-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/(b*x^3 + a)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2048 vs. 2(519) = 1038.
time = 0.41, size = 2048, normalized size = 2.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*d*x*cosh(d*x + c) - ((a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 +
 a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)
^2) + 2*(a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2
 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((
```

```
-a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^
3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) + 2*(-a*d^3/b)^(1/3)*((b*x^3
 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a
) + a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cos
h(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - ((a*d^3/b)^(2/3)*((b*x^3 + sqr
t(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)
*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*c
osh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*
x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3)
- 1) - c) + ((-a*d^3/b)^(2/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x
+ c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) + 2*(-a*d^3/b)
^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(
-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt
(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*((-a*d^3/b)^(2
/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) + 2*(-a*d^3
/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(-
d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*((a*d^3/b)^(2/3)*((b
*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3
)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(d*x + (a*
d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) - ((a*d^3/b)^(2/3)*((b*x^3 - sqrt(
-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*s
inh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cos
h(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x
- 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) +
1) + c) + ((-a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x +
c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) + 2*(-a*d^3/b)^(
1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3
)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-
3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + ((a*d^3/b)^(2/3)*(
(b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x
^3 + a) + a)*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3
 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x +
c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3
)*(sqrt(-3) - 1) - c) - ((-a*d^3/b)^(2/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) +
a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) +
2*(-a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b
*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)
^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*((
-a*d^3/b)^(2/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2)
 + 2*(-a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x +
 c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 2*((a*d^3/
b)^(2/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) + 2*(a
*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*
Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)))/((a^2*b*d*x^3 + a^3*d
)*cosh(d*x + c)^2 - (a^2*b*d*x^3 + a^3*d)*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)}{{\left(b\,x^3+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*x^3)^2,x)
```

```
[Out] int(cosh(c + d*x)/(a + b*x^3)^2, x)
```

3.106 $\int \frac{\cosh(c+dx)}{x\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=697

$$\frac{\cosh(c+dx)}{3abx^3}-\frac{\cosh(c+dx)}{3bx^3\left(a+bx^3\right)}+\frac{\cosh(c)\text{Chi}(dx)}{a^2}-\frac{\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a^2}-\frac{\cosh\left(c\right.}{}$$

```
[Out] Chi(d*x)*cosh(c)/a^2-1/3*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3
))/a^2-1/3*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3)*
d/b^(1/3))/a^2-1/3*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)
*a^(1/3)*d/b^(1/3))/a^2+1/3*cosh(d*x+c)/a/b/x^3-1/3*cosh(d*x+c)/b/x^3/(b*x^
3+a)+1/9*(-1)^(1/3)*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*
a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)-1/9*d*cosh(c-a^(1/3)*d/b^(1/3))*Shi(
a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)-1/9*(-1)^(2/3)*d*cosh(c-(-1)^(2/3)*a
^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)+Shi
(d*x)*sinh(c)/a^2-1/9*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3)
)/a^(5/3)/b^(1/3)-1/3*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/
a^2+1/9*(-1)^(1/3)*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c+(-1)^(1/3
)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)-1/3*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+
d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^2-1/9*(-1)^(2/3)*d*Chi(-(-1)^(2
/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(
1/3)-1/3*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/
b^(1/3))/a^2
```

Rubi [A]
time = 1.03, antiderivative size = 697, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {5399, 5401, 3378, 3384, 3379, 3382, 5400, 5388}

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x^3)^2),x]
```

```
[Out] Cosh[c + d*x]/(3*a*b*x^3) - Cosh[c + d*x]/(3*b*x^3*(a + b*x^3)) + (Cosh[c]*
CoshIntegral[d*x])/a^2 - (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshInte
gral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) - (Cosh[c - ((-1)^(2/3)
*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])
/(3*a^2) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3)
+ d*x])/(3*a^2) - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1
/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) + ((-1)^(1/3)*d*CoshIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a
^(5/3)*b^(1/3)) - ((-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/
```

```
3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) +
(Sinh[c]*SinhIntegral[d*x])/a^2 - ((-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3
)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/3
)*b^(1/3)) + (Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) - (d*Cosh[c - (a^(1/3)*d)/b^(1/3)]*
SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - (Sinh[c - (a
^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^2) - ((-1)
^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - (Sinh[c - ((-1)^(2/3)*a^(1
/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
```

```
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x(a+bx^3)^2}\,dx = -\frac{\cosh(c+dx)}{3bx^3(a+bx^3)} - \frac{\int \frac{\cosh(c+dx)}{x^4(a+bx^3)}\,dx}{b} + \frac{d\int \frac{\sinh(c+dx)}{x^3(a+bx^3)}\,dx}{3b}$$

$$= -\frac{\cosh(c+dx)}{3bx^3(a+bx^3)} - \frac{\int \left(\frac{\cosh(c+dx)}{ax^4} - \frac{b\cosh(c+dx)}{a^2x} + \frac{b^2x^2\cosh(c+dx)}{a^2(a+bx^3)}\right)dx}{b} + \frac{d\int \left(\frac{\sinh(c+dx)}{ax^3} - \frac{b}{\cdots}\right.}{3b}$$

$$= -\frac{\cosh(c+dx)}{3bx^3(a+bx^3)} + \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a^2} - \frac{\int \frac{\cosh(c+dx)}{x^4}\,dx}{ab} - \frac{b\int \frac{x^2\cosh(c+dx)}{a+bx^3}\,dx}{a^2} - \frac{d\int \frac{\sinh(c+dx)}{a+bx^3}\cdots}{3a}$$

$$= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3(a+bx^3)} - \frac{d\sinh(c+dx)}{6abx^2} - \frac{b\int \left(\frac{\cosh(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{\cosh\cdots}{3b^{2/3}\left(-\sqrt[3]{\cdots}\right.}\right.}{\cdots}$$

$$= \frac{\cosh(c+dx)}{3abx^3} - \frac{d^2\cosh(c+dx)}{6abx} - \frac{\cosh(c+dx)}{3bx^3(a+bx^3)} + \frac{\cosh(c)\mathrm{Chi}(dx)}{a^2} + \frac{\sinh(c)\mathrm{Shi}(dx)}{a^2}\cdots$$

$$= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3(a+bx^3)} + \frac{\cosh(c)\mathrm{Chi}(dx)}{a^2} + \frac{\sinh(c)\mathrm{Shi}(dx)}{a^2} - \frac{d^3\int \frac{\sinh(c+dx)}{x}\,dx}{6ab}\cdots$$

$$= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3(a+bx^3)} + \frac{\cosh(c)\mathrm{Chi}(dx)}{a^2} - \frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-}\cdots}{\cdots}\right.}{3a^2}$$

$$= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3(a+bx^3)} + \frac{\cosh(c)\mathrm{Chi}(dx)}{a^2} - \frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-}\cdots}{\cdots}\right.}{3a^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.43, size = 411, normalized size = 0.59

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^3)^2),x]
```

```
[Out] ((6*a*Cosh[c]*Cosh[d*x])/(a + b*x^3) + 18*Cosh[c]*CoshIntegral[d*x] - 3*Roo
tSum[a + b*#1^3 & , Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[
d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh
[c + d*#1]*SinhIntegral[d*(x - #1)] & ] - 3*RootSum[a + b*#1^3 & , Cosh[c +
 d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] +
```

```
Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x
 - #1)] & ] + (a*d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x
 - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhInte
gral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ])/b - (
a*d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshI
ntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)
] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ])/b + (6*a*Sinh[c]*Sin
h[d*x])/(a + b*x^3) + 18*Sinh[c]*SinhIntegral[d*x])/(18*a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.05, size = 338, normalized size = 0.48

method	result
risch	$\frac{e^{-dx-c} d^3}{6a\left((dx+c)^3 b-3(dx+c)^2 bc+3(dx+c)b\,c^2+a\,d^3-b\,c^3\right)}-\frac{e^{-c}\,\mathrm{expIntegral}(1,dx)}{2a^2}+\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3bc\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*exp(-d*x-c)*d^3/a/((d*x+c)^3*b-3*(d*x+c)^2*b*c+3*(d*x+c)*b*c^2+a*d^3-b*
c^3)-1/2/a^2*exp(-c)*Ei(1,d*x)+1/18/a^2/b*sum((-a*d^3+3*_R1^2*b-6*_R1*b*c+3
*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_
Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6*exp(d*x+c)*d^3/a/((d*x+c)^3*b-3*(d*x+c
)^2*b*c+3*(d*x+c)*b*c^2+a*d^3-b*c^3)-1/2/a^2*exp(c)*Ei(1,-d*x)+1/18/a^2/b*s
um((a*d^3+3*_R1^2*b-6*_R1*b*c+3*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d
*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^3 + a)^2*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1773 vs. 2(509) = 1018.
time = 0.48, size = 1773, normalized size = 2.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/x/(b*x^3+a)^2,x, algorithm="fricas")

[Out] -1/36*((6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + (6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(3*(b*x^3 + a)*cosh(d*x + c)^2 - 3*(b*x^3 + a)*sinh(d*x + c)^2 + (-a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*(3*(b*x^3 + a)*cosh(d*x + c)^2 - 3*(b*x^3 + a)*sinh(d*x + c)^2 + (a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + (6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - (6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(3*(b*x^3 + a)*cosh(d*x + c)^2 - 3*(b*x^3 + a)*sinh(d*x + c)^2 + (-a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 2*(3*(b*x^3 + a)*cosh(d*x + c)^2 - 3*(b*x^3 + a)*sinh(d*x + c)^2 + (a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) - 12*a*cosh(d*x + c) - 18*((b*x^3 + a)*Ei(d*x) + (b*x^3 + a)*Ei(-d*x))*cosh(c) - 18*((b*x^3 + a)*Ei(d*x) - (b*x^3 + a)*Ei(-d*x))*sinh(c))/((a^2*b*x^3 + a^3)*cosh(

```
d*x + c)^2 - (a^2*b*x^3 + a^3)*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x**3+a)**2,x)
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)}{x\,\left(b\,x^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(x*(a + b*x^3)^2),x)
[Out] int(cosh(c + d*x)/(x*(a + b*x^3)^2), x)
```

3.107 $\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3} \, dx$

Optimal. Leaf size=784

$$-\frac{x^3 \cosh(c+dx)}{6b\,(a+bx^3)^2}-\frac{\cosh(c+dx)}{6b^2\,(a+bx^3)}-\frac{(-1)^{2/3}d^2\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{54\sqrt[3]{a}\,b^{8/3}}+\frac{\sqrt[3]{-1}\,d^2\cosh}{}$$

```
[Out] -1/54*d^2*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(1/3)/b^(8
/3)-1/54*(-1)^(2/3)*d^2*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(
1/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^(8/3)+1/54*(-1)^(1/3)*d^2*Chi(-(-1)^(2/3)
*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^(8/3
)-1/6*x^3*cosh(d*x+c)/b/(b*x^3+a)^2-1/6*cosh(d*x+c)/b^2/(b*x^3+a)-2/27*(-1)
^(1/3)*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(
1/3)+d*x)/a^(2/3)/b^(7/3)+2/27*d*cosh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^
(1/3)+d*x)/a^(2/3)/b^(7/3)+2/27*(-1)^(2/3)*d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^
(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(7/3)+2/27*d*Chi(a^(
1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(7/3)-1/54*d^2*Shi(
a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(1/3)/b^(8/3)-2/27*(-1)^
(1/3)*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b
^(1/3))/a^(2/3)/b^(7/3)-1/54*(-1)^(2/3)*d^2*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/
3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^(8/3)+2/27*(-1)^(2/3
)*d*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1
/3))/a^(2/3)/b^(7/3)+1/54*(-1)^(1/3)*d^2*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d
*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^(8/3)-1/18*d*x*sinh(d*x+
c)/b^2/(b*x^3+a)
```

Rubi [A]
time = 1.16, antiderivative size = 784, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {5399, 5397, 5388, 3384, 3379, 3382, 5398, 5401}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(x^5*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -1/6*(x^3*Cosh[c + d*x])/(b*(a + b*x^3)^2) - Cosh[c + d*x]/(6*b^2*(a + b*x^
3)) - ((-1)^(2/3)*d^2*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral
[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(1/3)*b^(8/3)) + ((-1)^(1/3)*
d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(
1/3)*d)/b^(1/3)) - d*x])/(54*a^(1/3)*b^(8/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b
^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(1/3)*b^(8/3)) + (2*
d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(2
```

```
7*a^(2/3)*b^(7/3)) - (2*(-1)^(1/3)*d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^
(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(2/3)*b^(7/3))
 + (2*(-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Si
nh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(2/3)*b^(7/3)) - (d*x*Sinh[c
+ d*x])/(18*b^2*(a + b*x^3)) + (2*(-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)
*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(2/3
)*b^(7/3)) + ((-1)^(2/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhI
ntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(1/3)*b^(8/3)) + (2*d*
Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*
a^(2/3)*b^(7/3)) - (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)
*d)/b^(1/3) + d*x])/(54*a^(1/3)*b^(8/3)) + (2*(-1)^(2/3)*d*Cosh[c - ((-1)^(
2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]
)/(27*a^(2/3)*b^(7/3)) + ((-1)^(1/3)*d^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^
(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(1/3)*b^(8
/3))
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
```

```
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5398

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Sinh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3}\,dx = -\frac{x^3\cosh(c+dx)}{6b\,(a+bx^3)^2} + \frac{\int \frac{x^2\cosh(c+dx)}{(a+bx^3)^2}\,dx}{2b} + \frac{d\int \frac{x^3\sinh(c+dx)}{(a+bx^3)^2}\,dx}{6b}$$

$$= -\frac{x^3\cosh(c+dx)}{6b\,(a+bx^3)^2} - \frac{\cosh(c+dx)}{6b^2\,(a+bx^3)} - \frac{dx\sinh(c+dx)}{18b^2\,(a+bx^3)} + \frac{d\int \frac{\sinh(c+dx)}{a+bx^3}\,dx}{18b^2} + \frac{d\int \frac{\sinh(}{a+}}{6}$$

$$= -\frac{x^3\cosh(c+dx)}{6b\,(a+bx^3)^2} - \frac{\cosh(c+dx)}{6b^2\,(a+bx^3)} - \frac{dx\sinh(c+dx)}{18b^2\,(a+bx^3)} + \frac{d\int \left(-\frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}\right.}{}$$

$$= -\frac{x^3\cosh(c+dx)}{6b\,(a+bx^3)^2} - \frac{\cosh(c+dx)}{6b^2\,(a+bx^3)} - \frac{dx\sinh(c+dx)}{18b^2\,(a+bx^3)} - \frac{d\int \frac{\sinh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{54a^{2/3}b^2} - \frac{d\int}{}$$

$$= -\frac{x^3\cosh(c+dx)}{6b\,(a+bx^3)^2} - \frac{\cosh(c+dx)}{6b^2\,(a+bx^3)} - \frac{dx\sinh(c+dx)}{18b^2\,(a+bx^3)} - \frac{\left(d\cosh\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\right)\int}{54a^{2/3}b^2}$$

$$= -\frac{x^3\cosh(c+dx)}{6b\,(a+bx^3)^2} - \frac{\cosh(c+dx)}{6b^2\,(a+bx^3)} - \frac{(-1)^{2/3}d^2\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}}{\sqrt[3]{}}\right.}{54\sqrt[3]{a}\,b^{8/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.45, size = 397, normalized size = 0.51

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (d*RootSum[a + b*#1^3 & , (-4*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + 4*C
oshIntegral[d*(x - #1)]*Sinh[c + d*#1] + 4*Cosh[c + d*#1]*SinhIntegral[d*(x
 - #1)] - 4*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*Cosh
Integral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 - d*
Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[
d*(x - #1)]*#1)/#1^2 & ] + d*RootSum[a + b*#1^3 & , (4*Cosh[c + d*#1]*CoshI
ntegral[d*(x - #1)] + 4*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + 4*Cosh[c
+ d*#1]*SinhIntegral[d*(x - #1)] + 4*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)
] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 + d*CoshIntegral[d*(x - #1
)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sin
h[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] - (6*b*(3*(a + 2*b*x^3)*C
osh[c + d*x] + d*x*(a + b*x^3)*Sinh[c + d*x]))/(a + b*x^3)^2)/(108*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.61, size = 2448, normalized size = 3.12

method	result	size
risch	Expression too large to display	2448

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cosh(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/108/d^3*c^5/b/a^2*sum((_R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2
)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b
*c^3))+1/108/d^3/a^2/b^3*sum((10*_R1^2*a*b*c^2*d^3-_R1^2*b^2*c^5-_R1*a^2*d^
6-10*_R1*a*b*c^3*d^3+2*_R1*b^2*c^6-4*a^2*c*d^6+5*a*b*c^4*d^3-b^2*c^7-10*_R1
^2*a*b*c*d^3-20*_R1^2*b^2*c^4+20*_R1*a*b*c^2*d^3+34*_R1*b^2*c^5+4*a^2*d^6+1
0*a*b*c^3*d^3-14*b^2*c^6-10*_R1*a*b*c*d^3-20*_R1*b^2*c^4-10*a*b*c^2*d^3+10*
b^2*c^5)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*
_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6*d^6*exp(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*
d^6*x^3+a^2*d^6)*x^3+1/36*d^7*exp(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*
d^6)*x^4+1/36*d^7*exp(-d*x-c)*a/b^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x-5
/108/d^3*c^4/b^2/a^2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3+4*_R1^2*b-2*_R1
*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_
R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+5/54/d^3*c^3/b^2/a^2*s
um((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4+8*_R1^2*b*c-10*_R1*b*c^
2-2*a*d^3+2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x
-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+5/54/d^3*c^2/
b^2/a^2*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^2*d^3-b*c^
5-12*_R1^2*b*c^2+18*_R1*b*c^3+6*a*c*d^3-6*b*c^4-12*_R1*b*c^2-2*a*d^3+2*b*c^
3)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b
*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/12*d^6*exp(-d*x-c)*a/b^2/(b^2*d^6*x^6+2*a*b*d
^6*x^3+a^2*d^6)-5/108/d^3*c/a^2/b^3*sum((4*_R1^2*a*b*c*d^3-_R1^2*b^2*c^4-2*
_R1*a*b*c^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3*d^3-b^2*c^6-2*_R1^2*a*b*d^3
-16*_R1^2*b^2*c^3+4*_R1*a*b*c*d^3+26*_R1*b^2*c^4+10*a*b*c^2*d^3-10*b^2*c^5-
2*_R1*a*b*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^2*c^4)/(_R1^2-2*_R1*c+c^2)*exp
(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)
)+1/108/d^3*c^5/b/a^2*sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c
^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3
-b*c^3))-5/108/d^3*c^4/b^2/a^2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1
^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*
x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/108/d^3/a^
2/b^3*sum((10*_R1^2*a*b*c^2*d^3-_R1^2*b^2*c^5-_R1*a^2*d^6-10*_R1*a*b*c^3*d^
3+2*_R1*b^2*c^6-4*a^2*c*d^6+5*a*b*c^4*d^3-b^2*c^7+10*_R1^2*a*b*c*d^3+20*_R1
^2*b^2*c^4-20*_R1*a*b*c^2*d^3-34*_R1*b^2*c^5-4*a^2*d^6-10*a*b*c^3*d^3+14*b^
2*c^6-10*_R1*a*b*c*d^3-20*_R1*b^2*c^4-10*a*b*c^2*d^3+10*b^2*c^5)/(_R1^2-2*_
R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2
```

```
+a*d^3-b*c^3))-1/36*d^7*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^
4-1/36*d^7*exp(d*x+c)*a/b^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x-1/6*d^6*e
xp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^3+5/54/d^3*c^3/b^2/a^2*su
m((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+10*_R1*b*c^2
+2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+
_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+5/54/d^3*c^2/b
^2/a^2*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^2*d^3-b*c^5
+12*_R1^2*b*c^2-18*_R1*b*c^3-6*a*c*d^3+6*b*c^4-12*_R1*b*c^2-2*a*d^3+2*b*c^3
)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*
c+3*_Z*b*c^2+a*d^3-b*c^3))-1/12*d^6*exp(d*x+c)*a/b^2/(b^2*d^6*x^6+2*a*b*d^6
*x^3+a^2*d^6)-5/108/d^3*c/a^2/b^3*sum((4*_R1^2*a*b*c*d^3-_R1^2*b^2*c^4-2*_R
1*a*b*c^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3*d^3-b^2*c^6+2*_R1^2*a*b*d^3+1
6*_R1^2*b^2*c^3-4*_R1*a*b*c*d^3-26*_R1*b^2*c^4-10*a*b*c^2*d^3+10*b^2*c^5-2*
_R1*a*b*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^2*c^4)/(_R1^2-2*_R1*c+c^2)*exp(_
R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*((b*d^4*x^5*e^(2*c) + 4*b*d^3*x^4*e^(2*c) + 20*b*d^2*x^3*e^(2*c) + 120*
b*d*x^2*e^(2*c) - 3*(3*a*d^3*e^(2*c) - 280*b*e^(2*c))*x)*e^(d*x) - (b*d^4*x
^5 - 4*b*d^3*x^4 + 20*b*d^2*x^3 - 120*b*d*x^2 + 3*(3*a*d^3 + 280*b)*x)*e^(-
d*x))/(b^4*d^5*x^9*e^c + 3*a*b^3*d^5*x^6*e^c + 3*a^2*b^2*d^5*x^3*e^c + a^3*
b*d^5*e^c) - 1/2*integrate(3*(60*a*b*d^2*x^2*e^c - 3*a^2*d^3*e^c + 4*(9*a*b
*d^3*e^c - 560*b^2*e^c)*x^3 + 280*a*b*e^c - 3*(a^2*d^4*e^c - 120*a*b*d*e^c)
*x)*e^(d*x)/(b^5*d^5*x^12 + 4*a*b^4*d^5*x^9 + 6*a^2*b^3*d^5*x^6 + 4*a^3*b^2
*d^5*x^3 + a^4*b*d^5), x) - 1/2*integrate(-3*(60*a*b*d^2*x^2 + 3*a^2*d^3 -
4*(9*a*b*d^3 + 560*b^2)*x^3 + 280*a*b - 3*(a^2*d^4 + 120*a*b*d)*x)*e^(-d*x)
/(b^5*d^5*x^12*e^c + 4*a*b^4*d^5*x^9*e^c + 6*a^2*b^3*d^5*x^6*e^c + 4*a^3*b^
2*d^5*x^3*e^c + a^4*b*d^5*e^c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2980 vs. $2(562) = 1124$.
time = 0.47, size = 2980, normalized size = 3.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/216*(((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2
*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b
^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 +
 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 -
(b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x
 + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(
1/3)*(sqrt(-3) + 1) + c) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 -
sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^
3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(-a*d^
3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^
2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*
b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) +
1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + ((a*d^3/b)^(2/3)*((b^2*
x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)
^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sin
h(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b
^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - s
qrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + ((-
a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3
+ a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 +
2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x
^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6
 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2)
)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*
(sqrt(-3) - 1) + c) - 2*((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh
(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) - 4*(-a*d^3/b)^(
1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 +
a^2)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3
)) - 2*((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2
*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*
a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^
2))*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + ((a*d^3/b)^(2/3)
*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d
*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^
2))*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt
(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 +
a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x - 1/2*
(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c
) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a
*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2
*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(-a*d^3/b)^(1/3)*((b^2*x^6 +
2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (
b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x
+ c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)
```

```
^(1/3)*(sqrt(-3) + 1) - c) - ((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 +
 sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x
^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(a*d^
3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^
2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*
b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
)*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - ((-a*d^3/b)^(2/3)*((b^2*x^
6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2
 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(
d*x + c)^2) - 4*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^
2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sq
rt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2
*((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6
+ 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) - 4*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*
x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2))*
Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) + 2*((a*d^3/b)^(2/3)
*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)
*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x
 + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^
(1/3))*sinh(-c + (a*d^3/b)^(1/3)) - 36*(2*a*b*x^3 + a^2)*cosh(d*x + c) - 12
*(a*b*d*x^4 + a^2*d*x)*sinh(d*x + c))/((a*b^4*x...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cosh(d*x+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
[Out] integrate(x^5*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\cosh(c+d\,x)}{{\left(b\,x^3+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*cosh(c + d*x))/(a + b*x^3)^3,x)
```

```
[Out] int((x^5*cosh(c + d*x))/(a + b*x^3)^3, x)
```

3.108 $\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3} \, dx$

Optimal. Leaf size=1105

$$\frac{\cosh(c+dx)}{9ab^2x} - \frac{x^2\cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2x(a+bx^3)} - \frac{(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{27a^{4/3}b^{5/3}} -$$

```
[Out] -1/27*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/3)+
1/54*d^2*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(7/
3)-1/27*(-1)^(2/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*
a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/3)-1/54*(-1)^(1/3)*d^2*Chi((-1)^(1/3)*a^(1/
3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(7/3)+1/27
*(-1)^(1/3)*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3
)*d/b^(1/3))/a^(4/3)/b^(5/3)+1/54*(-1)^(2/3)*d^2*Chi(-(-1)^(2/3)*a^(1/3)*d/
b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(7/3)+1/9*cosh(
d*x+c)/a/b^2/x-1/6*x^2*cosh(d*x+c)/b/(b*x^3+a)^2-1/9*cosh(d*x+c)/b^2/x/(b*x
^3+a)-1/27*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d
/b^(1/3)+d*x)/a/b^2-1/27*d*cosh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+
d*x)/a/b^2-1/27*d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/
3)*d/b^(1/3)+d*x)/a/b^2-1/27*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/
b^(1/3))/a/b^2-1/27*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^
(4/3)/b^(5/3)+1/54*d^2*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))
/a^(2/3)/b^(7/3)-1/27*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c+(-1)^(
1/3)*a^(1/3)*d/b^(1/3))/a/b^2-1/27*(-1)^(2/3)*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(
1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/3)-1/54*(-1)^(1
/3)*d^2*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/
b^(1/3))/a^(2/3)/b^(7/3)-1/27*d*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh
(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a/b^2+1/27*(-1)^(1/3)*Shi((-1)^(2/3)*a^(1/
3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/3)+1/54
*(-1)^(2/3)*d^2*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(
1/3)*d/b^(1/3))/a^(2/3)/b^(7/3)-1/18*d*sinh(d*x+c)/b^2/(b*x^3+a)
```

Rubi [A]

time = 1.35, antiderivative size = 1105, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5399, 5401, 3378, 3384, 3379, 3382, 5400, 5396, 5389}

Antiderivative was successfully verified.

```
[In] Int[(x^4*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] Cosh[c + d*x]/(9*a*b^2*x) - (x^2*Cosh[c + d*x])/(6*b*(a + b*x^3)^2) - Cosh[
c + d*x]/(9*b^2*x*(a + b*x^3)) - ((-1)^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d
```

```
)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(4/3)*
b^(5/3)) - ((-1)^(1/3)*d^2*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(2/3)*b^(7/3)) + ((-1)^(
1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^
(1/3)*d)/b^(1/3)) - d*x])/(27*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d^2*Cosh[c - (
(-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)) - d*x])/(54*a^(2/3)*b^(7/3)) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegr
al[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(4/3)*b^(5/3)) + (d^2*Cosh[c - (a^(1/3
)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(2/3)*b^(7/3))
 - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)]
)/(27*a*b^2) - (d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c
 + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a*b^2) - (d*CoshIntegral[-(((-1)^(2
/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(2
7*a*b^2) - (d*Sinh[c + d*x])/(18*b^2*(a + b*x^3)) + (d*Cosh[c + ((-1)^(1/3)
*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(2
7*a*b^2) + ((-1)^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(4/3)*b^(5/3)) + ((-1)^(1/3)
*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1
/3)*d)/b^(1/3) - d*x])/(54*a^(2/3)*b^(7/3)) - (d*Cosh[c - (a^(1/3)*d)/b^(1/
3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a*b^2) - (Sinh[c - (a^(1/3
)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(4/3)*b^(5/3))
 + (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*
x])/(54*a^(2/3)*b^(7/3)) - (d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Sinh
Integral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a*b^2) + ((-1)^(1/3)*Si
nh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/
b^(1/3) + d*x])/(27*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d^2*Sinh[c - ((-1)^(2/3)
*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(5
4*a^(2/3)*b^(7/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5396

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_
)], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cosh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^4\cosh(c+dx)}{(a+bx^3)^3}\,dx = -\frac{x^2\cosh(c+dx)}{6b(a+bx^3)^2} + \frac{\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx}{3b} + \frac{d\int \frac{x^2\sinh(c+dx)}{(a+bx^3)^2}\,dx}{6b}$$

$$= -\frac{x^2\cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2x(a+bx^3)} - \frac{d\sinh(c+dx)}{18b^2(a+bx^3)} - \frac{\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx}{9b^2} + \frac{d\int \frac{\sinh(c+d}{x(a+bx^3}}{9b^2}$$

$$= -\frac{x^2\cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2x(a+bx^3)} - \frac{d\sinh(c+dx)}{18b^2(a+bx^3)} - \frac{\int\left(\frac{\cosh(c+dx)}{ax^2} - \frac{bx\cosh(c+dx)}{a(a+bx^3)}\right)}{9b^2}$$

$$= -\frac{x^2\cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2x(a+bx^3)} - \frac{d\sinh(c+dx)}{18b^2(a+bx^3)} - \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{9ab^2} + \frac{\int \frac{x\cosh(c+d}{a+bx^3}}{9ab}$$

$$= \frac{\cosh(c+dx)}{9ab^2x} - \frac{x^2\cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2x(a+bx^3)} - \frac{d\sinh(c+dx)}{18b^2(a+bx^3)} + \frac{\int\left(-\frac{c}{3\sqrt[3]{a}\sqrt[3]{}}\right.}{}$$

$$= \frac{\cosh(c+dx)}{9ab^2x} - \frac{x^2\cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2x(a+bx^3)} - \frac{\sqrt[3]{-1}\,d^2\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right.}{54a^{2/3}b}$$

$$= \frac{\cosh(c+dx)}{9ab^2x} - \frac{x^2\cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2x(a+bx^3)} - \frac{\sqrt[3]{-1}\,d^2\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right.}{54a^{2/3}b}$$

$$= \frac{\cosh(c+dx)}{9ab^2x} - \frac{x^2\cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2x(a+bx^3)} - \frac{(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{27a^{4/3}b^5}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.36, size = 675, normalized size = 0.61

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (RootSum[a + b*#1^3 & , (a*d^2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - a*
d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - a*d^2*Cosh[c + d*#1]*SinhInte
```

```
gral[d*(x - #1)] + a*d^2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + 2*b*Cosh
[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - 2*b*CoshIntegral[d*(x - #1)]*Sinh[
c + d*#1]*#1 - 2*b*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + 2*b*Sinh[c
+ d*#1]*SinhIntegral[d*(x - #1)]*#1 + 2*b*d*Cosh[c + d*#1]*CoshIntegral[d*(
x - #1)]*#1^2 - 2*b*d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1^2 - 2*b*d*
Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2 + 2*b*d*Sinh[c + d*#1]*SinhInt
egral[d*(x - #1)]*#1^2)/#1^2 & ] - RootSum[a + b*#1^3 & , (-(a*d^2*Cosh[c +
 d*#1]*CoshIntegral[d*(x - #1)]) - a*d^2*CoshIntegral[d*(x - #1)]*Sinh[c +
d*#1] - a*d^2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - a*d^2*Sinh[c + d*#1
]*SinhIntegral[d*(x - #1)] - 2*b*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1
 - 2*b*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 - 2*b*Cosh[c + d*#1]*Sinh
Integral[d*(x - #1)]*#1 - 2*b*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 +
2*b*d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1^2 + 2*b*d*CoshIntegral[d*(
x - #1)]*Sinh[c + d*#1]*#1^2 + 2*b*d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)
]*#1^2 + 2*b*d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2)/#1^2 & ] + (6*
b*Cosh[d*x]*(b*x^2*(-a + 2*b*x^3)*Cosh[c] - a*d*(a + b*x^3)*Sinh[c]))/(a +
b*x^3)^2 + (6*b*(-(a*d*(a + b*x^3)*Cosh[c]) + b*x^2*(-a + 2*b*x^3)*Sinh[c])
*Sinh[d*x])/(a + b*x^3)^2)/(108*a*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.40, size = 1927, normalized size = 1.74

method	result	size
risch	Expression too large to display	1927

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cosh(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18/d^2*c^2/b^2/a^2*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4+
8*_R1^2*b*c-10*_R1*b*c^2-2*a*d^3+2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+
c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^
3-b*c^3))+1/36*d^7*exp(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^3+1/
18*d^6*exp(-d*x-c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^5-1/36*d^6*exp(-
d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^2+1/36*d^7*exp(-d*x-c)*a/b^2
/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)+1/108/d^2/a^2/b^3*sum((4*_R1^2*a*b*c*d
^3-_R1^2*b^2*c^4-2*_R1*a*b*c^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3*d^3-b^2*
c^6-2*_R1^2*a*b*d^3-16*_R1^2*b^2*c^3+4*_R1*a*b*c*d^3+26*_R1*b^2*c^4+10*a*b*
c^2*d^3-10*b^2*c^5-2*_R1*a*b*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^2*c^4)/(_R1
^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z
*b*c^2+a*d^3-b*c^3))-1/108/d^2*c^4/b/a^2*sum((_R1^2-2*_R1*c+c^2+6*_R1-6*c+1
0)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b
*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/27/d^2*c^3/b^2/a^2*sum((_R1^2*b*c-2*_R1*b*c^2
-a*d^3+b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)
*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*
c^3))-1/27/d^2*c/b^2/a^2*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c
```

```
^4+a*c^2*d^3-b*c^5-12*_R1^2*b*c^2+18*_R1*b*c^3+6*a*c*d^3-6*b*c^4-12*_R1*b*c
^2-2*a*d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootO
f(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/108/d^2*c^4/b/a^2*sum((_R1^2
-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R
1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/27/d^2*c^3/b^2/a^2*su
m((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*
b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2
*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18/d^2*c^2/b^2/a^2*sum((_R1^2*b*c^2-_R1*a*d
^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+10*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1
*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3
*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/27/d^2*c/b^2/a^2*sum((_R1^2*a*d^3-
_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^2*d^3-b*c^5+12*_R1^2*b*c^2-18*_R1*b
*c^3-6*a*c*d^3+6*b*c^4-12*_R1*b*c^2-2*a*d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*ex
p(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3
))-1/36*d^7*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^3+1/18*d^6*e
xp(d*x+c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^5-1/36*d^6*exp(d*x+c)/b/(
b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^2-1/36*d^7*exp(d*x+c)*a/b^2/(b^2*d^6*x
^6+2*a*b*d^6*x^3+a^2*d^6)+1/108/d^2/a^2/b^3*sum((4*_R1^2*a*b*c*d^3-_R1^2*b^
2*c^4-2*_R1*a*b*c^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3*d^3-b^2*c^6+2*_R1^2
*a*b*d^3+16*_R1^2*b^2*c^3-4*_R1*a*b*c*d^3-26*_R1*b^2*c^4-10*a*b*c^2*d^3+10*
b^2*c^5-2*_R1*a*b*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^2*c^4)/(_R1^2-2*_R1*c+
c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^
3-b*c^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*((d^3*x^4*e^(2*c) + 5*d^2*x^3*e^(2*c) + 30*d*x^2*e^(2*c) + 210*x*e^(2*c
))*e^(d*x) - (d^3*x^4 - 5*d^2*x^3 + 30*d*x^2 - 210*x)*e^(-d*x))/(b^3*d^4*x^
9*e^c + 3*a*b^2*d^4*x^6*e^c + 3*a^2*b*d^4*x^3*e^c + a^3*d^4*e^c) - 1/2*inte
grate(3*(15*a*d^2*x^2*e^c + (3*a*d^3*e^c - 560*b*e^c)*x^3 + 90*a*d*x*e^c +
70*a*e^c)*e^(d*x)/(b^4*d^4*x^12 + 4*a*b^3*d^4*x^9 + 6*a^2*b^2*d^4*x^6 + 4*a
^3*b*d^4*x^3 + a^4*d^4), x) + 1/2*integrate(-3*(15*a*d^2*x^2 - (3*a*d^3 + 5
60*b)*x^3 - 90*a*d*x + 70*a)*e^(-d*x)/(b^4*d^4*x^12*e^c + 4*a*b^3*d^4*x^9*e
^c + 6*a^2*b^2*d^4*x^6*e^c + 4*a^3*b*d^4*x^3*e^c + a^4*d^4*e^c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4691 vs. $2(805) = 1610$.
time = 0.53, size = 4691, normalized size = 4.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/216*((4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - 4*
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - 2*(a*d^3/b)^(
2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^
2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6
+ 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (a*d^3/b)^(1/3)*((a*b^2*d^3*x^6
+ 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a
^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqr
t(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)^2))*Ei(d*x
 - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) +
 1) + c) - (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 -
 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 + 2*(-a*d^3/
b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3
+ a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*
x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (-a*d^3/b)^(1/3)*((a*b^2*d^3
*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^
3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3
+ sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)^2))*E
i(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sq
rt(-3) + 1) - c) + (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x
+ c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - 2*
(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b
^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3
)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (a*d^3/b)^(1/3)*((a*b
^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*
d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^
3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)
^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*
(sqrt(-3) - 1) - c) - (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d
*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 +
 2*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2
*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqr
t(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (-a*d^3/b)^(1/3)*
((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a
^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3
 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x
 + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b
)^(1/3)*(sqrt(-3) - 1) + c) - 2*(2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d
^3)*cosh(d*x + c)^2 - 2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*
x + c)^2 - 2*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*cosh(d*x + c
)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*(
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^
```

```
6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3)
)*cosh(c + (-a*d^3/b)^(1/3)) + 2*(2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*
d^3)*cosh(d*x + c)^2 - 2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d
*x + c)^2 + 2*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*cosh(d*x + c
)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((
a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6
 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*c
osh(-c + (a*d^3/b)^(1/3)) + (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*
cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x +
c)^2 - 2*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^
6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b
- sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (a*d^3/b)^(1
/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 +
 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3
*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh
(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) + 1) + c) - (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^
3)*cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x
 + c)^2 + 2*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^
3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^
2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (-a*d^3/
b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*
x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*
b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))
*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*
(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (4*(a*b^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cosh(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^4*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \cosh(c + d\,x)}{{\left(b\,x^3 + a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*cosh(c + d*x))/(a + b*x^3)^3,x)
```

```
[Out] int((x^4*cosh(c + d*x))/(a + b*x^3)^3, x)
```

3.109 $\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3} \, dx$

Optimal. Leaf size=776

$$\frac{\cosh(c+dx)}{18ab^2x^2} - \frac{x\cosh(c+dx)}{6b\,(a+bx^3)^2} - \frac{\cosh(c+dx)}{18b^2x^2\,(a+bx^3)} - \frac{\sqrt[3]{-1}\,\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}} - \cdots$$

[Out] 1/27*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/54*d^2*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a/b^2-1/27*(-1)^(1/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/54*d^2*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a/b^2+1/27*(-1)^(2/3)*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/54*d^2*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a/b^2+1/18*cosh(d*x+c)/a/b^2/x^2-1/6*x*cosh(d*x+c)/b/(b*x^3+a)^2-1/18*cosh(d*x+c)/b^2/x^2/(b*x^3+a)+1/27*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/54*d^2*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a/b^2-1/27*(-1)^(1/3)*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/54*d^2*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a/b^2+1/27*(-1)^(2/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/54*d^2*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a/b^2+1/18*d*sinh(d*x+c)/a/b^2/x-1/18*d*sinh(d*x+c)/b^2/x/(b*x^3+a)

Rubi [A]
time = 2.02, antiderivative size = 776, normalized size of antiderivative = 1.00, number of steps used = 71, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {5399, 5387, 5401, 3378, 3384, 3379, 3382, 5389, 5400, 5398}

Antiderivative was successfully verified.

[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^3)^3,x]

[Out] Cosh[c + d*x]/(18*a*b^2*x^2) - (x*Cosh[c + d*x])/(6*b*(a + b*x^3)^2) - Cosh[c + d*x]/(18*b^2*x^2*(a + b*x^3)) - ((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a*b^2) + ((-1)^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(54*a*b^2) + (Cos

```
h[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(
5/3)*b^(4/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)
/b^(1/3) + d*x])/(54*a*b^2) + (d*Sinh[c + d*x])/(18*a*b^2*x) - (d*Sinh[c +
d*x])/(18*b^2*x*(a + b*x^3)) + ((-1)^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/
b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5/3)*b^
(4/3)) + (d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a*b^2) + (Sinh[c - (a^(1/3)*d)/b^(1/3)]
*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Sinh[
c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a*b^2
) + ((-1)^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)
^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Sinh[c - ((-1
)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d
*x])/(54*a*b^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5387

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Si
mp[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1))), x] + (-Dis
t[(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Cosh[c + d*x])/x^n, x],
x] - Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x]
, x], x]) /; FreeQ[{a, b, c, d}, x] && IntegerQ[p] && IGtQ[n, 0] && LtQ[p,
```

```
-1] && GtQ[n, 2]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5398

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Sinh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^3)^3}\,dx = -\frac{x\cosh(c+dx)}{6b\,(a+bx^3)^2}+\frac{\int \frac{\cosh(c+dx)}{(a+bx^3)^2}\,dx}{6b}+\frac{d\int \frac{x\sinh(c+dx)}{(a+bx^3)^2}\,dx}{6b}$$

$$= -\frac{x\cosh(c+dx)}{6b\,(a+bx^3)^2}-\frac{\cosh(c+dx)}{18b^2x^2\,(a+bx^3)}-\frac{d\sinh(c+dx)}{18b^2x\,(a+bx^3)}-\frac{\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx}{9b^2}+\frac{d^2\int \frac{\cos\cdots}{x\cdots}}{\cdots}$$

$$= -\frac{x\cosh(c+dx)}{6b\,(a+bx^3)^2}-\frac{\cosh(c+dx)}{18b^2x^2\,(a+bx^3)}-\frac{d\sinh(c+dx)}{18b^2x\,(a+bx^3)}-\frac{\int \left(\frac{\cosh(c+dx)}{ax^3}-\frac{b\cosh(c+dx)}{a(a+bx^3)}\cdots\right.}{9b^2}$$

$$= -\frac{x\cosh(c+dx)}{6b\,(a+bx^3)^2}-\frac{\cosh(c+dx)}{18b^2x^2\,(a+bx^3)}-\frac{d\sinh(c+dx)}{18b^2x\,(a+bx^3)}-\frac{\int \frac{\cosh(c+dx)}{x^3}\,dx}{9ab^2}+\frac{\int \frac{\cosh(c\cdots}{a+\cdots}}{9\cdots}$$

$$= \frac{\cosh(c+dx)}{18ab^2x^2}-\frac{x\cosh(c+dx)}{6b\,(a+bx^3)^2}-\frac{\cosh(c+dx)}{18b^2x^2\,(a+bx^3)}-\frac{d\sinh(c+dx)}{18b^2x\,(a+bx^3)}+\frac{\int \left(-\frac{\cdots}{3a^{2/\cdots}}\cdots\right.}{\cdots}$$

$$= \frac{\cosh(c+dx)}{18ab^2x^2}-\frac{x\cosh(c+dx)}{6b\,(a+bx^3)^2}-\frac{\cosh(c+dx)}{18b^2x^2\,(a+bx^3)}+\frac{d^2\cosh(c)\text{Chi}(dx)}{18ab^2}+\frac{d\sinh(\cdots}{18a\cdots}$$

$$= \frac{\cosh(c+dx)}{18ab^2x^2}-\frac{x\cosh(c+dx)}{6b\,(a+bx^3)^2}-\frac{\cosh(c+dx)}{18b^2x^2\,(a+bx^3)}+\frac{d^2\cosh(c)\text{Chi}(dx)}{18ab^2}+\frac{d\sinh(\cdots}{18a\cdots}$$

$$= \frac{\cosh(c+dx)}{18ab^2x^2}-\frac{x\cosh(c+dx)}{6b\,(a+bx^3)^2}-\frac{\cosh(c+dx)}{18b^2x^2\,(a+bx^3)}-\frac{\sqrt[3]{-1}\,\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d\cdots}{\sqrt[3]{b}}\right.}{27a^{5/3}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.47, size = 429, normalized size = 0.55

$$-\frac{\text{RootSum}\left[a+b\#1^3\&,\frac{-2\cosh(c+d\#1)\text{Chi}(d(x-\#1))+2\text{Chi}(d(x-\#1))\sinh(c+d\#1)+2\cosh(c+d\#1)\text{Shi}(d(x-\#1))-2\sinh(c+d\#1)\text{Shi}(d(x-\#1))+d^2\cosh(c+d\#1)\text{Chi}(d(x-\#1))\#1^2-d^2\text{Chi}(d(x-\#1))\sinh(c+d\#1)\#1^2-d^2\cosh(c+d\#1)\text{Shi}(d(x-\#1))\#1^2+d^2\sinh(c+d\#1)\text{Shi}(d(x-\#1))\#1^2}{\#1^2}\&\right]+\text{RootSum}\left[a+b\#1^3\&,\frac{-2\cosh(c+d\#1)\text{Chi}(d(x-\#1))-2\text{Chi}(d(x-\#1))\sinh(c+d\#1)-2\cosh(c+d\#1)\text{Shi}(d(x-\#1))-2\sinh(c+d\#1)\text{Shi}(d(x-\#1))+d^2\cosh(c+d\#1)\text{Chi}(d(x-\#1))\#1^2+d^2\text{Chi}(d(x-\#1))\sinh(c+d\#1)\#1^2+d^2\cosh(c+d\#1)\text{Shi}(d(x-\#1))\#1^2+d^2\sinh(c+d\#1)\text{Shi}(d(x-\#1))\#1^2}{\#1^2}\&\right]-\frac{6bx((-2a+bx^3)\cosh(c+dx)+dx(a+bx^3)\sinh(c+dx))}{(a+bx^3)^2}}{108ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -1/108*(RootSum[a + b*#1^3 & , (-2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]
+ 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + 2*Cosh[c + d*#1]*SinhIntegral
[d*(x - #1)] - 2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d^2*Cosh[c + d*#
1]*CoshIntegral[d*(x - #1)]*#1^2 - d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*
#1]*#1^2 - d^2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2 + d^2*Sinh[c +
```

```
d*#1]*SinhIntegral[d*(x - #1)]*#1^2)/#1^2 & ] + RootSum[a + b*#1^3 & , (-2*
Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - 2*CoshIntegral[d*(x - #1)]*Sinh[c
 + d*#1] - 2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - 2*Sinh[c + d*#1]*Sin
hIntegral[d*(x - #1)] + d^2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1^2 +
d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1^2 + d^2*Cosh[c + d*#1]*SinhI
ntegral[d*(x - #1)]*#1^2 + d^2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2
)/#1^2 & ] - (6*b*x*((-2*a + b*x^3)*Cosh[c + d*x] + d*x*(a + b*x^3)*Sinh[c
+ d*x]))/(a + b*x^3)^2)/(a*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.30, size = 1456, normalized size = 1.88

method	result	size
risch	Expression too large to display	1456

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*d^6*exp(-d*x-c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^4-1/18*d^6*exp
(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x-1/36*d^7*exp(-d*x-c)/a/(b^
2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^5-1/36*d^7*exp(-d*x-c)/b/(b^2*d^6*x^6+2*
a*b*d^6*x^3+a^2*d^6)*x^2-1/36/d*c^2/b^2/a^2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^
3+b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(
-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
+1/36/d*c/b^2/a^2*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4+8*_R
1^2*b*c-10*_R1*b*c^2-2*a*d^3+2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)
*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*
c^3))+1/108/d/b^2/a^2*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+
a*c^2*d^3-b*c^5-12*_R1^2*b*c^2+18*_R1*b*c^3+6*a*c*d^3-6*b*c^4-12*_R1*b*c^2-
2*a*d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_
Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/108/d*c^3/b/a^2*sum((_R1^2-2*_R
1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=Roo
tOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/36*d^6*exp(d*x+c)/a/(b^2*d
^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^4-1/18*d^6*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*
d^6*x^3+a^2*d^6)*x+1/108/d/b^2/a^2*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3
+2*_R1*b*c^4+a*c^2*d^3-b*c^5+12*_R1^2*b*c^2-18*_R1*b*c^3-6*a*c*d^3+6*b*c^4-
12*_R1*b*c^2-2*a*d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c)
,_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/36/d*c/b^2/a^2*sum
((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+10*_R1*b*c^2+
2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_
R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/36*d^7*exp(d*
x+c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^5+1/36*d^7*exp(d*x+c)/b/(b^2*d
^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^2+1/108/d*c^3/b/a^2*sum((_R1^2-2*_R1*c+c^2-
6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3
*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/36/d*c^2/b^2/a^2*sum((_R1^2*b*c-2*
```

```
_R1*b*c^2-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_
R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2
+a*d^3-b*c^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*((d^2*x^3*e^(2*c) + 6*d*x^2*e^(2*c) + 42*x*e^(2*c))*e^(d*x) - (d^2*x^3
- 6*d*x^2 + 42*x)*e^(-d*x))/(b^3*d^3*x^9*e^c + 3*a*b^2*d^3*x^6*e^c + 3*a^2*
b*d^3*x^3*e^c + a^3*d^3*e^c) + 1/2*integrate(-3*(3*a*d^2*x^2*e^c - 112*b*x^
3*e^c + 18*a*d*x*e^c + 14*a*e^c)*e^(d*x)/(b^4*d^3*x^12 + 4*a*b^3*d^3*x^9 +
6*a^2*b^2*d^3*x^6 + 4*a^3*b*d^3*x^3 + a^4*d^3), x) - 1/2*integrate(-3*(3*a*
d^2*x^2 - 112*b*x^3 - 18*a*d*x + 14*a)*e^(-d*x)/(b^4*d^3*x^12*e^c + 4*a*b^3
*d^3*x^9*e^c + 6*a^2*b^2*d^3*x^6*e^c + 4*a^3*b*d^3*x^3*e^c + a^4*d^3*e^c),
x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2962 vs. $2(582) = 1164$.
time = 0.50, size = 2962, normalized size = 3.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/108*(((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b
^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 + (a*d^3/b)^(1/3)*(
(b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*
cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*
b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3)
+ 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((a*b^2*d^3*x^6 + 2*a^
2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 +
 a^3*d^3)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*
b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 +
2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x +
c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(
1/3)*(sqrt(-3) + 1) - c) + ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cos
h(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2
+ (a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a
*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(
-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/
```

b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + ((a
*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6
+ 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b^3*x^6 +
2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x +
c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 +
a^2*b))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*co
sh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + ((a*b^2*d^3*x^6 + 2*a^2*b*d^3
*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^
3)*sinh(d*x + c)^2 + 2*(-a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*co
sh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2))*Ei(-d*x +
(-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + ((a*b^2*d^3*x^6 + 2*a^2*b*d
^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*
d^3)*sinh(d*x + c)^2 - 2*(a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*c
osh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2))*Ei(d*x +
(a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + ((a*b^2*d^3*x^6 + 2*a^2*b*d^
3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d
^3)*sinh(d*x + c)^2 + (a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqr
t(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2
*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*
Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt
(-3) + 1) + c) + ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)
^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - (-a*d^3/
b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3
+ a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*
x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3
)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - ((a*b^2*d
^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^
2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 + (a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^
2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 -
(b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))
*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a
d^3/b)^(1/3)(sqrt(-3) - 1) - c) - ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3
*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*
x + c)^2 - (-a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3
*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2
b - sqrt(-3)(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*Ei(-d*x +
1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) -
1) + c) - ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a
*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 + 2*(-a*d^3/b)^(1
/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x
^3 + a^2*b)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/
b)^(1/3)) - ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 -
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - 2*(a*d^3/b)^(
1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*
x^3 + a^2*b)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b

```
)^(1/3)) - 6*(a*b^2*d*x^4 - 2*a^2*b*d*x)*cosh(d*x + c) - 6*(a*b^2*d^2*x^5 +
 a^2*b*d^2*x^2)*sinh(d*x + c))/((a^2*b^4*d*x^6 + 2*a^3*b^3*d*x^3 + a^4*b^2*
d)*cosh(d*x + c)^2 - (a^2*b^4*d*x^6 + 2*a^3*b^3*d*x^3 + a^4*b^2*d)*sinh(d*x
 + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \cosh(c+d\,x)}{{\left(b\,x^3+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*cosh(c + d*x))/(a + b*x^3)^3,x)
```

```
[Out] int((x^3*cosh(c + d*x))/(a + b*x^3)^3, x)
```

3.110 $\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3} \, dx$

Optimal. Leaf size=781

$$-\frac{\cosh(c+dx)}{6b\left(a+bx^3\right)^2}+\frac{(-1)^{2/3}d^2\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{54a^{4/3}b^{5/3}}-\frac{\sqrt[3]{-1}\,d^2\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right.}{54a^{4/3}}$$

```
[Out] 1/54*d^2*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/
3)+1/54*(-1)^(2/3)*d^2*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1
/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/3)-1/54*(-1)^(1/3)*d^2*Chi(-(-1)^(2/3)*
a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/3)
-1/6*cosh(d*x+c)/b/(b*x^3+a)^2-1/27*(-1)^(1/3)*d*cosh(c+(-1)^(1/3)*a^(1/3)*
d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)+1/27*d*co
sh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)+1/27*(-1
)^(2/3)*d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(
1/3)+d*x)/a^(5/3)/b^(4/3)+1/27*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*
d/b^(1/3))/a^(5/3)/b^(4/3)+1/54*d^2*Shi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/
3)*d/b^(1/3))/a^(4/3)/b^(5/3)-1/27*(-1)^(1/3)*d*Chi((-1)^(1/3)*a^(1/3)*d/b^
(1/3)-d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+1/54*(-1)^(
2/3)*d^2*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d
/b^(1/3))/a^(4/3)/b^(5/3)+1/27*(-1)^(2/3)*d*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/
3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/54*(-1)^(1/3
)*d^2*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(
1/3))/a^(4/3)/b^(5/3)+1/18*d*sinh(d*x+c)/a/b^2/x^2-1/18*d*sinh(d*x+c)/b^2/x
^2/(b*x^3+a)
```

Rubi [A]
time = 1.03, antiderivative size = 781, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5397, 5386, 5400, 3378, 3384, 3379, 3382, 5388, 5401}

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -1/6*Cosh[c + d*x]/(b*(a + b*x^3)^2) + ((-1)^(2/3)*d^2*Cosh[c + ((-1)^(1/3)
*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(5
4*a^(4/3)*b^(5/3)) - ((-1)^(1/3)*d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(54*a^(4/3)*b^(5/
3)) + (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) +
 d*x])/(54*a^(4/3)*b^(5/3)) + (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Si
nh[c - (a^(1/3)*d)/b^(1/3)])/(27*a^(5/3)*b^(4/3)) - ((-1)^(1/3)*d*CoshInteg
```

```
ral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b
^(1/3)])/(27*a^(5/3)*b^(4/3)) + ((-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^
(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(5
/3)*b^(4/3)) + (d*Sinh[c + d*x])/(18*a*b^2*x^2) - (d*Sinh[c + d*x])/(18*b^2
*x^2*(a + b*x^3)) + ((-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*
SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5/3)*b^(4/3)) -
((-1)^(2/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)
^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(4/3)*b^(5/3)) + (d*Cosh[c - (a^(1/
3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)
) + (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d
*x])/(54*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b
^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(
4/3)) - ((-1)^(1/3)*d^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegr
al[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(4/3)*b^(5/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> Si
mp[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1))), x] + (-Dis
t[(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Sinh[c + d*x])/x^n, x],
x] - Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x]
```

```
, x], x]) /; FreeQ[{a, b, c, d}, x] && IntegerQ[p] && IGtQ[n, 0] && LtQ[p,
-1] && GtQ[n, 2]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3}\,dx = -\frac{\cosh(c+dx)}{6b\,(a+bx^3)^2} + \frac{d\int \frac{\sinh(c+dx)}{(a+bx^3)^2}\,dx}{6b}$$

$$= -\frac{\cosh(c+dx)}{6b\,(a+bx^3)^2} - \frac{d\sinh(c+dx)}{18b^2x^2\,(a+bx^3)} - \frac{d\int \frac{\sinh(c+dx)}{x^3(a+bx^3)}\,dx}{9b^2} + \frac{d^2\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx}{18b^2}$$

$$= -\frac{\cosh(c+dx)}{6b\,(a+bx^3)^2} - \frac{d\sinh(c+dx)}{18b^2x^2\,(a+bx^3)} - \frac{d\int \left(\frac{\sinh(c+dx)}{ax^3} - \frac{b\sinh(c+dx)}{a(a+bx^3)}\right)dx}{9b^2} + \cdots$$

$$= -\frac{\cosh(c+dx)}{6b\,(a+bx^3)^2} - \frac{d\sinh(c+dx)}{18b^2x^2\,(a+bx^3)} - \frac{d\int \frac{\sinh(c+dx)}{x^3}\,dx}{9ab^2} + \frac{d\int \frac{\sinh(c+dx)}{a+bx^3}\,dx}{9ab} + \cdots$$

$$= -\frac{d^2\cosh(c+dx)}{18ab^2x} - \frac{\cosh(c+dx)}{6b\,(a+bx^3)^2} + \frac{d\sinh(c+dx)}{18ab^2x^2} - \frac{d\sinh(c+dx)}{18b^2x^2\,(a+bx^3)} + \cdots$$

$$= -\frac{\cosh(c+dx)}{6b\,(a+bx^3)^2} + \frac{d\sinh(c+dx)}{18ab^2x^2} - \frac{d\sinh(c+dx)}{18b^2x^2\,(a+bx^3)} - \frac{d\int \frac{\sinh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{27a^{5/3}b} - \cdots$$

$$= -\frac{\cosh(c+dx)}{6b\,(a+bx^3)^2} + \frac{d^3\text{Chi}(dx)\sinh(c)}{18ab^2} + \frac{d\sinh(c+dx)}{18ab^2x^2} - \frac{d\sinh(c+dx)}{18b^2x^2\,(a+bx^3)} + \cdots$$

$$= -\frac{\cosh(c+dx)}{6b\,(a+bx^3)^2} + \frac{(-1)^{2/3}d^2\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}} - dx\right)}{54a^{4/3}b^{5/3}} - \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.33, size = 423, normalized size = 0.54

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -1/108*(d*RootSum[a + b*#1^3 & , (2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]
 - 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - 2*Cosh[c + d*#1]*SinhIntegra
l[d*(x - #1)] + 2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1
]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#
1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhInt
```

```
egral[d*(x - #1)]*#1)/#1^2 & ] + d*RootSum[a + b*#1^3 & , (-2*Cosh[c + d*#1
]*CoshIntegral[d*(x - #1)] - 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - 2*
Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - 2*Sinh[c + d*#1]*SinhIntegral[d*(
x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 + d*CoshIntegral[d*
(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1
+ d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] - (6*b*Cosh[d*x]*(
-3*a*Cosh[c] + d*x*(a + b*x^3)*Sinh[c]))/(a + b*x^3)^2 - (6*b*(d*x*(a + b*x
^3)*Cosh[c] - 3*a*Sinh[c])*Sinh[d*x])/(a + b*x^3)^2)/(a*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.22, size = 994, normalized size = 1.27

method	result	size
risch	Expression too large to display	994

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/36*d^7*exp(-d*x-c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^4-1/36*d^7*ex
p(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x-1/12*d^6*exp(-d*x-c)/b/(b
^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)-1/108/b^2/a^2*sum((_R1^2*b*c^2-_R1*a*d^3-
2*_R1*b*c^3-a*c*d^3+b*c^4+8*_R1^2*b*c-10*_R1*b*c^2-2*a*d^3+2*b*c^3+8*_R1*b*
c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-
3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/108*c^2/b/a^2*sum((_R1^2-2*_R1*c+c^2+
6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3
*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/54*c/b^2/a^2*sum((_R1^2*b*c-2*_R1*
b*c^2-a*d^3+b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c
+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d
^3-b*c^3))+1/36*d^7*exp(d*x+c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^4+1/
36*d^7*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x-1/12*d^6*exp(d*x+
c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)-1/108/b^2/a^2*sum((_R1^2*b*c^2-_R1
*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+10*_R1*b*c^2+2*a*d^3-2*b*c^3+8
*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(
_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/108*c^2/b/a^2*sum((_R1^2-2*_R1
*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=Root
Of(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/54*c/b^2/a^2*sum((_R1^2*b*c
-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-
2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*
c^2+a*d^3-b*c^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*((d*x^2*e^(2*c) + 7*x*e^(2*c))*e^(d*x) - (d*x^2 - 7*x)*e^(-d*x))/(b^3*d
^2*x^9*e^c + 3*a*b^2*d^2*x^6*e^c + 3*a^2*b*d^2*x^3*e^c + a^3*d^2*e^c) + 1/2
*integrate((56*b*x^3*e^c - 9*a*d*x*e^c - 7*a*e^c)*e^(d*x)/(b^4*d^2*x^12 + 4
*a*b^3*d^2*x^9 + 6*a^2*b^2*d^2*x^6 + 4*a^3*b*d^2*x^3 + a^4*d^2), x) + 1/2*i
ntegrate((56*b*x^3 + 9*a*d*x - 7*a)*e^(-d*x)/(b^4*d^2*x^12*e^c + 4*a*b^3*d^
2*x^9*e^c + 6*a^2*b^2*d^2*x^6*e^c + 4*a^3*b*d^2*x^3*e^c + a^4*d^2*e^c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2972 vs. $2(559) = 1118$.
time = 0.50, size = 2972, normalized size = 3.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/216*(36*a^2*cosh(d*x + c) + ((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2
 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b
*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(a*
d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 +
a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*
a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) +
1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((-a*d^3/b)^(2/3)*((b^2*
x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)
^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sin
h(d*x + c)^2) + 2*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(
b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 +
sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d
^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) +
 ((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x
^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6
 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b
*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x
^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^
2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(
sqrt(-3) - 1) - c) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-
3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^
2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(-a*d^3/b)^(
1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*co
sh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3
+ a^2))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*co
sh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*((-a*d^3/b)^(2/3)*((b^2*x^6
 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x
+ c)^2) + 2*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 -
```

```
 (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*
cosh(c + (-a*d^3/b)^(1/3)) - 2*((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2
)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) + 2*(a*d^3
/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x
^3 + a^2)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(
1/3)) + ((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 +
2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(
b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b^2*x^6
+ 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 -
 (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*
x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^
(1/3)*(sqrt(-3) + 1) + c) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 -
 sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x
^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(-a*d
^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a
^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a
*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) +
 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - ((a*d^3/b)^(2/3)*((b^2
*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c
)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*si
nh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(
b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 -
sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - ((
-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3
 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 +
 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*
x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^
6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2
))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)
*(sqrt(-3) - 1) + c) + 2*((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cos
h(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) + 2*(-a*d^3/b)^
(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 +
 a^2)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/
3)) + 2*((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^
2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b^2*x^6 + 2
*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)
^2))*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) - 12*(a*b*d*x^4 +
 a^2*d*x)*sinh(d*x + c))/((a^2*b^3*x^6 + 2*a^3*...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\cosh(c+d\,x)}{{\left(b\,x^3+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(c + d*x))/(a + b*x^3)^3,x)
```

```
[Out] int((x^2*cosh(c + d*x))/(a + b*x^3)^3, x)
```

3.111 $\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=1147

$$-\frac{\cosh(c+dx)}{18ab^2x^4}+\frac{2\cosh(c+dx)}{9a^2bx}-\frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2}+\frac{\cosh(c+dx)}{18b^2x^4\left(a+bx^3\right)}-\frac{2(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\right.}{27a^{7/3}b^{2/3}}$$

```
[Out] -2/27*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)-
1/54*d^2*Chi(a^(1/3)*d/b^(1/3)+d*x)*cosh(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/
3)-2/27*(-1)^(2/3)*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*
a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)+1/54*(-1)^(1/3)*d^2*Chi((-1)^(1/3)*a^(1/
3)*d/b^(1/3)-d*x)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+2/27
*(-1)^(1/3)*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3
)*d/b^(1/3))/a^(7/3)/b^(2/3)-1/54*(-1)^(2/3)*d^2*Chi(-(-1)^(2/3)*a^(1/3)*d/
b^(1/3)-d*x)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/18*cosh
(d*x+c)/a/b^2/x^4+2/9*cosh(d*x+c)/a^2/b/x-1/6*cosh(d*x+c)/b/x/(b*x^3+a)^2+1
/18*cosh(d*x+c)/b^2/x^4/(b*x^3+a)-2/27*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3
))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^2/b-2/27*d*cosh(c-a^(1/3)*d/b^(
1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^2/b-2/27*d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b
^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^2/b-2/27*d*Chi(a^(1/3)*d/b^
(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^2/b-2/27*Shi(a^(1/3)*d/b^(1/3)+d*x)*
sinh(c-a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)-1/54*d^2*Shi(a^(1/3)*d/b^(1/3)+d*
x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-2/27*d*Chi((-1)^(1/3)*a^(1/3)*
d/b^(1/3)-d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^2/b-2/27*(-1)^(2/3)*S
hi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/
a^(7/3)/b^(2/3)+1/54*(-1)^(1/3)*d^2*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*
sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-2/27*d*Chi(-(-1)^(2/3)
*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^2/b+2/27*(-1
)^(1/3)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b
^(1/3))/a^(7/3)/b^(2/3)-1/54*(-1)^(2/3)*d^2*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3
)+d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+1/18*d*sinh(d*x
+c)/a/b^2/x^3-1/18*d*sinh(d*x+c)/b^2/x^3/(b*x^3+a)
```

Rubi [A]
time = 2.39, antiderivative size = 1147, normalized size of antiderivative = 1.00, number of steps used = 89, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {5399, 5401, 3378, 3384, 3379, 3382, 5400, 5398, 5389}

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -1/18*Cosh[c + d*x]/(a*b^2*x^4) + (2*Cosh[c + d*x])/(9*a^2*b*x) - Cosh[c +
d*x]/(6*b*x*(a + b*x^3)^2) + Cosh[c + d*x]/(18*b^2*x^4*(a + b*x^3)) - (2*(-
1)^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*
a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(7/3)*b^(2/3)) + ((-1)^(1/3)*d^2*Cosh[c +
((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
 - d*x])/(54*a^(5/3)*b^(4/3)) + (2*(-1)^(1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*
d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(27*a^(7
/3)*b^(2/3)) - ((-1)^(2/3)*d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Cos
hIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(54*a^(5/3)*b^(4/3)) -
(2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(
27*a^(7/3)*b^(2/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1
/3)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*CoshIntegral[(a^(1/3)*d)
/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(27*a^2*b) - (2*d*CoshIntegr
al[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^
(1/3)])/(27*a^2*b) - (2*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) -
d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^2*b) + (d*Sinh[c + d*x
])/(18*a*b^2*x^3) - (d*Sinh[c + d*x])/(18*b^2*x^3*(a + b*x^3)) + (2*d*Cosh[
c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(
1/3) - d*x])/(27*a^2*b) + (2*(-1)^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(
1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(7/3)*b^(2/
3)) - ((-1)^(1/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral
[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*Cosh[c
- (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^2*b)
- (2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])
/(27*a^(7/3)*b^(2/3)) - (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^
(1/3)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*Cosh[c - ((-1)^(2/3)*a
^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*
a^2*b) + (2*(-1)^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegra
l[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(7/3)*b^(2/3)) - ((-1)^(2/3)
*d^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1
/3)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5398

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Sinh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5399

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5400

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
```

```
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx = -\frac{\cosh(c+dx)}{6bx\,(a+bx^3)^2} - \frac{\int \frac{\cosh(c+dx)}{x^2(a+bx^3)^2}\,dx}{6b} + \frac{d\int \frac{\sinh(c+dx)}{x(a+bx^3)^2}\,dx}{6b}$$

$$= -\frac{\cosh(c+dx)}{6bx\,(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4\,(a+bx^3)} - \frac{d\sinh(c+dx)}{18b^2x^3\,(a+bx^3)} + \frac{2\int \frac{\cosh(c+dx)}{x^5(a+bx^3)}\,dx}{9b^2} - \frac{d\int \frac{\sin}{x^4}}{}$$

$$= -\frac{\cosh(c+dx)}{6bx\,(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4\,(a+bx^3)} - \frac{d\sinh(c+dx)}{18b^2x^3\,(a+bx^3)} + \frac{2\int \left(\frac{\cosh(c+dx)}{ax^5} - \frac{b\cosh(c+}{a^2x^2}\right.}{9b^2}$$

$$= -\frac{\cosh(c+dx)}{6bx\,(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4\,(a+bx^3)} - \frac{d\sinh(c+dx)}{18b^2x^3\,(a+bx^3)} + \frac{2\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx}{9a^2} + \frac{2\int}{}$$

$$= -\frac{\cosh(c+dx)}{18ab^2x^4} - \frac{d^2\cosh(c+dx)}{36ab^2x^2} + \frac{2\cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx\,(a+bx^3)^2} + \frac{\cosh(c+}{18b^2x^4\,(a+}$$

$$= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{d^2\cosh(c+dx)}{108ab^2x^2} + \frac{2\cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx\,(a+bx^3)^2} + \frac{\cosh(c+}{18b^2x^4\,(a+}$$

$$= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{2\cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx\,(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4\,(a+bx^3)} + \frac{d\sinh(c+}{18ab^2x}$$

$$= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{2\cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx\,(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4\,(a+bx^3)} + \frac{d^4\cosh(c)}{36a}$$

$$= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{2\cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx\,(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4\,(a+bx^3)} - \frac{d^4\cosh(c)}{108a}$$

$$= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{2\cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx\,(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4\,(a+bx^3)} - \frac{2(-1)^{2/3}c}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

time = 0.35, size = 669, normalized size = 0.58

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (RootSum[a + b*#1^3 & , (-(a*d^2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]) +
 a*d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + a*d^2*Cosh[c + d*#1]*SinhI
ntegral[d*(x - #1)] - a*d^2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + 4*b*C
osh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - 4*b*CoshIntegral[d*(x - #1)]*Si
nh[c + d*#1]*#1 - 4*b*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + 4*b*Sinh
[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + 4*b*d*Cosh[c + d*#1]*CoshIntegral[
d*(x - #1)]*#1^2 - 4*b*d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1^2 - 4*b
*d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2 + 4*b*d*Sinh[c + d*#1]*Sinh
Integral[d*(x - #1)]*#1^2)/#1^2 & ] - RootSum[a + b*#1^3 & , (a*d^2*Cosh[c
+ d*#1]*CoshIntegral[d*(x - #1)] + a*d^2*CoshIntegral[d*(x - #1)]*Sinh[c +
d*#1] + a*d^2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + a*d^2*Sinh[c + d*#1
]*SinhIntegral[d*(x - #1)] - 4*b*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1
 - 4*b*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 - 4*b*Cosh[c + d*#1]*Sinh
Integral[d*(x - #1)]*#1 - 4*b*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 +
4*b*d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1^2 + 4*b*d*CoshIntegral[d*(
x - #1)]*Sinh[c + d*#1]*#1^2 + 4*b*d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)
]*#1^2 + 4*b*d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2)/#1^2 & ] + (6*
b*Cosh[d*x]*(b*x^2*(7*a + 4*b*x^3)*Cosh[c] + a*d*(a + b*x^3)*Sinh[c]))/(a +
 b*x^3)^2 + (6*b*(a*d*(a + b*x^3)*Cosh[c] + b*x^2*(7*a + 4*b*x^3)*Sinh[c])*
Sinh[d*x])/(a + b*x^3)^2)/(108*a^2*b^2)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.13, size = 810, normalized size = 0.71

method	result
risch	$\frac{d^{6} \mathrm{e}^{-d x -c} b \,x^{5}}{9 a^{2} \left(b^{2} x^{6} d^{6}+2 a b \,d^{6} x^{3}+a^{2} d^{6}\right)}-\frac{d^{7} \mathrm{e}^{-d x -c} x^{3}}{36 a \left(b^{2} x^{6} d^{6}+2 a b \,d^{6} x^{3}+a^{2} d^{6}\right)}+\frac{7 d^{6} \mathrm{e}^{-d x -c} x^{2}}{36 a \left(b^{2} x^{6} d^{6}+2 a b \,d^{6} x^{3}+a^{2} d^{6}\right)}-\frac{d^{7} \mathrm{e}^{-d x -c}}{36 b \left(b^{2} x^{6} d^{6}+2 a b \,d^{6} x^{3}+a^{2} d^{6}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*d^6*exp(-d*x-c)/a^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*b*x^5-1/36*d^7*
exp(-d*x-c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^3+7/36*d^6*exp(-d*x-c)/
a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^2-1/36*d^7*exp(-d*x-c)/b/(b^2*d^6*x
^6+2*a*b*d^6*x^3+a^2*d^6)-1/108*d/b^2/a^2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+
b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_
```

```
R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1
/108*d*c/b/a^2*sum((_R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp
(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)
)+1/9*d^6*exp(d*x+c)/a^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*b*x^5+1/36*d^7
*exp(d*x+c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^3+7/36*d^6*exp(d*x+c)/a
/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^2+1/36*d^7*exp(d*x+c)/b/(b^2*d^6*x^6
+2*a*b*d^6*x^3+a^2*d^6)-1/108*d/b^2/a^2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*
c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)
*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/1
08*d*c/b/a^2*sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_
R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b^3*d*x^9*e^c + 3*a*b^2*d*x^6*e^c + 3*a
^2*b*d*x^3*e^c + a^3*d*e^c) + 1/2*integrate((8*b*x^3*e^c - a*e^c)*e^(d*x)/(
b^4*d*x^12 + 4*a*b^3*d*x^9 + 6*a^2*b^2*d*x^6 + 4*a^3*b*d*x^3 + a^4*d), x) -
 1/2*integrate((8*b*x^3 - a)*e^(-d*x)/(b^4*d*x^12*e^c + 4*a*b^3*d*x^9*e^c +
 6*a^2*b^2*d*x^6*e^c + 4*a^3*b*d*x^3*e^c + a^4*d*e^c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4691 vs. $2(843) = 1686$.
time = 0.50, size = 4691, normalized size = 4.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/216*((8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - 8*
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - 4*(a*d^3/b)^(
2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^
2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6
+ 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((a*b^2*d^3*x^6
+ 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a
^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqr
t(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)^2))*Ei(d*x
 - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) +
 1) + c) - (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 -
 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 + 4*(-a*d^3/
```

b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3
+ a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*
x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*((a*b^2*d^3
*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^
3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3
+ sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)^2))*E
i(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sq
rt(-3) + 1) - c) + (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x
+ c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - 4*
(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b
^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3
)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((a*b
^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*
d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^
3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)
^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*
(sqrt(-3) - 1) - c) - (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d
x + c)^2 - 8(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 +
4*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2
*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqr
t(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*
((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a
^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3
+ a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x
+ c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b
)^(1/3)*(sqrt(-3) - 1) + c) - 2*(4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d
^3)*cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*
x + c)^2 - 4*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*cosh(d*x + c
)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2) + (-a*d^3/b)^(1/3)*(
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^
6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3)
)*cosh(c + (-a*d^3/b)^(1/3)) + 2*(4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*
d^3)*cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d
x + c)^2 + 4(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*cosh(d*x + c
)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2) + (a*d^3/b)^(1/3)*((
a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6
+ 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*c
osh(-c + (a*d^3/b)^(1/3)) + (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*
cosh(d*x + c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x +
c)^2 - 4*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^
6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b
- sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (a*d^3/b)^(1
/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 +
2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3
*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh

```
(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) + 1) + c) - (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^
3)*cosh(d*x + c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x
 + c)^2 + 4*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^
3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^
2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (-a*d^3/
b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*
x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*
b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))
*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*
(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (8*(a*b^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\cosh(c+d\,x)}{{\left(b\,x^3+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(c + d*x))/(a + b*x^3)^3,x)
```

```
[Out] int((x*cosh(c + d*x))/(a + b*x^3)^3, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
            # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```